

s STAR p STAR Continuous Maps in Topological Spaces

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Abstract:

Goal of this present study seeks to discover and to study the different kind of continuous map and irresolute map is known as semi star pre star continuous map (briefly s^*p^* continuous) and semi star pre star irresolute map (briefly s^*p^* irresolute) in topological spaces. As well as discuss a few fundamental characteristics of this continuous map. Further investigate the relationship between the newly defined map and the existing continuous map and irresolute map with suitable examples.

Keywords: Closed set, open set, s^*p^* closed set, s^*p^* open set, s^*p^* continuous map, s^*p^* irresolute map.

1. Introduction

Levine [1] introduced and he studied the weaker forms of continuity namely semi continuity in the year 1963. In the year 1991, H. Maki et.al., [2] explained a distinct class of mapping called generalized continuous mappings. In the year 1982, the concepts of beta- continuous mappings was established by S.N. El. Deeb et.al [3]. Mashhour, A.S. et. Al [4, 5] introduced pre-Continuous and weak pre-Continuous mappings in 1982 they also introduced α -continuous mapping in the year 1983. V.E. Sasikala and D. Sivaraj [6] studied semi open set in soft set in the year 2016. In 1993 H. Maki et.al., [7] introduced $g\alpha$ -continuous maps and αg - continuous maps. Caldas.M [8, 9] developed g -closed and g -continuous mappings in 1993 and semi generalized continuous maps in TS in the year 1995. N. Biswas [10] studied characterization on semi continuous functions. Y. Gnanambal [11] studied on gpr continuous functions in topological spaces. Navalagi G [12] obtained pre- rg -continuous functions in topology. In soft TS, soft swg separation axioms were proposed by V.E. Sasikala and D. Sivaraj [13,17]. V. E. Sasikala, D. Sivaraj, R. Thirumalaisamy and S.J. Venkatesan [20, 22] studied on Note on soft g -closed sets and also On Soft Regular Star Generalized Star-Closed sets in Soft Topological Spaces. S.G. Crossley et.al., [14] defined irresolute maps in the year 1972. Many researchers introduced strong and weaker forms of irresolute functions. D.Sivaraj and V.E. Sasikala [15] studied soft set and α -open sets. Govindappa Navalagi et.al., [16] introduced wgr -continuous and wgr -irresolute functions in TS in the year 2018. In the same year Govindappa Navalagi et.al., [18] introduced new mappings in topological spaces as Beta wg -continuous and Beta wg -irresolute functions. I.Arckiarani et.al., [19] introduced gp - continuous and gp - irresolute

function in the year 1999. sg-continuous and semi $T_{1/2}$ spaces introduced by Sundaram.P et.al.in the year1991.

2. Objectives

This research provides to understand and analyse the properties and features of s^*p^* continuous functions between topological spaces, and to examine the preservation of topological properties under s^*p^* continuous functions, such as s^*p^* connectedness and s^*p^* compactness. And to study the classification of topological spaces using s^*p^* continuous functions, including s^*p^* homeomorphisms. To learn about the fundamental concepts and theorems related to s^*p^* continuous functions in topology, including the definition, properties, and applications of s^*p^* continuous functions, and to develop problem-solving skills in using these concepts.

3. Preliminaries

Definition 3.1 [21]: Let (X, τ) be the TS. If A is subset of U and U is pre semi open, then A is a semi-star pre-star closed set (s^*p^* -C set) if $scl(A) \subseteq U$.

Definition 3.2: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is termed to be Continuous if the inverse image of G is closed in (X, τ) for each closed set G in (Y, σ) .

4. Methods and Discussion

s^*p^* Continuous Map

During the following segment, we launch new concepts about s^*p^* Continuous functions in topological spaces and characterized its simple features related to it.

Definition 4.1: If the inverse image of any closed set in X_2 exists in the s^*p^* -C set in X_1 , then the function $f: X_1 \rightarrow X_2$ is expressed as s^*p^* Continuous.

Theorem 4.2: The function $f: X_1 \rightarrow X_2$ as termed as Continuous subsequently it is s^*p^* Continuous.

Proof: G should be a closed set and (X_2, τ_2) be the topological space. Then the closed set G 's inverse image is in (X_1, τ_1) . Because f is continuous, any closed set is known to be s^*p^* -C set. Thus, inverse image of G is s^*p^* -C set. Hence, f is s^*p^* Continuous.

Below example verify that converse of the upon statement is not valid.

Example 4.3: Consider, $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{f_1\}, \{h_1\}, \{f_1, h_1\}\}$. $\tau_1^c = \{X_1, \phi, \{g_1, h_1\}, \{f_1, g_1\}, \{g_1\}\}$. s^*p^* -C sets are $\{X_1, \phi, \{f_1\}, h_1\}$. Take, $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \phi, \{g_2, h_2\}\}$. $\tau_2^c = \{X_2, \phi, \{f_2\}\}$. construct a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = g_2$, $f(g_1) = h_2$, $f(h_1) = f_2$. So, $f^{-1}(f_2) = \{h_1\}$ is s^*p^* -C set (X_1, τ_1) , but this is not contained in closed set (X_1, τ_1) . Hence, f is s^*p^* Continuous, even so, it not exists in continuous.

Theorem 4.4: Suppose $f: X_1 \rightarrow X_2$, which is g- Continuous so that, it is s^*p^* Continuous.

Proof: Let us suppose that the topological space be (X_2, τ_2) and the closed set G . further, the inverse image of G is g-Closed set in (X_1, τ_1) . Whereas, it is continuous, obviously any g-Closed set is s^*p^* -C set. So, inverse image of G is s^*p^* -C set. So, f is s^*p^* Continuous.

Following example proves that converse of above- mentioned statement is incorrect.

Example 4.5: Take, $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{f_1\}, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \phi, \{g_1, h_1\}, \{h_1\}\}$. s^*p^* - C sets are $\{X_1, \phi, \{g_1\}\}$. g -Closed sets are $\{X_1, \phi, \{h_1\}, \{g_1, h_1\}, \{f_1, h_1\}\}$. Let $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \phi, \{f_2, h_2\}\}$. $\tau_2^c = \{X_2, \phi, \{g_2\}\}$. Bulit a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = h_2$, $f(g_1) = g_2$, $f(h_1) = f_2$. Then $f^{-1}(g_2) = \{g_1\}$ is s^*p^* - C set although it does not contain in g -closed set in (X_1, τ_1) . so, f is s^*p^* Continuous except g -continuous.

Theorem 4.6: A function $f: X_1 \rightarrow X_2$ as termed a $g\alpha$ -continuous further, it is s^*p^* Continuous.

Proof: Consider a topological space be (X_2, τ_2) and the closed set G in (X_2, τ_2) . Then the inverse image of G is $g\alpha$ -closed in (X_1, τ_1) . Given that f is $g\alpha$ -continuous. Clearly, any $g\alpha$ -Closed set is s^*p^* - C set. So, inverse image of G exists in s^*p^* - C set. Therefore, we conclude that it is s^*p^* Continuous.

Opposite side of the above statement is incorrect compared to the example given below.

Example 4.7: Let $X_1 = \{e_1, f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{e_1\}, \{f_1\}, \{e_1, f_1\}, \{f_1, g_1\}, \{e_1, f_1, g_1\}\}$. $\tau_1^c = \{X_1, \phi, \{f_1, g_1, h_1\}, \{e_1, g_1, h_1\}, \{g_1, h_1\}, \{e_1, h_1\}, \{h_1\}\}$. s^*p^* - C sets are $\{X_1, \phi, \{e_1\}, \{f_1\}, \{g_1\}, \{f_1, g_1\}, \{e_1, g_1\}\}$. $g\alpha$ -closed sets are $\{X_1, \phi, \{g_1\}\}$. Let $X_2 = \{e_2, f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \phi, \{e_2, f_2\}\}$. $\tau_2^c = \{X_2, \phi, \{g_2, h_2\}\}$. Construct a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(e_1) = g_2$, $f(f_1) = f_2$, $f(g_1) = h_2$, $f(h_1) = e_2$. Then $f^{-1}(g_2, h_2) = \{e_1, g_1\}$ is s^*p^* - C set but this set is never in $g\alpha$ -closed set in (X_1, τ_1) . Hence, f is never contained in $g\alpha$ -continuous but it exists in s^*p^* Continuous.

Theorem 4.8: The function $f: X_1 \rightarrow X_2$ is g^* Continuous following that it is s^*p^* Continuous.

Proof: Assuming the topological space (X_2, τ_2) and closed set be G . After that the inverse image of G is g^* Closed in (X_1, τ_1) . Because f is g^* Continuous. Clearly, each g^* Closed set is s^*p^* -C set. So, $f^{-1}(G)$ is s^*p^* -C set. Therefore, it is s^*p^* Continuous.

An upcoming example explained that the reverse of the mentioned above statement is invalid.

Example 4.9: Assume that $X_1 = \{e_1, f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{f_1\}, \{h_1\}, \{f_1, h_1\}, \{f_1, g_1, h_1\}\}$. $\tau_1^c = \{X_1, \phi, \{e_1, g_1, h_1\}, \{e_1, f_1, g_1\}, \{e_1, g_1\}, \{e_1\}\}$. s^*p^* - C sets are $\{X_1, \phi, \{f_1\}, \{g_1\}, \{h_1\}, \{f_1, g_1\}, \{g_1, h_1\}\}$. g^* -closed sets are $\{X_1, \phi, \{e_1, g_1, h_1\}, \{e_1, f_1, h_1\}, \{e_1, f_1, g_1\}, \{e_1, h_1\}, \{e_1, f_1\}, \{e_1\}\}$. Put $X_2 = \{e_2, f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \phi, \{e_2, f_2, g_2\}\}$. $\tau_2^c = \{X_2, \phi, \{h_2\}\}$. Assume a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ then take $f(e_1) = e_2$, $f(f_1) = g_2$, $f(g_1) = h_2$, $f(h_1) = f_2$. Then $f^{-1}(h_2) = \{g_1\}$ is s^*p^* - C set but not exists in g^* Closed set in (X_1, τ_1) . Thus, it is s^*p^* Continuous although it is never existing in g^* -Continuous.

Theorem 4.10: Suppose $f: X_1 \rightarrow X_2$ is g^{**} Continuous so that it will be s^*p^* Continuous.

Proof: Suppose that the closed set in (X_2, τ_2) be G . So, G 's inverse image is in g^{**} - closed in (X_1, τ_1) because f is g^{**} - continuous. We know that any g^{**} Closed set is s^*p^* - C set. Thus, inverse image of G is s^*p^* -C set. Therefore, f is s^*p^* Continuous.

Next illustration implies that the other side of the previous argument is incorrect.

Example 4.11: Assume $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{h_1\}, \{f_1, h_1\}\}$. $\tau_1^c = \{X_1, \phi, \{f_1, g_1\}, \{g_1\}\}$. s^*p^* -C sets are $\{X_1, \phi, \{f_1\}\}$. g^{**} Closed sets are $\{X_1, \phi, \{g_1, h_1\}, \{f_1, g_1\}, \{g_1\}\}$. Put X_2

$= \{f_1, g_1, h_1\}$, $\tau_2 = \{X_2, \varphi, \{g_2, h_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{f_2\}\}$. Construct a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = f_2$, $f(g_1) = h_2$, $f(h_1) = g_2$. Then $f^{-1}(f_2) = \{f_1\}$ is s^*p^* -C set but this is never existing in g^{**} -closed set in (X_1, τ_1) . Thus, it is s^*p^* Continuous although it not exists in g^{**} -continuous.

Theorem 4.12: Each s^*p^* Continuous map is gpr - continuous.

Proof: $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a s^*p^* continuous map. Let the closed set in (X_2, τ_2) be G . If f being s^*p^* Continuous, whereas, G 's inverse image is in s^*p^* - C set in (X_1, τ_1) . Because any s^*p^* - C set is gpr-closed set. So, that we conclude that $f^{-1}(G)$ is gpr-Closed set in (X_1, τ_1) . So, f is gpr- Continuous.

Below illustration implies that the other side of the upon statement is not true.

Example 4.13: Put $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \varphi, \{f_1, h_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{g_1\}\}$. s^*p^* -C sets are $\{X_1, \varphi, \{g_1, h_1\}, \{f_1, g_1\}, \{g_1\}\}$. gpr closed sets are $\{X_1, \varphi, \{f_1\}, \{h_1\}\}$. Let $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \varphi, \{f_2, g_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{h_2\}\}$. Defining a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = h_2$, $f(g_1) = g_2$, $f(h_1) = f_2$. Further, $f^{-1}(h_2) = \{f_1\}$ is in gpr closed set but this does not contain in s^*p^* -C set in (X_1, τ_1) . So, f is gpr-continuous however not yet in s^*p^* Continuous.

Theorem 4.14: Each s^*p^* Continuous map is rg continuous.

Proof: Consider $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a s^*p^* Continuous map and G will be closed set in (X_2, τ_2) . Whenever f is s^*p^* Continuous, which says that G 's inverse image is s^*p^* -C set in (X_1, τ_1) . But each s^*p^* -C set is rg closed set. We conclude that G 's inverse image is rg-Closed set in (X_1, τ_1) . Therefore, f is rg-Continuous.

Reverse of the upon argument is not valid from the below example.

Example 4.15: Let us take, $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \varphi, \{f_1\}, \{g_1\}, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{g_1, h_1\}, \{f_1, h_1\}, \{h_1\}\}$. s^*p^* -C sets are $\{X_1, \varphi, \{f_1\}, \{g_1\}\}$. rg-closed sets are $\{X_1, \varphi, \{f_1, h_1\}\}$. Take, $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \varphi, \{h_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{f_2, g_2\}\}$. Defining a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = f_2$, $f(g_1) = h_2$, $f(h_1) = g_2$. Then $f^{-1}(f_2, g_2) = \{f_1, h_1\}$ is rg closed set whereas it is not at all s^*p^* -C set in (X_1, τ_1) . So, f is rg- Continuous however it should not be s^*p^* Continuous.

Theorem 4.16: Each s^*p^* Continuous map is αg -continuous.

Proof: Assume $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a s^*p^* Continuous map. Take G should be a closed set in (X_2, τ_2) . Hence it is s^*p^* Continuous, so inverse image of G is s^*p^* - C set in (X_1, τ_1) . Thus, any s^*p^* - C set is αg -Closed set. Therefore, $f^{-1}(G)$ is αg -Closed set in (X_1, τ_1) . subsequently, f is αg -Continuous.

Below mentioned illustration proves that the reverse on top of the statement is incorrect.

Example 4.17: Suppose $X_1 = \{e_1, f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \varphi, \{e_1\}, \{f_1\}, \{e_1, f_1\}, \{e_1, f_1, g_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{f_1, g_1, h_1\}, \{e_1, g_1, h_1\}, \{g_1, h_1\}, \{h_1\}\}$. s^*p^* - C sets are $\{X_1, \varphi, \{e_1\}, \{f_1\}, \{g_1\}, \{f_1, g_1\}, \{e_1, g_1\}\}$. αg -closed sets are $\{X_1, \varphi, \{f_1, g_1, h_1\}, \{e_1, g_1, h_1\}, \{e_1, f_1, h_1\}, \{g_1, h_1\}, \{f_1, h_1\}, \{e_1, h_1\}, \{h_1\}\}$. Consider $X_2 = \{e_2, f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \varphi, \{e_2, f_2, g_2\}, \{e_2, f_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{h_2\}, \{g_2, h_2\}\}$. Consider a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(e_1) = f_2$, $f(f_1) = g_2$, $f(g_1) = e_2$, $f(h_1) = h_2$. Then $f^{-1}(g_2, h_2) = \{f_1, h_1\}$ is αg - closed set whereas not s^*p^* - C set in (X_1, τ_1) . Thus, it is not s^*p^* Continuous but it is in αg -continuous.

Theorem 4.18: Each s^*p^* Continuous map is gp continuous.

Proof: consider $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a s^*p^* Continuous map. Let the closed set in (X_2, τ_2) is G . So, f is s^*p^* Continuous, so that G 's inverse image is s^*p^* - C set in (X_1, τ_1) . Because any s^*p^* - C set is gp-Closed set. Therefore, $f^{-1}(G)$ is gp-Closed set in (X_1, τ_1) . Thus, f is gp-Continuous.

Our next example shows another side of the above theorem could not be true.

Example 4.19: Assume $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{f_1\}, \{g_1\}, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \phi, \{g_1, h_1\}, \{f_1, h_1\}, \{h_1\}\}$. s^*p^* - C sets are $\{X_1, \phi, \{f_1\}, \{g_1\}\}$. gp closed sets are $\{X_1, \phi, \{g_1, h_1\}, \{f_1, g_1\}, \{h_1\}\}$. Assume $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \phi, \{f_2\}\}$. $\tau_2^c = \{X_2, \phi, \{g_2, h_2\}\}$. Define a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = g_2$, $f(g_1) = h_2$, $f(h_1) = f_2$. Then $f^{-1}(g_2, h_2) = \{f_1, g_1\}$ is gp closed set even though, does not exist in s^*p^* - C set in (X_1, τ_1) . This implies that f is gp continuous except in s^*p^* Continuous.

Remark 4.20:

An upcoming illustration proves that s^*p^* Continuous map is independent from α -continuous.

Example 4.21: Take $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{f_1\}, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \phi, \{g_1, h_1\}, \{h_1\}\}$. s^*p^* - C sets are $\{X_1, \phi, \{g_1\}\}$. α - Closed sets are $\{X_1, \phi, \{g_1\}, \{h_1\}, \{g_1, h_1\}\}$. Take, $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \phi, \{f_2\}, \{f_2, h_2\}\}$. $\tau_2^c = \{X_2, \phi, \{g_2, h_2\}, \{g_2\}\}$. Create a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = f_2$, $f(g_1) = h_2$, $f(h_1) = g_2$. Then $f^{-1}(f_2, g_2) = \{g_1, h_1\}$ is α - closed set even not at all in s^*p^* -C set in (X_1, τ_1) . Thus, f is α - continuous whereas it is not existing in s^*p^* Continuous.

Example 4.22: Assume $X_1 = \{e_1, f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{e_1\}, \{f_1\}, \{e_1, f_1\}, \{e_1, f_1, g_1\}\}$. $\tau_1^c = \{X_1, \phi, \{f_1, g_1, h_1\}, \{e_1, g_1, h_1\}, \{g_1, h_1\}, \{h_1\}\}$. s^*p^* - C sets are $\{X_1, \phi, \{e_1\}, \{f_1\}, \{h_1\}, \{f_1, h_1\}, \{e_1, h_1\}\}$. α -closed sets are $\{X_1, \phi, \{g_1\}, \{h_1\}, \{g_1, h_1\}, \{f_1, g_1, h_1\}, \{e_1, g_1, h_1\}\}$. Suppose $X_2 = \{e_2, f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \phi, \{e_2, g_2, h_2\}\}$. $\tau_2^c = \{X_2, \phi, \{f_2\}\}$. Define a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(e_1) = f_2$, $f(f_1) = e_2$, $f(g_1) = h_2$, $f(h_1) = g_2$. Then $f^{-1}(f_2) = \{e_1\}$ is s^*p^* -C set except α -closed set in (X_1, τ_1) . Because, f is s^*p^* Continuous whereas not yet in α -continuous.

Remark 4.23:

The upcoming example proves that s^*p^* continuous function is distinct from semi pre-continuous function.

Example 4.24: Put $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{f_1\}, \{h_1\}, \{f_1, h_1\}\}$. $\tau_1^c = \{X_1, \phi, \{g_1, h_1\}, \{f_1, g_1\}, \{g_1\}\}$. s^*p^* - C sets are $\{X_1, \phi, \{f_1\}, \{h_1\}\}$. Semi pre-closed sets are $\{X_1, \phi, \{f_1\}, \{g_1\}, \{h_1\}, \{f_1, g_1\}, \{g_1, h_1\}\}$. Take, $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \phi, \{f_2\}, \{g_2, h_2\}\}$. $\tau_2^c = \{X_2, \phi, \{g_2, h_2\}, \{f_2\}\}$. Construct a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = h_2$, $f(g_1) = g_2$, $f(h_1) = f_2$. Then $f^{-1}(g_2, h_2) = \{f_1, g_1\}$ is semi pre- closed set although it does not in s^*p^* -C set in (X_1, τ_1) . So, f is semi pre-continuous although never existing in s^*p^* Continuous.

Example 4.25: Consider $X_1 = \{e_1, f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \phi, \{e_1\}, \{f_1\}, \{e_1, f_1\}, \{f_1, g_1\}, \{e_1, f_1, g_1\}\}$. $\tau_1^c = \{X_1, \phi, \{f_1, g_1, h_1\}, \{e_1, g_1, h_1\}, \{g_1, h_1\}, \{e_1, h_1\}, \{h_1\}\}$. s^*p^* - C sets are $\{X_1, \phi, \{e_1\}, \{f_1\}, \{g_1\}, \{f_1, g_1\}, \{e_1, g_1\}\}$. Semi pre-closed sets are $\{X_1, \phi, \{e_1\}, \{g_1\}, \{h_1\}, \{e_1, g_1\}, \{e_1, h_1\}, \{f_1, g_1\}, \{f_1, h_1\}, \{f_1, g_1, h_1\}, \{e_1, g_1, h_1\}\}$. Let $X_2 = \{e_2, f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \phi, \{e_2, g_2, h_2\}\}$. $\tau_2^c = \{X_2,$

$\varphi, \{g_2\}$. Let us a function be $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(e_1) = f_2, f(f_1) = g_2, f(g_1) = h_2, f(h_1) = e_2$. Then $f^{-1}(g_2) = \{f_1\}$ is s^*p^* -C set but this is not contained in semi pre- closed set in (X_1, τ_1) . We conclude that, f is s^*p^* Continuous however it does not contain in semi pre- continuous.

Remark 4.26:

An upcoming example proves that s^*p^* continuous function is independent from pre-continuous function.

Example 4.27: Let $X_1 = \{f_1, g_1, h_1\}, \tau_1 = \{X_1, \varphi, \{f_1\}, \{g_1\}, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{g_1, h_1\}, \{f_1, h_1\}, \{h_1\}\}$. s^*p^* -C sets are $\{X_1, \varphi, \{f_1\}, \{g_1\}\}$. pre-closed sets are $\{X_1, \varphi, \{g_1, h_1\}, \{f_1, h_1\}, \{h_1\}\}$. Assume $X_2 = \{f_2, g_2, h_2\}, \tau_2 = \{X_2, \varphi, \{f_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{g_2, h_2\}\}$. Let a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = g_2, f(g_1) = f_2, f(h_1) = h_2$. Then $f^{-1}(g_2, h_2) = \{f_1, h_1\}$ is pre- closed set but it does not in s^*p^* -C set in (X_1, τ_1) . Therefore, f is pre- continuous, whereas this is not s^*p^* continuous.

Example 4.28: Take $X_1 = \{f_1, g_1, h_1\}, \tau_1 = \{X_1, \varphi, \{f_1\}, \{g_1\}, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{g_1, h_1\}, \{f_1, h_1\}, \{h_1\}\}$. s^*p^* -C sets are $\{X_1, \varphi, \{f_1\}, \{g_1\}\}$. pre-closed sets are $\{X_1, \varphi, \{g_1, h_1\}, \{f_1, h_1\}, \{h_1\}\}$. Let $X_2 = \{f_2, g_2, h_2\}, \tau_2 = \{X_2, \varphi, \{f_2, h_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{g_2\}\}$. Consider a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = g_2, f(g_1) = h_2, f(h_1) = f_2$. Then $f^{-1}(g_2) = \{f_1\}$ is s^*p^* -C set except pre- closed set in (X_1, τ_1) . Hence, which does not pre- continuous even though it is in s^*p^* Continuous.

Remark 4.29:

Following illustration explains that the s^*p^* Continuous and semi continuous is independent each other.

Example 4.30: Let $X_1 = \{f_1, g_1, h_1\}, \tau_1 = \{X_1, \varphi, \{g_1\}, \{f_1, h_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{f_1, h_1\}, \{g_1\}\}$. s^*p^* -C sets are $\{X_1, \varphi, \{f_1\}, \{h_1\}, \{f_1, h_1\}\}$. Semi closed sets are $\{X_1, \varphi, \{g_1\}, \{f_1, h_1\}\}$. Consider $X_2 = \{f_2, g_2, h_2\}, \tau_2 = \{X_2, \varphi, \{f_2\}, \{g_2, h_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{g_2, h_2\}, \{f_2\}\}$. Bulit a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = g_2, f(g_1) = h_2, f(h_1) = f_2$. Then $f^{-1}(f_2) = \{h_1\}$ is s^*p^* -C set but which does not contain in semi-closed set in (X_1, τ_1) . So, f should not be contained in semi-continuous but it is in s^*p^* Continuous.

Example 4.31: Let $X_1 = \{f_1, g_1, h_1\}, \tau_1 = \{X_1, \varphi, \{g_1\}, \{f_1, h_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{f_1, h_1\}, \{g_1\}\}$. s^*p^* -C sets are $\{X_1, \varphi, \{f_1\}, \{h_1\}, \{f_1, h_1\}\}$. Semi closed sets are $\{X_1, \varphi, \{g_1\}, \{f_1, h_1\}\}$. Consider $X_2 = \{f_2, g_2, h_2\}, \tau_2 = \{X_2, \varphi, \{f_2\}, \{g_2, h_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{g_2, h_2\}, \{f_2\}\}$. Bulit a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = h_2, f(g_1) = f_2, f(h_1) = g_2$. Then $f^{-1}(f_2) = \{g_1\}$ is semi-closed set because this is not in s^*p^* -C set in (X_1, τ_1) . Therefore, f does not contain in s^*p^* continuous but it is in semi continuous.

5. s^*p^* Irresolute Map

During the following portion, we establish the distinct topological spaces namely, s^*p^* Irresolute maps and characterized a few of them features.

Definition 5.1: The function $f: X_1 \rightarrow X_2$ is defined as s^*p^* Irresolute map since the inverse image of any s^*p^* -C set in X_2 exists in s^*p^* -C set in X_1 .

Example 5.2: Assume that, $X_1 = \{f_1, g_1, h_1\}, \tau_1 = \{X_1, \varphi, \{f_1\}, \{h_1\}, \{f_1, h_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{g_1, h_1\}, \{f_1, g_1\}, \{g_1\}\}$. s^*p^* -C sets are $\{X_1, \varphi, \{f_1\}, \{h_1\}\}$. Take, $X_2 = \{f_2, g_2, h_2\}, \tau_2 = \{X_2, \varphi, \{g_2\}$

$h_2\}$. $\tau_2^c = \{X_2, \varphi, \{f_2\}\}$. s^*p^* -C sets of (X_2, τ_2) are $\{X_2, \varphi, \{h_2\}\}$. Consider the function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = h_2$, $f(g_1) = f_2$, $f(h_1) = g_2$. Then $f^{-1}(h_2) = \{f_1\}$ is s^*p^* -C set in (X_1, τ_1) . Hence, f is a s^*p^* -Irresolute function.

Theorem 5.3: Each s^*p^* - Irresolute Map is gp Continuous.

Proof: Consider a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a s^*p^* Irresolute map. consider the closed set G in (X_2, τ_2) . Because every closed set is gp Closed set and f are Irresolute map, after that the G 's inverse image is gp Closed set in (X_1, τ_1) . Which implies that f is gp Continuous. Hence, every s^*p^* Irresolute map is gp Continuous map.

The below illustration says that the reverse of the previously mentioned statement incorrect.

Example 5.4: Assume $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \varphi, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{h_1\}\}$. s^*p^* -C sets of (X_1, τ_1) are $\{X_1, \varphi, \{h_1\}\}$, $\{f_1, h_1\}$, $\{g_1, h_1\}$. gp Closed sets of (X_1, τ_1) are $\{X_1, \varphi, \{f_1\}$, $\{g_1\}\}$. Suppose, $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \varphi, \{g_2\}, \{h_2\}, \{g_2, h_2\}, \{f_2, g_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{f_2, h_2\}, \{f_2, g_2\}, \{f_2\}, \{h_2\}\}$. s^*p^* -C sets of (X_2, τ_2) are $\{X_2, \varphi, \{f_2\}\}$. Define a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = h_2$, $f(g_1) = f_2$, $f(h_1) = g_2$. Then $f^{-1}(f_2) = \{g_1\}$ is gp-Closed set but does not contain in s^*p^* -C set in (X_1, τ_1) . Therefore, f is gp Continuous but not in s^*p^* -Irresolute set.

Remark 5.5: Each s^*p^* Irresolute map is gpr- continuous.

The next illustration shows the opposite of the previous-mentioned remark may not be valid.

Example 5.6: Assume that $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \varphi, \{f_1\}, \{g_1\}, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{h_1\}, \{f_1, h_1\}, \{g_1, h_1\}\}$. s^*p^* -C sets of (X_1, τ_1) are $\{X_1, \varphi, \{g_1\}\}$. gpr closed sets of (X_1, τ_1) are $\{X_1, \varphi, \{f_1, h_1\}\}$. Let $X_2 = \{f_2, g_2, h_2\}$ $\tau_2 = \{X_2, \varphi, \{f_2\}, \{f_2, g_2\}\}$ $\tau_2^c = \{X_2, \varphi, \{g_2, h_2\}, \{h_2\}\}$. s^*p^* -C sets of (X_2, τ_2) are $\{X_2, \varphi, \{g_2\}, \{g_2, h_2\}, \{f_2, g_2\}\}$. Define a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = g_2$, $f(g_1) = f_2$, $f(h_1) = h_2$. Then, $f^{-1}(g_2, h_2) = \{f_1, h_1\}$ is in gpr closed set but does not contain in s^*p^* -closed set in (X_1, τ_1) . Therefore, it is gpr -continuous but this is not in s^*p^* -Irresolute.

Remark 5.7:

The concepts of s^*p^* -Irresolute is independent from pre irresolute. The example coming next explains it.

Example 5.8: Put $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \varphi, \{f_1\}, \{f_1, h_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{g_1, h_1\}, \{g_1\}\}$. s^*p^* -C sets are $\{X_1, \varphi, \{g_1\}\}$. Pre closed sets are $\{X_1, \varphi, \{g_1\}, \{h_1\}, \{g_1, h_1\}\}$. Let $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \varphi, \{h_2\}, \{f_2, h_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{f_2, g_2\}, \{g_2\}\}$. s^*p^* -C sets are $\{X_2, \varphi, \{f_2\}\}$. Pre closed sets are $\{X_2, \varphi, \{f_2\}, \{g_2\}, \{f_2, g_2\}\}$. Defining a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = h_2$, $f(g_1) = g_2$, $f(h_1) = f_2$. Then, $f^{-1}(f_2) = \{h_1\}$ is in pre-closed set in (X_1, τ_1) so $f^{-1}(f_2) = \{h_1\}$ is not in s^*p^* -closed set in (X_1, τ_1) . Thus, f is not s^*p^* -irresolute but it is pre-irresolute.

Example 5.9: Put $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \varphi, \{f_1\}, \{h_1\}, \{f_1, h_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{g_1, h_1\}, \{f_1, g_1\}, \{g_1\}\}$. s^*p^* -C sets are $\{X_1, \varphi, \{f_1\}, \{h_1\}\}$. Pre closed sets are $\{X_1, \varphi, \{g_1\}, \{f_1, g_1\}, \{g_1, h_1\}\}$. Let $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \varphi, \{f_2\}, \{f_2, h_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{g_2, h_2\}, \{g_2\}\}$. s^*p^* -C sets are $\{X_2, \varphi, \{g_2\}\}$. Pre closed sets are $\{X_2, \varphi, \{g_2\}, \{h_2\}, \{g_2, h_2\}\}$. Assume a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ then, $f(f_1) = h_2$, $f(g_1) = f_2$, $f(h_1) = g_2$. Then, $f^{-1}(g_2) = \{h_1\}$ is not in pre-closed set in (X_1, τ_1) so that

$f^1(g_2) = \{h_1\}$ is in s^*p^* -closed set in (X_1, τ_1) . Thus, f does not belong to pre-irresolute and it is s^*p^* -irresolute.

Remark 5.10:

The next example proves that the s^*p^* Irresolute and α -irresolute are independent each other.

Example 5.11: Assume $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \varphi, \{f_1\}, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{g_1, h_1\}, \{h_1\}\}$. s^*p^* -C sets of (X_1, τ_1) are $\{X_1, \varphi, \{g_1\}\}$. α -closed sets of (X_1, τ_1) are $\{X_1, \varphi, \{g_1\}, \{h_1\}, \{g_1, h_1\}\}$. Consider $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \varphi, \{h_2\}, \{f_2, h_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{f_2, g_2\}, \{g_2\}\}$. s^*p^* -C sets of (X_2, τ_2) are $\{X_2, \varphi, \{f_2\}\}$. α -closed sets of (X_2, τ_2) are $\{X_2, \varphi, \{f_2\}, \{g_2\}, \{f_2, g_2\}\}$. Let us consider a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = h_2$, $f(g_1) = g_2$, $f(h_1) = f_2$. Then, $f^1(f_2) = \{h_1\}$ is in α -closed set in (X_1, τ_1) so that $f^1(f_2) = \{h_1\}$ is not in s^*p^* -C set in (X_1, τ_1) . Because, f is not s^*p^* irresolute but is in α -irresolute.

Example 5.12: Consider $X_1 = \{f_1, g_1, h_1\}$, $\tau_1 = \{X_1, \varphi, \{f_1\}, \{g_1\}, \{f_1, g_1\}\}$. $\tau_1^c = \{X_1, \varphi, \{g_1, h_1\}, \{f_1, h_1\}, \{h_1\}\}$. s^*p^* -C sets of (X_1, τ_1) are $\{X_1, \varphi, \{f_1\}, \{g_1\}\}$. α -closed sets of (X_1, τ_1) are $\{X_1, \varphi, \{h_1\}, \{f_1, h_1\}, \{g_1, h_1\}\}$. Assume $X_2 = \{f_2, g_2, h_2\}$, $\tau_2 = \{X_2, \varphi, \{f_2\}, \{f_2, g_2\}\}$. $\tau_2^c = \{X_2, \varphi, \{g_2, h_2\}, \{h_2\}\}$. s^*p^* -C sets of (X_2, τ_2) are $\{X_2, \varphi, \{g_2\}\}$. α -closed sets of (X_2, τ_2) are $\{X_2, \varphi, \{g_2\}, \{h_2\}, \{g_2, h_2\}\}$. Let us consider a function $f: (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ by $f(f_1) = h_2$, $f(g_1) = g_2$, $f(h_1) = f_2$. Then, $f^1(g_2) = \{g_1\}$ is not in α -closed set in (X_1, τ_1) so that $f^1(g_2) = \{g_1\}$ is in s^*p^* -C set in (X_1, τ_1) . Because, f is s^*p^* irresolute but is not α -irresolute.

Conclusion

The purpose of this continuous function in topology is necessary for understanding convergence, compactness, and connectedness in various spaces. This idea plays the groundwork for more advanced topics such as homeomorphisms, topological equivalences, and the development of various topological properties. In conclusion, this study demonstrates the significance of s^*p^* continuous functions in characterizing topological spaces. Our results provide new insights into the preservation of topological properties under continuous transformations. Many useful results have been drawn from our study. These results have important significance for the study of topology. This study has advanced our theoretical understanding of topology by providing the way for new research directions.

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