Nanotechnology Perceptions
ISSN 1660-6795
www.nano-ntp.com

Shortest Path Problem Using Pentagonal Fuzzy Number

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The shortest path problem is one of the fundamental network optimization problem connected with real world applications. In this paper a creative algorithm is computed for shortest path problem and solved using pentagonal fuzzy number. Initially the path length is calculated and the minimum length is finalized and then the Euclidean distance is applied to evaluate the shortest path. A numerical example is given to illustrate the algorithm. In real life situations the application of shortest path algorithm helps emergency services to optimize their operations and ultimately saves lives by ensuring swift and efficient deployment of resources in critical situations.

Keywords: Pentagonal fuzzy number, Minimum length, Euclidean distance.

1. Introduction

However, in real world applications many types of uncertainty happen due to imprecise data and vagueness. In such cases shortest path problem under fuzzy environment plays a vital role in scheduling and it helps to navigate during natural disasters. Estimating the shortest path between two vertices was practiced in many fields such as Artificial intelligence, telecommunications, transportations etc. and also it plays a vital role in searching shortest distance to reach the goal considering the path and cost. In searching all its neighborhood nodes to check the lowest score path by choosing a priority queue of alternative path to reach the goal.

Many experts focused on shortest path problems to overcome the uncertainties. In 1980, Dubois and Prade was the first to focus on shortest path problem. Then Flyod Warshall algorithm has been developed for all pairs of shortest path problem. Inspite of various algorithms Dijkstra algorithm (1959) is the common algorithm to calculate the shortest path from source vertex to destination vertex for road network. Chanas in 1994 contributed a new approach based on α – cut. P.K. Raut, Dr.S.P. Behara, Dr.J.K. Pati[7] accumulated a new algorithm for shortest path in a closed network in Fuzzy Environment. A.Nagoorgani and A.Mumtaj Begam[3] introduced a new approach on Shortest path problem using graded mean integration. Tarek E. Jerbi and Mohamed Muamer[9] computed Fuzzy Rough Shortest path problem using Euclidean distance method. Aye Lai Lai soe, Myo kay Khaing[1] determined a new searching algorithm for improving delivery service applying shortest path algorithm for Large Road Network. Maheswari.D and Harshini.K[2] presented an algorithm for shortest path using α – cut and Euclidean distance method in finding shortest path in a network using Trapezoidal Intuitionistic Fuzzy Number. R.Sophia Porchelvi and

G. Sudha[4] determined shortest path by order relation between fuzzy numbers by computation of shortest path in a Fuzzy Network using Triangular Intuitionistic Fuzzy Network. A.Thamarai selvi and R. Santhi[10] presented transportation problem in Neutrosophic Environment as a new approach for optimization of real life Transportation problem in Neutrosophic Enviornment. Said Broumi, Assia Bakali, Mohamed Talea, Florentin Smarandache[8] illustrated the shortest path under Trapezoidal Neutrosophic Information.

In this paper the shortest path is calculated by using the new algorithm. Intially the possible path length is calculated for the network and by using the formula the minimum length is finalized then the Euclidean distance is applied for Lmin and Li from that the lowest value is assumed to be as the shortest path. And also it is compared with the real life application by rendering service in emergency operations to the disaster victims.

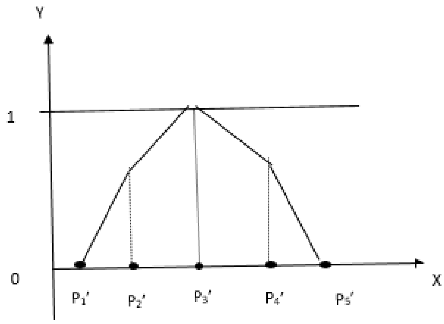
2. Preliminaries:

1. Definition:

1.1 Pentagonal Fuzzy Number(PFN):

The membership function for PFN is given by $\tilde{P} = (p_1', p_2', p_3', p_4', p_5')$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 \; for \; x < p_1' \\ \frac{x-p_1'}{p_2'-p_1'} \; for \; p_1' \le x \le p_2' \\ \frac{x-p_2'}{p_3'-p_2'} \; for \; p_2' \le x \le p_3' \\ 1 \; for \; x = p_3' \\ \frac{p_4'-x}{p_4'-p_3'} \; for \; p_3' \le x \le p_4' \\ \frac{p_5'-x}{p_5'-p_4'} \; for \; p_4' \le x \le p_5' \\ 0 \; for \; x > p_5' \end{cases}$$



Let $\tilde{P} = (p_1', p_2', p_3', p_4', p_5')$ and by $\tilde{Q} = (q_1', q'_2, q_3', q'_4, q_5')$ be any two intervals then the Euclidean distance

$$D_i = \sqrt{(p-p_i)^2 + (q-q_i)^2 + (r-r_i)^2 + (s-s_i)^2 + (t-t_i)^2}$$

1.2. Arithmetic Operations Under Pentagonal Fuzzy Number:

Let $\tilde{P} = (p_1', p_2', p_3', p_4', p_5')$ and $\tilde{Q} = (q_1', q_2', q_3', q_4', q_5')$ are two functions then the arithmetic operations are given by

1. Addition of $\tilde{P}$ and $\tilde{Q}$ are defined as

$$\tilde{P} + \tilde{Q} = (p_1' + q_1', p_2' + q_2', p_3' + q_3', p_4' + q_4', p_5' + q_5')$$

2. Subtraction of $\tilde{P}$ and $\tilde{Q}$ are defined as

$$\tilde{P} - \tilde{Q} = (p_1' - q_1', p_2' - q_2', p_3' - q_3', p_4' - q_4', p_5' - q_5')$$

3. Algorithm for Shortest path problem:

Step 1: Find all the possible paths from source node to destination node for the given network

Step 2: Intialize the path of P_1 as $d_{min}(p_1', q_1', r'_1, s'_1, t_1')$

Step 3: For i = 2

Calculate (p, q, r, s, t) using $d_{min}(p_1', q_1', r'_1, s'_1, t_1')$ and

$P_2 = (p_2'', q_2'', r_2'', s_2'', t''_2)$ where

$$p = \min(p_1', p_2'')$$

$$q = \begin{cases} q & if\ q_1'' < p_2'' \\ \frac{(q_1'' \times q_2'') - (p_1'' \times p_2'')}{(q_1'' + q_2'') - (p_1'' + p_2'')} & if\ q_1'' > p_2'' \end{cases}$$

$$r = \begin{cases} r & if\ r_1'' < q_2'' \\ \frac{(r_1'' \times r_2'') - (q_1'' \times q_2'')}{(r_1'' + r_2'') - (q_1'' + q_2'')} & if\ r_1'' > q_2'' \end{cases}$$

$$s = \min(s_1'', r_2'')$$

$$t = \begin{cases} t & if\ t_1'' < s_2'' \\ \frac{(t_1'' \times t_2'') - (s_1'' \times s_2'')}{(t_1'' + t_2'') - (s_1'' + s_2'')} & if\ t_1'' > s_2'' \end{cases}$$

Step 4: Now set d_{min} (p, q, r,s, t)

Step 5 : Continue the procedure till i = 5

Step 6: Finalize L_{min} (p, q, r,s, t)

Step 7: Then the Euclidean distance is computed between L_{min} and for all possible path

P_i's where

$$D_i = \sqrt{(p - p_i)^2 + (q - q_i)^2 + (r - r_i)^2 + (s - s_i)^2 + (t - t_i)^2}$$

Step 8: The shortest path is calculated by the shortest Euclidean distance value

3. Real Time Example

During large scale natural disasters such as earthquakes or wild fires, floods, pandemic situations etc. In such cases search and rescue missions need to navigate through challenging terrain to reach standard individuals or disaster victims. In such cases disaster command centres are established to co-ordinate response efforts, allocate resources and disseminate information to affected populations. These command centres serve as central nodes for managing emergency operations and decision making. They serve as nodes for communication and co-ordination among different emergency services, including fire departments, police agencies and medical responders.

Hospitals, clinics and medical centres serve as critical nodes in emergency response efforts providing medical care to disaster victims, treating injuries and managing public health concerns. Specialized medical facilities such as trauma centres or mobile field hospitals may also be established as temporary nodes in disaster affected areas.

Emergency management agencies use shortest path algorithm plan evacuation nodes for affected population. By considering factors such as population density, road capacity and the location of emergency shelters. The authorities can identify the most efficient evacuation routes to move people safely away from danger zone and minimizes congestion. Shelter and evacuation centres are established to provide temporary housing, food and basic necessities to individuals and families. These facilities serve as nodes for providing humanitarian assistance and support to affected people population.

Node 1: Search and rescue mission Node 2: Emergency response centres Node 3: Disaster command centres

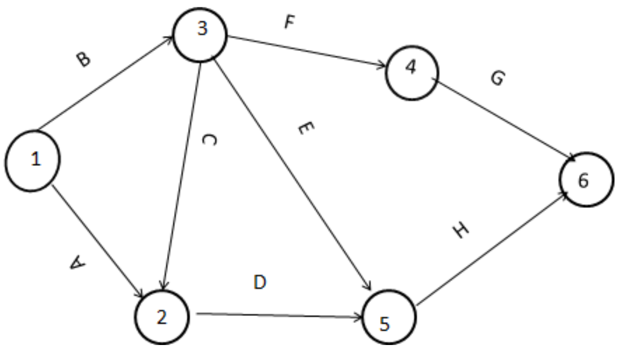
Node 4: Police – patrol route optimization Node 5: Medical supply distribution Node 6: Shelters and evacuation centres.

1. Numerical Example:

Activity	Duration
A: 1 - 2	(15, 22, 35, 46, 53)
B: 1 - 3	(20, 21, 34, 42, 48)
C: 2 - 3	(24, 31, 47, 58, 61)
D: 2 - 5	(26, 37, 43, 52, 60)
E: 3 - 5	(18, 26, 36, 56, 64)
F: 3 - 4	(22, 29, 39, 48, 54)
G: 4 - 6	(17, 25, 36, 53, 63)
H: 5 - 6	(23, 31, 47, 56, 60)

Solution:

The following table is represented in the following network diagram



Let the possible paths be

P1 = 1 → 2 → 5 → 6 = (64, 90, 125, 154, 173)

P2 = 1 → 2 → 3 → 5 → 6 = (80, 110, 165, 216, 238)

P3 = 1 → 3 → 4 → 6 = (59, 75, 109, 143, 165)S

P4 = 1 → 3 → 5 → 6 = (61, 78, 117, 154, 172)

P5 = 1 → 2 → 3 → 4 → 6 = (78, 107, 157, 205, 231)

Let us calculate dmin using the path length. Let us assume that

$P_1 = L_1 = (p''_1, q''_1, r''_1, s''_1, t''_1) = (64, 90, 125, 154, 173)$

Now set $d_{min} = (64, 90, 125, 154, 173)$ and $P_2 = (p''_2, q''_2, r''_2, s''_2, t''_2)$

For i = 2

Let p = min (p_1, p_2)

$p = \min(64, 80) = 64$

$$q = \frac{(90 \times 110) - (80 \times 64)}{(90+110) - (80+64)} = \frac{9900-5120}{200-144} = \frac{4780}{56} = 85.35$$

$$r = \frac{(165 \times 125) - (90 \times 110)}{(165+125) - (90+110)} = \frac{20625-9900}{290-200} = \frac{10725}{90} = 119.16$$

$s = \min(154, 165) = 154$

$t = 173$ since $s''_2 > t''_1$

Now set $d_{min} = (64, 85.35, 119.16, 154, 173)$

For i = 3

Calculate (p, q, r, s, t) where $d_{min} = (64, 85.35, 119.16, 154, 173)$ and $P_3 = (59, 75, 109, 143, 165)$

$p = \min(59, 64) = 59$

$$q = \frac{(85.35 \times 75) - (64 \times 59)}{(85.35+75) - (64+59)} = \frac{6401.25-3776}{160.35-123} = \frac{2625.25}{37.35} = 72.4$$

$$r = \frac{(119.16 \times 109) - (85.35 \times 75)}{(119.16+109) - (85.35+75)} = \frac{12988.44-6401.25}{228.16-160.35} = \frac{6587.19}{67.81} = 97.14$$

$s = \min(154, 109) = 109$

$$t = \frac{(175 \times 165) - (154 \times 143)}{(175+165) - (154+143)} = \frac{28875-22022}{340-297} = \frac{6853}{43} = 159.37$$

For i = 4

Calculate (p, q, r, s, t) where dmin = (59, 72.42, 97.14, 109, 159.37)

P4 = (61, 78, 117, 154, 172)

p = min(59, 61) = 59

$$q = \frac{(72.42 \times 78) - (59 \times 61)}{(72.42+78) - (59+61)} = \frac{5648.76-3599}{150.42-120} = \frac{2049.76}{30.42} = 67.38$$

$$r = \frac{(97.14 \times 117) - (72.42 \times 78)}{(97.14+117) - (72.42+78)} = \frac{11365.38-648.76}{214.14-150.42} = \frac{5716.62}{63.72} = 89.71$$

$s = \min(109, 117) = 109$

$$t = \frac{(159.37 \times 172) - (109 \times 54)}{(159.37+172) - (109+54)} = \frac{27411.64-16786}{331.37-263} = \frac{10625.64}{68.37} = 155.413$$

where dmin = (59, 67.38, 89.71, 109, 155.413)

for i = 5

Calculate (p, q, r, s, t) where dmin = (59, 67.38, 89.71, 109, 155.413) and P5 = (78, 107, 157, 205, 231)

p = min(59, 78) = 59

q = 67.38 since $p_5 > q_4$

r = 89.71 since $q''_5 > r''_4$

s = min (109, 157) = 109

t = 155.413 sine $s_5 > t_4$

Now the Lmin = (59, 67.38, 89.71, 109, 155.413)

Now the finalized value of Lmin = (59, 67.38, 89.71, 109, 155.413) Then the Euclidean distance value has to be calculated for

P1 = (64, 90, 125, 154, 173) and Lmin = (59, 67.38, 89.71, 109, 155.413)

$$D_1 = \sqrt{(5)^2 + (22.62)^2 + (35.29)^2 + (45)^2 + (17.59)^2}$$

$$= \sqrt{25 + 511.66 + 1245.38 + 2025 + 309.40}$$

$$= \sqrt{4116.44} = 64.16$$

Now the Euclidean distance for P2 = (59, 67.38, 165, 216, 238) and Lmin = (59, 67.38, 89.71, 109, 155.413)

$$D_2 = \sqrt{(21)^2 + (42.62)^2 + (75.29)^2 + (107)^2 + (82.59)^2}$$

$$= \sqrt{441 + 1816.46 + 5668.58 + 11449 + 6821.11}$$

$$= \sqrt{26196.15} = 161.85$$

Now the Euclidean distance for P3 = (59, 75, 109, 143, 165) and Lmin = (59, 67.38, 89.71, 109, 155.413)

$$D_3 = \sqrt{(0)^2 + (7.62)^2 + (19.29)^2 + (34)^2 + (9.59)^2}$$

$$= \sqrt{0 + 58.06 + 372.10 + 1156 + 91.97}$$

$$= \sqrt{1678.13} = 40.96$$

Now the Euclidean distance for P4 = (61, 78, 117, 154, 172) and Lmin = (59, 67.38, 89.71, 109, 155.413)

$$D_4 = \sqrt{(2)^2 + (10.62)^2 + (27.29)^2 + (45)^2 + (16.59)^2}$$

$$= \sqrt{4 + 112.78 + 744.74 + 2025 + 275.22}$$

$$= \sqrt{3161.74} = 56.23$$

Now the Euclidean distance for P5 = (78, 107, 157, 205, 231) and Lmin = (59, 67.38, 89.71, 109, 155.413)

$$D_5 = \sqrt{(19)^2 + (39.62)^2 + (67.29)^2 + (96)^2 + (75.59)^2}$$

$$= \sqrt{361 + 1569.74 + 4527.94 + 9216 + 5713.85}$$

$$= \sqrt{21388.53} = 146.24$$

Therefore the shortest distance is D3 = 40.96 and the shortest path is 1 → 3 → 4 → 6

Hence this result illustrates that the search rescue mission will direct the disaster victims to the disaster command centres. They communicate and co-ordinate the emergency to the police patrol then they protect them from the danger zone to shelter evacuation centres where they provide food and basic necessities for the affected people.

4. Conclusion:

In this paper the shortest path problem introduces a new algorithm for the decision makers to rescue the disaster victims and how to navigate them to evacuation centres. Here the shortest path is calculated by using the pentagonal fuzzy number. So it is concluded that the developed algorithm is an alternative source to find the shortest path in fuzzy environment.

References

1. Aye Lai Lai Soe, Myo Kay Khaing, " Improving delivery service applying shortest path algorithm for large Road Network".
2. D.Maheswari and K.Harshini, "Finding Shortest path in a network using Trapezoidal Intuitionistic fuzzy number", Research Review International Journal of Multidisciplinary, www.rrjournals.com[UGC Listed journal, Volume -04, Issue – 02 February 2019.

3. A. Nagoorgani and A. MumtajBegam, " A New Approach on shortest path in Fuzzy Environment",
4. R.SophiaPorchelvi, G. Sudha,"Computation of Shortest path in a Fuzzy Network using Triangular Intuitionistic Fuzzy Number", International Journal of Science and Engineering Research, Volume -5,ISSN 2229 – 5518.
5. L. Sujatha and J. DapheneHyacinta, "The Shortest path problem on Networks with Intuitionistic Fuzzy Edge weights", Global journal of Pure and Applied Mathematics, ISSN 0973 – 1768 Volume 13, Number 7 (2017), pp.3285 -3300 , Research India Publications, http://www.ripublication.com.
6. L. Sujatha and S. Elizabeth , " Fuzzy Shortest path problem based on similarity degree" journal in Applied Mathematics, publication at http://www.researchgate.net/publication/258555366
7. P.K. Raut, Dr.S.P. Behera , Dr.J.K. Pati, Calculation of Shortest Path in a Closed Network in Fuzzy Environment, International Journal of Mathematics Trends and Technology, ISSN: 2231 – 5373/ doi:10.14445/22315373/IJMIT – V67I11P504
8. Said Broumi, Mohamed Talea, AssiaBakali, FlorentinSmarandache, "Shortest path problem under Trapezoidal Neutrosophic Information, Computing Conference 2017, 18- 20 July 2017/ London,UK
9. Tarek Eljerbi and Mohamed Muamer, " Fuzzy Rough Shortest path Problems" Libyan Journal of Basic Sciences, Special issue for 5th International Conference for Basic Sciences and their Applications(5th ICBSTA, 2022) Vol: 19, Issue: 1, P:13- 25, 22-24/10/2022 http://ljbs.omu.edu.lv/eISSN 2707 - 6261
10. A. Thamaraiselvi and R. Santhi," A new approach for optimization of real life transportation problem in Neutrosophic Environment", Hindawi publishing corporation, Mathematical problems in Engineering, Volume 2016, Article ID 5950747, 9 pages, http://dx.doi.org/10.1155/2016/5950747
11. Xiaoqun Liao, Jia Yi Wang, Li Ma, "An algorithmic approach for finding the fuzzy constrained shortest path in a fuzzy graph", Complex and Intelligent Systems. https:// doi.org/10.1007/s 40747 – 620 – 00413 -6