

Entropy Method of Multi-Attribute Decision-Making Problems

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Abstract Food choice aims to identify the solution to the exigency of the food. It is possible to do this through diversification. Finding the best selection of food alternatives is the goal of food diversification. Multi-attribute decision-making (MADM) methods attempt to choose the best alternative from a group of possibilities based on several parameters. This paper provides the description of the entropy process use in the analysis and application of multi attribute decision-making (MADM) for choosing foods.

1 Introduction

Decision-making is broadly defined to include any choice or evaluation of choices and is consequently relevant in a wide range of domains, including the soft social sciences as well as the hard sciences and engineering. [10, 11] Traditional decision making involves a collection of possible states of nature, a set of possible alternatives available to the decision maker, a relation showing the state or consequence to be expected from each alternative action, and a utility or optimization method that evaluates the outcomes according to their attractiveness. [1, 3, 9]. When the only evidence accessible about the results is their conditional probability distributions, one for every operation, a decision is made under risky circumstances. In this scenario, the difficulty of making a decision is transformed into an optimization problem of maximizing predicted utility.

When we analyze a discrete collection of alternatives characterized by some goals, we confront multiple attribute decision-making difficulties in a variety of circumstances where several alternatives, actions, or candidates must be picked based on a set of attributes. Determine the best option in terms of all relevant goals with the highest degree of attractiveness. [6, 8]. Multi-attribute decision-making (MADM) problems depict a circumstance in which decision-makers compare a definite collection of predetermined choices that are clearly known at the outset of the solution process to a range of frequently conflicting assessment criteria. It would not be an understatement to suggest that nearly

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everyone experiences decision-making issues on a regular basis, whether in their personal or professional lives. A resource that gathers and describes the many MADM approaches in a clear and systematic manner is still lacking, making it difficult for those who need to use these techniques to understand, compare, and apply them. [12]

This is despite the fact that decision-makers have access to a variety of MADM approaches deal with real-life decision-making issues. [2] Most current materials on MADM approaches are primarily concerned with the outcomes of their applications while overlooking fundamental and unifying ideas. In game theory, a maximax procedure where a player chooses an action that provides the "best of the best" outcomes when faced with uncertainty. A maximax approach is one that looks for where the biggest advantage can be discovered. All decisions will have costs and benefits. [7] John von Neumann developed the maximax theorem for the first time in 1928. It is frequently described as an assertive or optimistic approach.

2 Preliminaries

2.1 Normalized Values

Measure all the characteristics in dimensionless units and make inter-attribute analysing easier, use the following formulas to normalise every attribute value a_{ij} from the decision matrix $A = (a_{ij})_{n \times m}$:

$$\hat{r}_{ij} = \frac{r_{ij}}{\sum_{i=1}^n r_{ij}}$$

where $j = 1, 2, 3, \dots, m$

2.2 Entropy Weight

The weight of crisp criteria will change as an interval criterion's degree of diversification in its interval degree of diversification changes.

$$w_j = -\frac{d_j}{\sum_{k=1}^m d_k}$$

where $d_j = 1 - e_j$.

2.3 Weight Vector

Let $w = (w_1, w_2, \dots, w_n)$ represents the weight matrix of attributes $w_j \geq 0, j = 1, 2, 3, \dots, m$ that satisfy the constraint condition.

$$\sum_{j=1}^n w_j^2 = 1$$

Then it may evaluate the attribute value of every alternative.

$$z_i(w) = \sum_{j=1}^n r_{ij} w_j^2$$

2.4 Multiple-Attribute Decision-Making Algorithm

Defining system evaluation qualities that link system capabilities to objectives.

Creating alternate systems for achieving the objectives (generating alternatives)
Comparing and contrasting alternatives in terms of qualities (the values of the attributes functions)
Using a normative method of multiple attribute analysis
Accepting one option as "ideal."
If the final solution is rejected, collect new data and proceed to the next round of multiple attribute optimization.

Fig 1 explained Multiple characteristics of decision-making offer a number of powerful and effective strategies for dealing with sorting issues. The key to making a decision is to weigh the options. In the case of conflicting alternatives, however, a decision maker must also take into account imprecise or ambiguous facts, which is common in these types of dilemmas. [2]

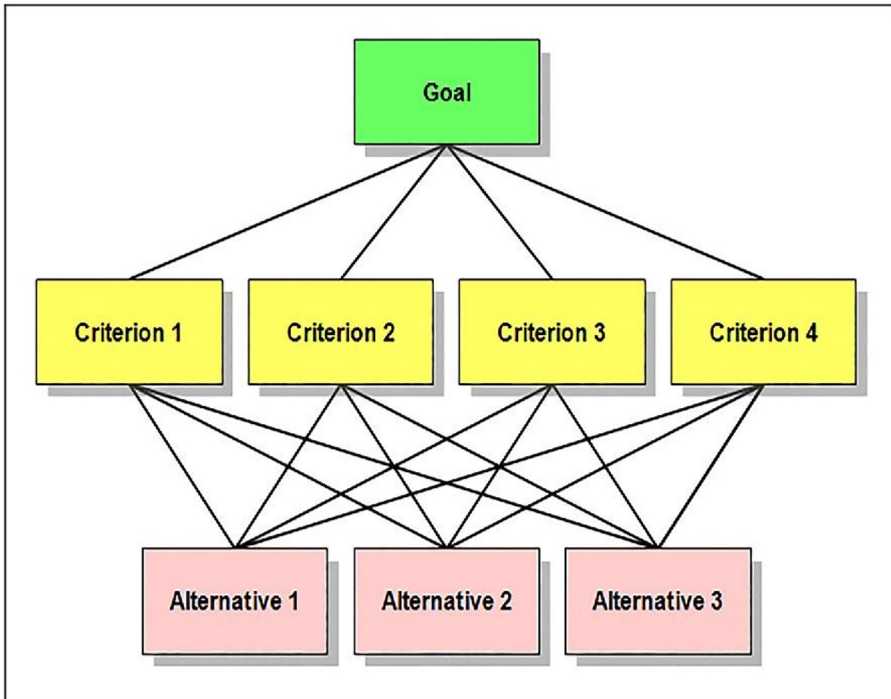


Figure 1.1 MADM method

2.5 Real life Application in Decision Making

In real life application Decision Making Plays an important role in many fields such as:
Traditional Speciality Areas: aerodynamics, controls, structures, trajectory analysis and heat transfer.

Other Application Areas: industries, chemistry, electromagnetics, forestry management, systems integration, multidisciplinary optimization, mission, trade space analyses, structures life cycle, cost analyses, management, robust, applications area, uncertainty (or) risk analysis reliability design methods, technology assessments, research portfolio analyses.

2.6 Algorithm for MADM problem using Entropy

The term "entropy" comes from the field of thermodynamics. It was first used to define the irreversible phenomena that occur throughout the movement process. Entropy are later utilized to illustrate the uncertainty objects that exhibit the information theory.

Step 1: A decision matrix $A = (a_{ij})_{n \times m}$ is first generated for a MADM problem, and then it is normalized into the matrix $R = (r_{ij})_{n \times m}$ using the appropriate formulas.

Step 2: Convert the matrix $R = (r_{ij})_{n \times m}$ into the matrix $\hat{R} = (\hat{r}_{ij})_{n \times m}$ by using the formula of normalized values.

Step 3: Determine the attributes equivalent information entropy u_j :

$$E_j = -\frac{1}{\ln n} \sum_{i=1}^n \hat{r}_{ij} \ln \hat{r}_{ij}$$

where $j = 1, 2, 3, \dots, m$

Step 4: Calculate the weight vector of attribute $w = (w_1, w_2, w_3, \dots, w_n)$ by using weight vector.

Step 5: Utilize $z_i(w) = \sum_{j=1}^m r_{ij} w_j$ to determine the alternatives x_i for entire attribute value, $z_i(w)$.

Step 6: Rank and select the alternatives $x_i (i = 1, 2, \dots, n)$ choices based on $z_i(w) (i = 1, 2, \dots, n)$.

3 Numerical Examples

A food firm selects eight categories, such as $\{a_1, a_2, a_3, \dots\}$ and their features are utilised to evaluate food $\{x_1, x_2, x_3, \dots\}$.

a_1 = Analog Rice	x_1 = Carbo-hydrate (g)
a_2 = Corn	x_2 = Fat (g)
a_3 = Rice	x_3 = Fiber (g)
a_4 = Wheat	x_4 = Protein (g)
a_5 = Cassava	x_5 = Energy (kcal)
a_6 = Sago	x_6 = Ca (mg)
a_7 = Potato	x_7 = Fe (mg)
a_8 = Sorghum	x_8 = Glycemix Index (GI)

These characteristics are classified into two parts: cost attributes (0 attributes) and benefit attributes (8 attributes) (8 attributes).

FA	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
a_1	82.2 8	5.66	2.80	0.66	378.00	33.00	1.80	44.19
a_2	73.0 0	4.60	2.80	9.20	358.00	26.00	2.70	72.00
a_3	76.0	2.70	1.00	7.90	362.00	33.00	1.80	91.00

	0							
a_4	71.0 0	2.00	2.00	11.60	348.00	30.00	3.50	68.00
a_5	38.0 6	0.28	0.90	1.36	180.00	33.00	30.00	96.46
a_6	85.0 0	0.20	0.50	0.70	353.00	10.00	1.20	51.00
a_7	17.4 7	0.09	2.50	2.02	77.00	11.00	1.00	67.71
a_8	70.7 0	3.10	2.00	10.40	329.00	25.00	5.40	32.00

Table 3.1: Food Firm.

FA	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
a_1	82.28	5.66	2.80	0.66	378.00	33.00	1.80	44.19
a_2	73.00	4.60	2.80	9.20	358.00	26.00	2.70	72.00
a_3	76.00	2.70	1.00	7.90	362.00	33.00	1.80	91.00
a_4	71.00	2.00	2.00	11.60	348.00	30.00	3.50	68.00
a_5	38.06	0.28	0.90	1.36	180.00	33.00	30.00	96.46
a_6	85.00	0.20	0.50	0.70	353.00	10.00	1.20	51.00

a_7	17.47	0.09	2.50	2.02	77.00	11.00	1.00	67.71
a_8	70.70	3.10	2.00	10.40	329.00	25.00	5.40	32.00
$\sum x_{ij}$	513.51	18.63	14.5	43.84	2385.00	201.00	47.4	522.36

Table 3.2: Addition of each column

Find the normalized values for each column.

FA	NV for x_1	NV for x_2	NV for x_3	NV for x_4	NV for x_5	NV for x_6	NV for x_7	NV for x_8
a_1	0.16	0.30	0.19	0.02	0.16	0.16	0.04	0.08
a_2	0.14	0.25	0.19	0.21	0.15	0.13	0.06	0.14
a_3	0.14	0.14	0.07	0.18	0.15	0.16	0.04	0.17
a_4	0.07	0.11	0.14	0.26	0.15	0.15	0.07	0.13
a_5	0.07	0.02	0.06	0.03	0.08	0.16	0.63	0.18
a_6	0.17	0.01	0.03	0.02	0.15	0.05	0.03	0.10
a_7	0.03	0.00	0.17	0.05	0.03	0.05	0.02	0.13
a_8	0.14	0.17	0.14	0.24	0.14	0.12	0.11	0.06

Table 3.3: Normalized Value

Determine the information entropy associated with the attribute u_j .

FA	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
a_1	-0.08731	-0.24918	-0.11441	-0.00511	-0.08731	-0.08731	-0.01243	-0.03167
a_2	-0.07121	-0.18034	-0.11441	-0.13456	-0.07907	-0.06372	-0.02133	-0.07121
a_3	-0.07121	-0.07121	-0.02632	-0.10497	-0.07907	-0.08731	-0.01243	-0.09594
a_4	-0.02632	-0.04984	-0.07121	-0.19301	-0.07907	-0.07907	-0.02632	-0.06372
a_5	-0.02632	0.00511	-0.02133	-0.00856	-0.01243	-0.08731	-1.36353	-0.10497
a_6	-0.09594	-0.00217	0.00856	-0.00511	0.07907	-0.01669	-0.00856	-0.04343
a_7	-0.00856	0	-0.09594	-0.01669	-0.00856	-0.01669	-0.00511	-0.06372
a_8	-0.07121	-0.09594	-0.07121	-0.16817	-0.07121	-0.05660	-0.04984	-0.02133
E_j	0.22029	0.31440	0.25169	0.30593	0.58064	0.23789	0.72112	0.23852
d_j	0.77971	0.8656	0.74831	0.69407	0.41936	0.76211	0.27888	0.76148
W_j	0.14685	0.16303	0.14094	0.13072	0.07898	0.14354	0.05252	0.14342

Table 3.4: EntropyWeight

Obtain the entire attribute values by utilizing definition 2.3 of the alternative x_j .

FA	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	$z_1(w)$
a_1	0.16	0.30	0.19	0.02	0.16	0.16	0.04	0.08	0.15098
a_2	0.14	0.25	0.19	0.21	0.15	0.13	0.06	0.14	0.16928
a_3	0.14	0.14	0.07	0.18	0.15	0.16	0.04	0.17	0.13807
a_4	0.07	0.11	0.14	0.26	0.15	0.15	0.07	0.13	0.13793
a_5	0.07	0.02	0.06	0.03	0.08	0.16	0.63	0.18	0.11411
a_6	0.17	0.01	0.03	0.02	0.15	0.05	0.03	0.10	0.06838
a_7	0.03	0.00	0.17	0.05	0.03	0.05	0.02	0.13	0.06414
a_8	0.14	0.17	0.14	0.24	0.14	0.12	0.11	0.06	0.14204

Table 3.5: WeightVector

Range of entire alternatives $x_j (j = 1, 2, 3, \dots, 8)$ according to $z_i(w)$
 $a_2 > a_1 > a_8 > a_3 > a_4 > a_5 > a_6 > a_7$

4 Result for Entropy

Based on the above evaluation of entropy methods shows a_2 will be the best option for food choice.

4.1 Algorithm for Maximax Method

Step 1: First, place the attribute in the appropriate category.
 Beneficiary Attributes
 Price Attributes

Step 2: Apply any suitable normalising technique. Here, we're employing a method called "Linear scale transformation (Sum)," which uses normalised values for Beneficiary characteristics. Values for cost attributes that are normalised

Normalized value for benefit attributes

$$\mu_m^{NV} = \frac{c_{lm}}{\sum c_{lm}}$$

Normalized value for cost attributes

$$\mu_m^{NV} = \frac{1}{\sum \frac{1}{c_{lm}}}$$

Step 3: After obtaining the normalised decision matrix. Find the highest number for each row.

Step 4: Examine the highest value in the group of highest values. then discovered the ideal alternative.

4.2 Numerical Example

Here we using the same problem data to evaluate the maximax method and identify the best alternative.

FA	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
a_1	82.28	5.66	2.80	0.66	378.00	33.00	1.80	44.19
a_2	73.00	4.60	2.80	9.20	358.00	26.00	2.70	72.00
a_3	76.00	2.70	1.00	7.90	362.00	33.00	1.80	91.00
a_4	71.00	2.00	2.00	11.60	348.00	30.00	3.50	68.00
a_5	38.06	0.28	0.90	1.36	180.00	33.00	30.00	96.46
a_6	85.00	0.20	0.50	0.70	353.00	10.00	1.20	51.00
a_7	17.47	0.09	2.50	2.02	77.00	11.00	1.00	67.71
a_8	70.70	3.10	2.00	10.40	329.00	25.00	5.40	32.00

Table 3.6: Food Firm

By using table 3.4

FA	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Max
a_1	0.16	0.30	0.19	0.02	0.16	0.16	0.04	0.08	0.30
a_2	0.14	0.25	0.19	0.21	0.15	0.13	0.06	0.14	0.25
a_3	0.14	0.14	0.07	0.18	0.15	0.16	0.04	0.17	0.18
a_4	0.07	0.11	0.14	0.26	0.15	0.15	0.07	0.13	0.26
a_5	0.07	0.02	0.06	0.03	0.08	0.16	0.63	0.18	0.63
a_6	0.17	0.01	0.03	0.02	0.15	0.05	0.03	0.10	0.17
a_7	0.03	0.00	0.17	0.05	0.03	0.05	0.02	0.13	0.17
a_8	0.14	0.17	0.14	0.24	0.14	0.12	0.11	0.06	0.24

Table 3.7: Analyze maximum value

Maxi Value is,

$$a_5 = 0.63$$

4.3 Result for Entropy

Based on the above evaluation of Maximax methods shows a_5 will be the best option for food choice.

5 Conclusion

Today's decision-making takes place in more complex situations. In the situation of incomplete information, the entropy and maximax technique involves a structured method for comparing and weighting multiple features, as well as alternatives to decision making. The challenge of being superior to others has yet to be solved. In this study, we examine the

meal options available and recommend the best alternatives. We can also use other algorithms such as AHP, Maximin, Maximax, TOPSIS, and so on.

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