Pythagorean Fuzzy Sets and Its Application in Crop Cultivation

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Abstract In any decision-making problem, ambiguity is an important factor in finding a solution. Many advanced methods and techniques have been introduced to tackle this unreliable environment. Pythagorean fuzzy sets, a generalization of IFS, is a most flexible tool to handle this situation in decision-making problems. Various factors such as soil type, land type, crop pattern, and investment influence the crop cultivation in a region. This article analyzes the properties of Pythagorean fuzzy sets and Pythagorean fuzzy functions taking the above factors into account and applies the same for the problem of crop cultivation. Finally, a decision-making approach to crop cultivation process is carried out using a hypothetical database using the Pythagorean fuzzy sets and max–min–max operator.

Keywords Crop cultivation · Fuzzy set · Intuitionistic fuzzy set · Pythagorean fuzzy set · Pythagorean fuzzy relations

1 Introduction

Zadeh introduced the fuzzy set theory in which he gave importance of membership value than the non-membership value [15]. The extension of the fuzzy set theory is the intuitionistic fuzzy set (IFS) developed by Atanassov [1–3]. He considered membership, non-membership along with the hesitation index values in his definition [4]. The concept of IFS became a powerful tool in solving many real-life problems like medical diagnosis, career placement, and personal appointments [4–6, 9–12]. The limitations in the above concepts are tackled by Pythagorean fuzzy set proposed by Yager [13, 14]. The Pythagorean fuzzy relations and their composition constructed by Ejegwa [7, 8] are more reliable to find the solutions of the above problems.

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In this paper, Sect. 2 discusses the basic definitions of fuzzy and Pythagorean fuzzy sets, Sect. 3 gives the notions of Pythagorean fuzzy relations a max–min–max rule, Sect. 4 deals with the construction of the problem using composition rule, Sect. 5 shows the numerical illustration to the problem, and remaining sections conclude that crop cultivation can be achieved with the Pythagorean fuzzy relations approach.

2 Preliminaries

Some basic definitions are discussed.

Definition 2.1 Let U be a fixed set. A fuzzy set F in U is defined as set $A = \{\langle p, \lambda_F(p)\rangle / p \in U\}\}$ where the function $\lambda_F(p)$: $U \to [0, 1]$ defines the degree of membership of the element, $p \in U$.

Definition 2.2 Let U be fixed set. An IFS F in U is a set $A = \{\langle p, \lambda_F(p), \eta_F(p)\rangle \mid p \in U\}$.

where the functions $\lambda_F(p)$: $U \to [0, 1]$ and $\eta_F(p)$: $U \to [0, 1]$ are the degree of membership and non-membership, respectively. The hesitation index $\pi_F(p)$ is the degree of non-determinacy of $p \in U$, to the set F and $\pi_F(p) \in [0, 1]$. Thus, $\lambda_F(p) + \eta_F(p) + \pi_F(p) = 1$.

Definition 2.3 Let U be a fixed set. Then a Pythagorean fuzzy set is defined as $F = \{\langle p, \lambda_F(p), \eta_F(p)\rangle \mid p \in U\}$ is a subset of U, and for every $p \in U$: $0 \leq (\lambda_F(p))^2 + (\eta_F(p))^2 \leq 1$. We denote the set of all PFSs over U by PFS(U).

Theorem 2.1 *Let* $U = \{u_j\}$ *be a non-empty set, for* $j = 1, \ldots, n$ *and* $F \in$ PFS(U). *Let* $\pi_F(u_j) = 0$. *Then,*

(i) $|\lambda_F(u_j)| = \sqrt{|(\eta_F(u_j) + 1)(\eta_F(u_j) - 1)|}$.

(ii) $|\eta_F(u_j)| = \sqrt{|(\lambda_F(u_j) + 1)(\lambda_F(u_j) - 1)|}$.

Proof Suppose that $u_j \in U$ and $F \in$ PFS(U).

Assume that $\pi_F(u_j) = 0$, for $u_j \in U$.

we have,

$$\begin{aligned}
&(\lambda_F(u_j))^2 + (\eta_F(u_j))^2 = 1 \Rightarrow -(\lambda_F(u_j))^2 = (\eta_F(u_j))^2 - 1\\
&\quad\Rightarrow -(\lambda_F(u_j))^2 = (\eta_F(u_j) + 1)(\eta_F(u_j) - 1)\\
&\quad\Rightarrow \left|(\lambda_F(u_j))^2\right| = |(\eta_F(u_j) + 1)(\eta_F(u_j) - 1)|\\
&\quad\Rightarrow |\lambda_F(u_j)|^2 = |(\eta_F(u_j) + 1)(\eta_F(u_j) - 1)|\\
&\quad\Rightarrow |\lambda_F(u_j)| = \sqrt{|(\eta_F(u_j) + 1)(\eta_F(u_j) - 1)|}.
\end{aligned}$$

Definition 2.5 The complement of a PFS F is defined as follows: Fc $= \{\langle p, \eta_F(p), \lambda_F(p)\rangle | p \in U\}$.

Definition 2.6 The operations union and intersection of two PFSs F and G are defined as.

(i) $F \cup G = \{\langle p, \max(\lambda_F(p), \lambda_G(p)), \min(\eta_F(p), \eta_G(p))\rangle | \, p \in U\}$.
(ii) $F \cap G = \{\langle p, \min(\lambda_F(p), \lambda_G(p)), \max(\eta_F(p), \eta_G(p))\rangle | \, p \in U\}$.

Definition 2.7 Let F be a PFS. Its score is given by the function $sc(F) = (\lambda_F(p))^2 - (\eta_F(p))^2$, where $sc(F) \in [-1, 1]$.

Definition 2.8 The accuracy function of a PFS F is defined as $ac(F) = (\lambda_F(p))^2 + (\eta_F(p))^2$ for $ac(F) \in [0, 1]$.

Theorem 2.2 *Let* $F \in$ PFS(U). *Then*:

(i) $ac(F) = 1 \Leftrightarrow \pi_F(p) = 0$.
(ii) $ac(F) = 0 \Leftrightarrow |\lambda_F(p)| = |n_E(p)|$.

Proof **I Part**.

(i) Suppose $ac(F) = 1$.

Then, $(\lambda_F(u))^2 + (\eta_F(u))^2 = 1$, $(\because \pi_F(u) = 0)$.

II Part.

Assume that $\pi_F(u) = 0$.

Then, it follows that: $(\lambda_E(u))^2 + (\eta_E(u))^2 = 1 \Rightarrow ac(F) = 1$.

(ii) Suppose $ac(F) = 0$. Then, $(\lambda_F(u))^2 = -(\eta_F(u))^2$ or $(\eta_F(u))^2 = -(\lambda_F(u))^2 \Leftrightarrow |\lambda_F(u)|^2 = |\eta_F(u)|^2 \Leftrightarrow |\lambda_F(u)| = |\eta_F(u)|$.

Theorem 2.3 *Let* $F, G \in$ PFS(U). Then*, the following holds*:

(i) $sc(F) = sc(G) \Leftrightarrow F = G$.
(ii) $sc(F) \le sc(G) \Leftrightarrow F \subseteq G$.
(iii) $sc(F) < sc(G) \Leftrightarrow$ F $\subseteq G$ *and* $F \ne G$.

The proof is obvious.

3 Pythagorean Fuzzy Relation

Basic definitions of Pythagorean fuzzy relations are given.

Definition 3.1 Let U and V be fixed sets. Let $f: U \to V$ be a function. Let F and G be PFS over U and V. Then,

(i) The image of F is $f(F)$, is a Pythagorean fuzzy set of V defined as

$$\lambda_{f(F)}(q) = \begin{cases} \bigvee_{p \in f^{-1}(q)} \lambda_F(p), & f^{-1}(q) \neq \emptyset \\ 0, \quad \text{otherwise} & \end{cases}$$

and

$$\eta_{f(F)}(q) = \begin{cases} \bigwedge_{p \in f^{-1}(q)} \eta_F(p), & f^{-1}(q) \neq \emptyset \\ 1, \quad \text{otherwise} & \end{cases}$$

for each $q \in V$.

(ii) The inverse image of G is $f^{-1}(G)$, is a Pythagorean fuzzy set of U defined as.
$\lambda_{f-1(G)}(p) = \lambda_G(f(p))$ and
$\eta_{f-1(G)}(p) = \eta_G(f(p)) \; \forall \; p \in U$.

Definition 3.2 A Pythagorean fuzzy relation (PFR), R, from U to V is a PFS of U $\times$ V given by the membership function, λ_R and non-membership function, η_R. A PFR from U to V is denoted by R $(U \to V)$.

Definition 3.3 Let F be a PFS over U. Let R be a PFR from U to V. Then, the composite mapping of $R(U \to V)$ with F is a PFS G of V denoted by $G = R \cdot F$, such that its membership and non-membership functions are defined by the following: $\lambda_G(y) = \bigvee_p(\min[\lambda_F(p), \lambda_R(p, q)])$ and $\eta_G(q) = \bigwedge_p(\max[\eta_F(p), \eta_R(p, q)]) \forall \, p \in U$ and $q \in V$, where $\bigvee$ = maximum, $\bigwedge$ = minimum.

Definition 3.4 Let Q $(U \to V)$ and R $(V \to W)$ be two PFRs.

Then, the (max–min–max) composition $R \cdot Q$ is a PFR from U to W, such that its membership and non-membership functions are defined by the following: $\lambda_{R \cdot Q}(p, r) = \bigvee_q \left(\min\left[\lambda_Q(p, q), \lambda_R(q, r)\right]\right)$ and $\eta_{R \cdot Q}(p, r) = \bigwedge_q \left(\max\left[\eta_Q(p, q), \eta_R(q, r)\right]\right), \forall \, (p, r) \in U \times W$ and $\forall \, q \in V$.

Result 3.1 From Definitions 4.3 and 4.4, the PFR $B = R \bullet$ Q is found by the following:
$B = \lambda_B(q) - \eta_B(p)\pi_B(q), \forall \, q \in V$.

4 Crop Cultivation by Composite Relations of Pythagorean Fuzzy Sets

Let us construct the problem as follows.

Let $P = \{p_1, p_2, \ldots, p_t\}$, $S = \{s_{1,} s_2, \ldots, s_m\}$, $C = \{c_1, c_2, \ldots, c_n\}$ be the collection of states, soils, and crops, respectively.

Let $R_1(P \to S)$ and $R_2(S \to C)$ be two Pythagorean fuzzy relations s.t $R_1 = \{(p, s), \lambda_R(p, s), \eta_R(p, s)/(p, s) \in P \times S\}$ and $R_2 =$

$\{(s, c), \lambda_U(s, c), \eta_U(s, c)/(s, c) \in S \times C\}$, where $\lambda_{R_1}(p, s)$ denotes the degree to which the state p related to the soil s, $\eta_{R_1}(p, s)$ denotes the degree to which the state p not related to the soils.

Similarly, $\lambda_{R_2}(s, c)$ denotes the degree to which the soil s related to the crop c, $\eta_{R_2}(s, c)$ denotes the degree to which the soil s not related to the crops, respectively.

The composite mapping M of R_1 and R_2 is denoted by $R_1 \bullet R_2$ which describes the state in which the states p_i concerning the soil s_j the cultivated crop c_k.

$\lambda_M(p_i, c_k) = \bigvee_{s_j \in S}\{\min[\lambda_{R_1}(p_i, c_j), \lambda_{R_2}(s_j, c_k)]\}$ and $\eta_M(p_i, c_k) = \bigwedge_{s_j \in S}\{\max[\eta_{R_1}(p_i, s_j), \eta_{R_2}(s_j, c_k)]\}$.

5 Case Study

Consider the following factors as sets,

the set of states P is

$$P = \{\text{Assam, Gujarat, Karnataka, Uttar Pradesh, West Bengal}\},$$

the set of soils S is

$$S = \{\text{Alluvial Soil, Black Soil, Red Soil, Desert Soil, Laterite Soil, Mountain Soil}\}.$$

the set of crops C is

$$C = \{\text{Rice, Wheat, Sugarcane, Vegetables, Tea, Coffee}\}$$

The Pythagorean fuzzy relations $R_1(P \to S)$ and R_2 $(S \to C)$ are given hypothetically in Tables 1 and 2. λ_M and η_M function values of the composite relation $M = R_1 \bullet R_2$ are given in Table 3. After finding the hesitation index ($\pi = \sqrt{1 - (u^2 + v^2)}$), we calculate the values of M given in Table 4, respectively.

Table 1 $R_1(P \to S)$ denotes the degree of states to different types of soil

R_1	Alluvial soil	Black soil	Red soil	Desert soil	Laterite soil	Mountain soil
Assam	⟨0.6, 0.1⟩	⟨0.5, 0.4⟩	⟨0.3, 0.4⟩	⟨0.7, 0.2⟩	⟨0.3, 0.4⟩	⟨0.3, 0.4⟩
Gujarat	⟨0.8, 0.1⟩	⟨0.6, 0.1⟩	⟨0.2, 0.8⟩	⟨0.6, 0.1⟩	⟨0.1, 0.6⟩	⟨0.2, 0.8⟩
Karnataka	⟨0.0, 0.8⟩	⟨0.4, 0.4⟩	⟨0.6, 0.1⟩	⟨0.1, 0.7⟩	⟨0.1, 0.8⟩	⟨0.5, 0.4⟩
Uttar Pradesh	⟨0.8, 0.1⟩	⟨0.8, 0.1⟩	⟨0.0, 0.6⟩	⟨0.2, 0.7⟩	⟨0.0, 0.5⟩	⟨0.6, 0.1⟩
West Bengal	⟨0.2, 0.8⟩	⟨0.6, 0.1⟩	⟨0.0, 0.8⟩	⟨0.4, 0.4⟩	⟨0.6, 0.1⟩	⟨0.1, 0.7⟩

Table 2 $R_2(S \to C)$ denotes the degree of soil related to different types of crop

R_2	Rice	Wheat	Sugarcane	Vegetables	Tea	Coffee
Alluvial soil	$\langle 0.1, 0.7\rangle$	$\langle 0.0, 0.9\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.8, 0.0\rangle$	$\langle 0.2, 0.8\rangle$	$\langle 0.7, 0.0\rangle$
Black soil	$\langle 0.4, 0.3\rangle$	$\langle 0.7, 0.0\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.2, 0.8\rangle$	$\langle 0.7, 0.0\rangle$
Red soil	$\langle 0.4, 0.0\rangle$	$\langle 0.7, 0.0\rangle$	$\langle 0.3, 0.3\rangle$	$\langle 0.1, 0.7\rangle$	$\langle 0.1, 0.8\rangle$	$\langle 0.2, 0.6\rangle$
Desert soil	$\langle 0.3, 0.5\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.2, 0.4\rangle$	$\langle 0.0, 0.8\rangle$	$\langle 0.2, 0.4\rangle$
Laterite soil	$\langle 0.1, 0.7\rangle$	$\langle 0.1, 0.8\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.0, 0.8\rangle$	$\langle 0.2, 0.7\rangle$
Mountain soil	$\langle 0.7, 0.0\rangle$	$\langle 0.3, 0.3\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.7, 0.0\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.2, 0.8\rangle$

Table 3 $\lambda_M(p, c)$ and $\eta_M(p, c)$ denote the function values of M

λ_M, η_M	Rice	Wheat	Sugarcane	Vegetables	Tea	Coffee
Assam	$\langle 0.60, 0.05\rangle$	$\langle 0.40, 0.35\rangle$	$\langle 0.50, 0.25\rangle$	$\langle 0.45, 0.45\rangle$	$\langle 0.70, 0.05\rangle$	$\langle 0.40, 0.35\rangle$
Gujarat	$\langle 0.50, 0.25\rangle$	$\langle 0.75, 0.05\rangle$	$\langle 0.70, 0.10\rangle$	$\langle 0.50, 0.25\rangle$	$\langle 0.45, 0.30\rangle$	$\langle 0.50, 0.25\rangle$
Karnataka	$\langle 0.70, 0.10\rangle$	$\langle 0.45, 0.45\rangle$	$\langle 0.55, 0.20\rangle$	$\langle 0.55, 0.20\rangle$	$\langle 0.55, 0.25\rangle$	$\langle 0.70, 0.10\rangle$
Uttar Pradesh	$\langle 0.60, 0.05\rangle$	$\langle 0.60, 0.10\rangle$	$\langle 0.70, 0.10\rangle$	$\langle 0.50, 0.25\rangle$	$\langle 0.50, 0.25\rangle$	$\langle 0.60, 0.05\rangle$
West Bengal	$\langle 0.55, 0.20\rangle$	$\langle 0.45, 0.35\rangle$	$\langle 0.50, 0.25\rangle$	$\langle 0.75, 0.05\rangle$	$\langle 0.75, 0.05\rangle$	$\langle 0.60, 0.05\rangle$

Table 4 $M = \lambda_M - \eta_M \pi_M$ denotes the composition values of R_1 and R_2

M	Rice	Wheat	Sugarcane	Vegetables	Tea	Coffee
Assam	0.5600	0.1035	0.2927	0.1029	0.6644	0.1035
Gujarat	0.3508	0.7170	0.6293	0.2927	0.1977	0.2927
Karnataka	0.6293	0.1029	0.3878	0.3878	0.3508	0.6293
Uttar Pradesh	0.5601	0.5206	0.6293	0.2927	0.2927	0.5601
West Bengal	0.3878	0.1624	0.2927	0.7170	0.7170	0.5601

6 Decisions on the States' Crop Yield

Here, we made the following two decisions on the crop yield:

Row-wise observations: This is based on chances of the state that it can cultivate one or more crop.

Assam can cultivate tea, Gujarat can cultivate wheat, Karnataka can cultivate rice and coffee, Uttar Pradesh can cultivate sugarcane, West Bengal can cultivate tea and vegetables.

Column-wise observations: This decision is centered on the relation that the maximum number of state that can cultivate a crop.

It is noticed that rice can be cultivated in Karnataka, wheat can be cultivated in Gujarat, sugarcane can be cultivated in Gujarat and Uttar Pradesh, vegetables can be cultivated in West Bengal, tea can be cultivated in West Bengal, and coffee can be cultivated in Karnataka.

7 Conclusion

From the above discussion, it is concluded that PFR has vast applications in solving many multi-decision-making problems. Also, it is proved that this approach yields better crop production for economic benefits. Agriculture is the main occupation of our country, yielding a variety of crops in a state in this pandemic, and escalates the country's economy. The same problem can be extended by taking climatic conditions and degree of rainfall as additional input factors and applying the fuzzy neural network to obtain improved results.

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