Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 2 (2024)

A Study of s Star p Star Homeomorphism in Topological Spaces

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Article History:

Received: *30-01-2024*

Revised: *10-04-2024*

Accepted: *28-04-2024*

Abstract:

The present study aims to provide an overview and explore the new class of closed map and open map is termed as semi star pre star closed map (Briefly s*p*closed map), semi star pre star open map (Briefly s*p*open map) and some of its characterizations are studied. More over semi star pre star homeomorphism (Briefly s*p*Homeomorphism) in topological spaces is defined via s*p*open map and s*p*continuous map and to get a few of their specific features. Also, compared with existing one.

Keywords: s*p*Closed set, s*p*Open set, s*p*closed map, s*p* open map, s*p*Homeomorphism.

1. Introduction

In Topology, the idea of homeomorphism is necessary. Homeomorphism is the process of continuously stretching and bending an object into a new shape. A homeomorphism is a function which is bijection between topological spaces so that the map and its inverse are both continuous.

S.R.Malghan explores as well as establishes the Generalized closed maps [1]. Benchalli [2] developed regular closed maps, rw-closed maps and αrw-closed maps. rgα-closed map also rgα-open map was introduced by A.Vadivel and K.Vairamanikcam [3]. Sg homomorphism and gs homomorphism in topological spaces was studied by Devi et al [4]. Generalized homeomorphism were initially stated and examined by Maki et al. [5]. Rs Wali et.al [6] have introduced rgwα-homeomorphism in topological spaces. Gnanambal [7] has defined gpr-closed maps and studied some of their unique features. D.Iyappan and N.Nagaveni [8] was delivered on sgb-continuous map, sgb-closed maps in topological space. Studies on generalization of homeomorphism in topological spaces introduced by N.Nagaveni [9]. T.Shyla Isac Mary and P.Thangavelu [10] studied and developed RPS-Homeomorphism in topological spaces. In topological spaces, A. Pushpalatha and K. Anitha [11] established properties of g*s-closed set and A. Pushpalatha [12] looked on research on topological space generalizations of mapping. In topological spaces, generalizations of generalized closed sets and maps was developed by I.Arockiarani [13]. S.Sekar and B.Jothilakshmi [14] were created and explored the sg*b-closed maps in topological spaces.

Fundamental topological ideas are explored in this paper with a special focus to the s star p star homeomorphism. The field of topological spaces and its fundamental properties are expected to be better understood with the development of s star p star homeomorphism ideas, which offer a new point of view.

In the present analysis, topological spaces are treated as TS, s*p*closed set as s*p*-C set, s*p* open set as s*p*-O set, s*p* homeomorphism as s*p*-H.

2. Objectives

This research aims to define and analyses s star p star homeomorphism in topological spaces in an exact way. Formulate a number of theorems that demonstrate these sets features and implications. Utilize the illustrations how these concepts are effective.

3. Preliminaries

Definition: [15] Let the TS be X. Let A ⊆ X is known as semi star pre star closed sets (Briefly s* p * closed sets) if scl of A is subset of U when A is ⊆ of U and U is pre-open set.

The Complement of s*p*-C set is known as s*p*-O set.

Definition: The term s*p* continuous refers to a function f: $X_1 \to X_2$ where each closed set in X_2 has an inverse image that is also s*p*-C set in of every closed set in X_1.

4. s*p* closed map

Using the basic ideas of s*p*-C sets, we bring about s*p*-C map in topological spaces in this part and go through some of its essential features.

Definition 4.1: A Function f: $X_1 \to X_2$ is referred to be a s*p*-C (s*p*-O) map if all closed (open) set in X_1 has an image is in s*p*-C (s*p*-O) set in X_2.

Example 4.2: Let $X_1 = X_2 = \{r, s, t\}$; $\tau = \{X_1, \varphi, \{r\}, \{t\}, \{r, t\}\}$ and $\tau^c = \{X_1, \varphi, \{s, t\}, \{r, s\}, \{s\}\}$. $\sigma = \{X_2, \varphi, \{s\}, \{r, t\}\}$; $\sigma^c = \{X_2, \varphi, \{r, t\}, \{s\}\}$ and s*p*-C sets of X_2 are $\{X_2, \varphi, \{r\}\}$. Define a Map f: $X_1 \to X_2$ by f(r) = s; f(s) = r; f (t) = t. Then f is s*p*-C map because the image of closed map {s} in (X_1, τ), f {s} = {r} is in s* p*-C set in X_2.

Theorem 4.3: A closed map is always a s*p*-C map.

Proof: Consider f: $X_1 \to X_2$ be a closed map. Assume that V is in X_1. And it will be closed set. It Therefore, the image of V is in X_2 and that is closed set. All closed set is known to be s*p*- C set. Subsequently, f(V) is s*p*-C set. Thus, f is a s*p*-C map.

This theorem reverse implication may not hold, as shown by the example that follows.

Example 4.4: Let $X_1 = X_2 = \{r, s, t\}$; $\tau = \{X_1, \varphi, \{r\}, \{s, t\}\}$ and $\tau^c = \{X_1, \varphi, \{s, t\}, \{r\}\}$. $\sigma = \{X_2, \varphi, \{t\}, \{r, t\}\}$; $\sigma^c = \{X_2, \varphi, \{r, s\}, \{s\}\}$ and s*p*-C sets of X_2 are $\{X_2, \varphi, \{r\}\}$. Define a Map f: $X_1 \to X_2$ by f(r) = r; f(s) = s; f (t) = t. Hence f does not a closed map rather a s*p*-C map. Because the image of closed map {a} in (X_1, τ), f {r} = {r} is not in closed set in X_2 however it in s* p*-C set in X_2.

Theorem 4.5: Every map that belongs to pre- closed is s*p*-C map.

Proof: Let us consider f: $X_1 \to X_2$ be a pre-closed map. Let us consider the closed set in X_1 which is denoted by V. Thus, its image f(V) is pre closed set in X_2. Because each pre-closed set is s*p*-C set. So, the image of V is in s*p*-C set. Hence, f is a s*p*-C map.

Upcoming example shows the reverse of the earlier theorem never hold.

Example 4.6: Let $X_1 = X_2 = \{r, s, t\}$; $\tau = \{X_1, \varphi, \{r\}, \{s\}, \{r, s\}, \{s, t\}\}$ and $\tau^c = \{X_1, \varphi, \{s, t\}, \{r, t\}, \{t\}, \{r\}\}$. $\sigma = \{X_2, \varphi, \{r\}, \{s\}, \{r, s\}\}$; $\sigma^c = \{X_2, \varphi, \{s, t\}, \{r, t\}, \{t\}\}$ and s*p*-C sets of X_2 are $\{X_2, \varphi, \{r\}, \{s\}\}$. Pre-closed sets are $\{X_2, \varphi, \{s, t\}, \{r, t\}, \{t\}\}$. Define a Map f: $X_1 \to X_2$ by f(r) = s; f(s) = t; f (t) = r. Hence f is s*p*-C map. However, it is not pre-closed map. Because the closed map {r} in (X_1, τ), its image f (r) = {s} is not in pre- closed set in X_2 but it in s*p*-C set in X_2.

Theorem 4.7: Every map which is g*-closed map is also s*p*-C map.

Proof: Consider f: $X_1 \to X_2$ be a g*-closed map. Let us assume the closed set in X_1 it is denoted by V. After that the image of V that is f(V) in X_2 is g*closed set. Each g*closed set is s*p*-C set. Thus, the image of V is s*p*-C set. Hence, it is s*p*-C map.

Reverse implication of the previously stated theorem does not valid by the following example.

Example 4.8: Let $X_1 = X_2 = \{r, s, t\}$; $\tau = \{X_1, \varphi, \{r\}, \{t\}, \{r, t\}\}$ and $\tau^c = \{X_1, \varphi, \{s, t\}, \{r, s\}, \{s\}\}$. $\sigma = \{X_2, \varphi, \{r\}, \{r, s\}\}$; $\sigma^c = \{X_2, \varphi, \{s, t\}, \{t\}\}$ and s*p*-C sets of X_2 are $\{X_2, \varphi, \{s\}\}$. g*closed sets are $\{X_2, \varphi, \{s, t\}, \{r, t\}, \{t\}\}$. Assume f: $X_1 \rightarrow X_2$ by f(r) = t; f(s) = s; f (t) = r. Hence f is not g*closed map however it is in s*p*-C map. Hence the image of closed map {r} in (X_1, τ), f{r} = {s} is not in g* closed set in X_2 but it in s* p*-C set in X_2.

Theorem 4.9: s*p*-C maps are all gpr closed maps.

Proof: Suppose f: $X_1 \rightarrow X_2$ be a gpr closed map. Assume that V be in X_1. It is closed set. Then image of V that is f(V) is gpr closed set in X_2. Since any gpr-closed set is s*p*-C set thus image of V is gpr-closed set. So that f is s*p*-C map.

Upcoming Illustration shows the opposite of the previously stated theorem is not always right.

Example 4.10: Let $X_1 = X_2 = \{r, s, t\}$; $\tau = \{X_1, \varphi, \{s\}, \{t\}, \{r, s\}, \{s, t\}\}$ and $\tau^c = \{X_1, \varphi, \{r, t\}, \{r, s\}, \{t\}, \{r\}\}$. $\sigma = \{X_2, \varphi, \{r\}, \{s\}, \{r, s\}\}$; $\sigma^c = \{X_2, \varphi, \{s, t\}, \{r, t\}, \{t\}\}$ and s*p*-C sets of X_2 are $\{X_2, \varphi, \{s\}\}$. gpr closed sets of X_2 are $\{X_2, \varphi, \{r, t\}\}$. Define a Map f: $X_1 \rightarrow X_2$ by f(r) = t; f(s) = r; f (t) = s. Hence f does not a gpr closed map rather than s*p*-C map. Consequently, the image of closed map {c} in (X_1, τ), f{t} = {s} is not in gpr closed set in X_2 but it in s*p*- C set in X_2.

Theorem 4.11: Every αg closed map is a s*p*-C map.

Proof: Consider f: $X_1 \rightarrow X_2$ as an αg closed map. Take V in X_1 be a closed set. Thus, its image is in αg closed set in X_2. Because all αg closed set is s*p*-C set, then the image of V is in s*p*- C in X_2. Hence, f is s*p*-C map.

Our next illustration reveals why the other side of the above-mentioned theorem may not be correct.

Example 4.12: Let $X_1 = X_2 = \{q, r, s, t\}$; $\tau = \{X_1, \varphi, \{q\}, \{r\}, \{q, r\}, \{q, r, s\}\}$ and $\tau^c = \{X_1, \varphi, \{r, s, t\}, \{q, s, t\}, \{s, t\}, \{t\}\}$. $\sigma = \{X_2, \varphi, \{q\}, \{s\}, \{t\}, \{q, s\}, \{q, t\}, \{s, t\}, \{q, s, t\}\}$; $\sigma^c = \{X_2, \varphi, \{r, s, t\}, \{q, r, t\}, \{q, r, s\}, \{r, t\}, \{r, s\}, \{q, r\}, \{r\}\}$ and s*p*-C sets of X_2 are $\{X_2, \varphi, \{q\}, \{s\}, \{t\}, \{q, s\}, \{q, t\}, \{s, t\}\}$. αg closed sets of X_2 are $\{X_2, \varphi, \{r\}, \{q, r\}, \{r, t\}, \{q, r, t\}, \{q, r, t\}, \{r, s, t\}\}$. Declare a Map f: $X_1 \rightarrow X_2$ by f(q) = s; f(r) = r; f (s) = q; f(t) = t. Hence f does not a αg closed map rather than s*p*-C map. Because the image of closed map {s, t} in (X_1, τ), f {s, t} = {q, t} is not in αg closed set in X_2 but it in s*p*-C set in X_2.

Theorem 4.13: All ω-closed maps are s*p*-C map.

Proof: Given the idea and fact that any ω-closed set is s*p*-C set, the proof is obvious.

As this next illustration clarifies, the opposite of given theorem is not valid.

Example 4.14: Let $X_1 = X_2 = \{r, s, t\}$; $\tau = \{X_1, \varphi, \{r\}, \{r, s\}\}$ and $\tau^c = \{X_1, \varphi, \{s, t\}, \{t\}\}$. $\sigma = \{X_2, \varphi, \{r\}, \{r, t\}\}$; $\sigma^c = \{X_2, \varphi, \{s, t\}, \{s\}\}$ and s*p*-C sets of X_2 are $\{X_2, \varphi, \{t\}\}$. ω closed sets of X_2 are $\{X_2, \varphi, \{s\}, \{r, s\}, \{s, t\}\}$. Define a Map f: $X_1 \rightarrow X_2$ by f(r) = s; f(s) = r; f (t) = t. Hence f is not ω closed map but it is s*p*-C map. Because the image of closed map {t} in (X_1, τ), f {t} = {t} is not in ω closed set in X_2 but it in s*p*-C set in X_2.

Remark 4.15: An upcoming example shows that g-closed map and s*p*-C map are not dependent.

Let $X_1 = X_2 = \{q, r, s, t\}$; $\tau = \{X_1, \varphi, \{q\}, \{r\}, \{q, r\}, \{r, s\}, \{q, r, s\}\}$ and $\tau^c = \{X_1, \varphi, \{r, s, t\}, \{q, s, t\}, \{s, t\}, \{q, t\}, \{t\}\}$. $\sigma = \{X_2, \varphi, \{r\}, \{t\}, \{r, t\}, \{r, s, t\}\}$; $\sigma^c = \{X_2, \varphi, \{q, s, t\}, \{q, r, s\}, \{q, s\}, \{q\}\}$ and s*p*-C sets of X_2 are $\{X_2, \varphi, \{r\}, \{s\}, \{t\}, \{r, s\}, \{s, t\}\}$. g closed sets of X_2 are $\{X_2, \varphi, \{q\}, \{q, r\}, \{q, s\}, \{q, t\}, \{q, r, s\}, \{q, r, t\}, \{q, s, t\}\}$. Define a Map f: $X_1 \rightarrow X_2$ by f(q) = r; f(r) = q; f (s) = s; f(t) = t. Hence f does not a g closed map rather than s*p*-C map. When the image of closed map {s, t} in (X_1, τ), f {s, t} = {s, t} is not in g closed set in X_2 but it in s*p*-C set in X_2. Similarly, f is g closed but not s*p*-C map. Thus, the closed set {r, s, t} in (X_1, τ), f {r, s, t} = {q, s, t} is in g closed set in X_2 but not in s*p*-C set in X_2.

5. s*p* - HOMEOMORPHISM

This portion deals with s*p*-Homeomorphism (s*p*-H) using s*p*-C maps and s*p*-O maps.

Definition 5.1: A Bijection maps f: $X_1 \rightarrow X_2$ is known as s*p*Homeomorphism(s*p*-H) if f and its inverse are s*p*continuous maps.

Example 5.2: Let X_1 = X_2= {r, s, t}; τ = {X_1, φ, {r}, {s}, {r, s}} and τ^c = {X_1, φ, {s, t}, {r, t}, {t}}. s*p*-C sets of X_1 are {X_1, φ, {r}, {s}}. σ = {X_2, φ, {s}, {t}, {r, s}, {s, t}}; σ^c = {X_2, φ, {r, t}, {r, s}, {t}, {r}} and s*p*-C sets of X_2 are {X_2, φ, {r}, {r, s}}. Define a Map f: $X_1 \rightarrow X_2$ by f(r) = s; f(s) = t; f (t) = r. Here the inverse image of the closed set {c} in X_2 is s*p*-C set in X_1 and $(f^{-1})^{-1}$ (t) = f(t) = {r} is in s*p*-C set in X_2. Hence, f and its inverse are s*p*continuous. Which leads to f is s*p*-H.

Theorem 5.3: All homeomorphism may be expressed as s*p*Homeomorphism.

Proof: A homeomorphism is defined as f: $X_1 \rightarrow X_2$. Consequently, f and its inverse are continuous and bijection. So that f is s*p*-H if any continuous function is s*p*continuous.

The subsequent example explains that the theorems in contradiction does not always valid.

Example 5.4: Let X_1 = X_2= {r, s, t}; τ = {X_1, φ, {t}, {r, t}} and τ^c = {X_1, φ, {r, s}, {s}}. s*p*- C sets of X_1 are {X_1, φ, {r}}. σ = {X_2, φ, {r}, {t}, {s, t}, {r, t}}; σ^c = {X_2, φ, {s, t}, {r, s}, {r}, {s}} and s*p*-C sets of X_2 are {X_2, φ, {s}, {t}, {s, t}}. Assume a Map f: $X_1 \rightarrow X_2$ by f(r) = r; f(s) = t; f (t) = s. Then f and its inverse are s*p*continuous. So, f does not a homeomorphism rather than a s*p*-H. Because the inverse image of closed map {r, s} in (X_1, τ), $(f^{-1})^{-1}$ (r, s) = f (r, s) = {r, t} is not in closed set X_2.

Theorem 5.5: Every α-homeomorphism is a s*p*-Homeomorphism.

Proof: A α homeomorphism is $X_1 \rightarrow X_2$. Following that f and its inverse are α-continuous and f is bijection. f and its inverse are s*p* continuous obtained through each α- continuous function that is s*p*-continuous. Accordingly, f is s*p*-Homeomorphism.

A Further illustration demonstrates that the previously stated theorems reversal is not generally correct.

Example 5.6: Let X_1 = X_2= {r, s, t}; τ = {X_1, φ, {r}, {s}, {r, s}} and τ^c = {X_1, φ, {s, t}, {r, t}, {t}}. s*p*-C sets of X_1 are {X_1, φ, {r}, {s}} and α-closed sets of X_1 are {X_1, φ, {r}, {s}, {t}, {s, t}, {r, t}}. σ = {X_2, φ, {s}, {t}, {r, s}, {s, t}}; σ^c = {X_2, φ, {r, t}, {r, s}, {t}, {r}} and s*p*- C sets of X_2 are {X_2, φ, {r}, {r, s}}. α-closed sets of X_2 are {X_2, φ, {r}, {s}, {r, s}, {r, t}}. Define a Map f: $X_1 \rightarrow X_2$ by f(r) = s; f(s) = r; f (t) = t. Here f is s*p*-H but not α- homeomorphism because the inverse image of the closed set {t} in X_1 and $(f^{-1})^{-1}$ (t) = f(t) = {t} is not in α- closed set in X_2.

Theorem 5.7: All g- homeomorphisms are equivalent to s*p*-Homeomorphism.

Proof: Consider a g- homeomorphism $X_1 \rightarrow X_2$. Thus, f and its inverse are g continuous as well as bijection. f and its inverse are s*p* continuous this comes from any g-continuous function being s*p*-continuous. Because of this f is s*p*Homeomorphism.

The reverse implications are not valid as demonstrated from the below example.

Example 5.8: Let X_1 = X_2= {r, s, t}; τ = {X_1, φ, {t}, {r, t}} and τ^c= {X_1, φ, {r, s}, {s}} and s*p*closed sets of X_1 = {X_1, φ, {r}}. g closed sets of X_1 = {X_1, φ, {s}, {r, s}, {s, t}}. σ = {X_2, φ, {r}, {t}, {s, t}, {r, t}}; σ^c = {X_2, φ, {s, t}, {r, s}, {r}, {s}} and s*p*-C sets of X_2 are {X_2, φ, {s}, {t}, {s, t}}. g closed sets of X_2 are {X_2, φ, {s}, {t}, {s, t}}. Declare a Map f: $X_1 \rightarrow X_2$ by f(r) = r; f(s) = s; f (t) = t. Hence f does not a g-homeomorphism rather than a s*p*-H. Because the inverse image of closed map {s} in (X_1, τ), $(f^{-1})^{-1}$ (r, s) = f (r, s) = {r, s} is not in g- closed set in X_2.

Remark 5.9: The example that comes next points out the independence of s*p*-H and αg-homeomorphism.

Example 5.10: Let X_1= X2= {q, r, s, t}; τ = {X_1, φ, {q}, {r}, {q, r}, {q, r, s}} and $τ^c$ = {X_1, φ, {r, s, t}, {q, s, t}, {s, t}, {t}}. s*p*-C sets of X_1 are {X_1, φ, {q}, {r}, {t}, {r, t}, {q, t}}. σ = {X_2, φ, {q}, {s}, {t}, {q, s}, {q, t}, {s, t}, {q, s, t}} and $σ^c$ = {X_2, φ, {r, s, t}, {q, r, t}, {q, r, s}, {r, t}, {r, s}, {q, r}, {r}} and s*p*-C sets of X_2 are { X_2, φ, {q}, {s}, {t}, {q, s}, {q, t}, {s, t}}. αg closed sets of X_2 are {X_2, φ, {r}, {q, r}, {r, t}, {q, r, s}, {q, r, t}, {r, s, t}}. Assume a Map f: X_1→ X_2 by f(q) = q; f(r) = t; f (s) = r; f(t) = s. Hence f does not a αg homeomorphism rather than s*p*-H. Hence the image of closed set {u} in (X_1, τ), $(f^{-1})^{-1}$ (t) = f (t) = {s} is does not in αg closed set in X_2. Also, f is αg homeomorphism but not s*p*-H, consider the closed set {q, s, t} in (X_1, τ), then its inverse image $(f^{-1})^{-1}$ (q, s, t) = f (q, s, t) = {q, r, s} is not in s*p*-C set in X_2.

6. Conclusion

In this paper, we derived unique features of s*p*closed map and even s*p*homeomorphism via s*p*continuous and many of the implications, relations and independence of relationship with few of the existing closed sets are studied.

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