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Article *in* Materials Research Innovations · August 2014

DOI: 10.1179/1432891714Z.000000000983

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When designing a gear reducer, there are many important factors to be considered, such as weight, size, strength, durability material and geometry. The material of the gear reducer has a key impact on its weight. In this paper, a two-stage gear reducer is optimised with major conflict functions like minimisation of gear material volume, minimisation of centre distance, maximisation of power and maximisation of efficiency as objectives with design stresses as the constraints. We have considered two different types of materials for this study. A new population-based evolutionary algorithm named selective breeding algorithm is considered to solve this design problem, with two different types of materials. In selective breeding algorithm, solutions are made to breed, mutate, sort and multiple better solutions are formed. Then fitness conditions are placed, best solutions are generated and results are compared with an existing design.

Keywords: Evolutionary algorithm, Gear material, Gear reducer, Optimisation, Volume reduction

Notation

H_s	Specific sliding velocity at start of approach action
H_t	Specific sliding velocity at end of recess action
P	Power transmitted (kW)
$P^{(L)}$, $P^{(U)}$	Lower and upper limit of power (kW)
R_0	Addendum circle radius of gear (mm)
r_0	Addendum circle radius of pinion (mm)
R	Pitch circle radius of gear (mm)
r	Pitch circle radius of pinion (mm)
K_c	Stress concentration factor, 1·5
K_d	Dynamic load factor, 1·1
r_i	Gear or transmission ratio
w_i	Speed of the wheel in rpm
a_i	Centre distance for the corresponding gear pair (mm)
b_i	Thickness of the gear pair (mm)
$b^{(L)}$, $b^{(U)}$	Lower and upper limit of thickness of the gear pair (mm)
y_i	Form factor
Z_{pi}, Z_{wi}	Number of teeth in pinion, gear
d_1, d_2	Pitch circle diameter of pinion and gear (mm)
m	Module (mm)
i	Gear pair
ρ	Density of the material (kg mm^{-3})
E	Young's modulus (N mm^{-2})
η	Percentage efficiency
P_L	Percentage of power loss
f	Average coefficient of friction, 0·08
Φ	Pressure angle in degrees, 20°
NW	Normalised weight, 0·25

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Introduction

In engineering design, the best possible design is achieved by comparing some alternative design solutions using previous problem information. Evolutionary algorithms offer efficient and systematic ways of creating and comparing new design solutions in order to generate an optimal design. Optimisation plays a vital role in various engineering design applications, to minimise the various values and to maximise the most significant desirable effect. Liu *et al.*[1] designed an artificial neural network modelling to evaluate the behaviour of zinc–aluminium alloy. Huang *et al.*[2] described a goal programming optimisation mathematical model in order to improve the efficiency of designing point-line meshing gears. Majid and Esmaile[3] evaluated an engineering optimisation problem with continuous design variables using two new, harmony search heuristic algorithms. Ebnonnasir *et al.*[4] used a new artificial neural network model to find the hardness of friction stir process on aluminium plate. Liu and Yin[5] proposed an optimisation topology for microstructure of composites with multi-objectives. Zhang *et al.*[6] proposed a response surface method to optimise the design of aluminium radiator extrusion die.

Gear reducer is a universally used mechanical element in various areas, like automobiles, machine tools, aircraft, etc. A single-speed gearbox has a gear pair with a constant speed ratio, contained in a separate casing and intended to reduce or increase the angular speed of the

 DOI 10.1179/1432891714Z.000000000983

output shaft as compared with that of input shaft. In a multi-speed gearbox, the speed of the output shaft is varied in discrete steps; there may be increased or decreased speeds, from the input speed. Li *et al.*[7] developed a multi-objective for gear transmission with design variables and choice restrictive constraints. Li *et al.*[8] designed a three-stage wind turbine gearbox and optimized it with genetic algorithm. Wu *et al.*[9] established a multi-objective optimisation model of the gear train with minimum quality and minimum centre distance of the gear using particle swarm optimisation algorithm.

Gear reducer design optimisation involves compact selection of gear geometry with suitable materials. Many researchers have proposed the solution for compact gearbox through various optimisation techniques. Savsani *et al.*[10] developed the particle swarm optimisation and simulated annealing heuristic algorithms to minimise the weight of a spur gear train. Mogal and Wakchaure[11] evaluated on worm and worm wheel with multiple objectives like minimise volume of worm and worm wheel and its centre distance. Buiga and Popa[12] proposed that gear materials have a key impact on the gear reducer's weight. Hence, we have considered two different types of materials. Vipin and Chauhan[13] discussed and minimised the surface fatigue life factor and volume of gearbox with classical sequential quadratic programming algorithm and other non-traditional NSGA-II with other geometric conditions. Padmanabhan *et al.*[14] used a modified immune system algorithm to optimise the design of helical gear pair with combined objective function to maximise the power, efficiency and minimise weight, centre distance for two different gear materials.

Evolutionary algorithm is the optimisation algorithm used to reduce the complexity of the design and it is a subset of evolutionary computation. It is also a generic population-based metaheuristic optimisation algorithm. Sriramya *et al.*[15] proposed a novel evolutionary algorithm based on selective breeding algorithm (SBA) for the bin packing application.

In this study, SBA is used to optimise a two-stage gear reducer with major conflict functions, like minimisation of gear material volume, minimisation of centre distance, maximisation of power and maximisation of efficiency as objectives with design stresses as the constraints.

Design optimisation

The major root cause for premature failure of many mechanical components like gears and gearboxes are due to improper design. In addition to premature failures, designers are focusing in designing compact gearboxes capable of transmitting maximum power with minimum weight and appreciable life. Consequently, a lot of optimisation works have been carried out in this area. But many works do not involve more than one objective. Evolutionary computation seems to be a promising approach to deal with the multi-objective optimisation problems with more realistic constraints.

This design optimisation involves the determination of optimum values for all the decision variables such as number of teeth, thickness, module and power. This would increase the power output and efficiency and also reduce the volume and centre distance of the gear reducer. The feasibility of a design solution depends on satisfaction of number of equality and inequality constraints. The resulting optimisation problem becomes more complicated due to the presence of multiple conflicting objectives.

In addition, the size and the power delivered would improve the performance of automobiles or machine tools. Hence, the present work is carried out to increase the power output and reduce the volume of the gear reducer.

To meet the above objective, a two-stage gear reducer test problem is adopted as follows: 'A two-stage gear spur gear reduction unit with 20° full depth involute teeth. The input shaft rotates at 1440 rev min^{-1} and receives 10 kW power through a flexible coupling. The speed of output shaft should be approximately 180 rev min^{-1}. All the gears are made of plain carbon steel 45C8.'

For the above problem, the objective functions and design constraints are as described below.

Objective function

Objective function is formed with design variables such as power, module, gear thickness and number of teeth, whose values determine the solution to the problem. The objective functions listed below were adopted from Deb and Jain:[16]

(i) Maximisation of power delivered by the gear reducer.
(ii) Minimisation of overall gear material used, which is directly related to the weight and cost of the gear reducer.
(iii) Maximisation of efficiency of the gear reducer.
(iv) Minimisation of centre distance between input and output shafts.

The various assumptions involved in the work are described below:

(i) All the gears in the gear reducer are spur gears.
(ii) The thicknesses of the gears are the same in a gear pair.
(iii) In a gear pair, wheel is assigned the larger number of teeth between the mating gears.

Equations (1)–(3) and (7) represent the above said objective functions

$$\text{Maximise } f_1 = P, \quad \text{where } P^{\mathrm{L}} \le P \le P^{\mathrm{U}} \tag{1}$$

$$\text{Minimise } f_2 = \frac{\pi}{4}\sum_{i=1}^{2} m^2(Z_{\mathrm{p}i}^2 + Z_{\mathrm{w}i}^2)b_i \quad \text{for } i = 1,\ 2 \tag{2}$$

$$\text{Maximise } f_3 = 100 - P_{\mathrm{L}} \tag{3}$$

$$P_{\mathrm{L}} = \frac{50f}{\cos\Phi}\times\frac{(H_{\mathrm{s}}^2 + H_{\mathrm{t}}^2)}{(H_{\mathrm{s}} + H_{\mathrm{t}})} \tag{4}$$

where $f = 0{\cdot}08$, $\Phi = 20°$

$$H_t = \frac{(r_i+1)}{r_i} \times \sqrt{\left(\left[\frac{r_0}{r}\right]^2 - \cos^2 \Phi\right)} - \sin \Phi \quad (5)$$

$$H_s = (r_i+1) \times \sqrt{\left(\left[\frac{R_0}{R}\right]^2 - \cos^2 \Phi\right)} - \sin \Phi \quad (6)$$

$$\text{Minimise } f_4 = \frac{m}{2}(Z_{wi} + Z_{pi}) \quad (7)$$

The power loss 'P_L' is calculated for each gear pairs from Dudley.[17]

Design constraints

For the gear reducer layout shown in Fig. 1, there are two gear pairs. There also exists a number of constraints associated with this problem. By considering fixed number of teeth Z_i and varying thickness values b_i and power delivered P, each gear must satisfy two constraints mentioned in equations (8) and (9), which were adopted from Deb and Jain.[16]

$$\text{Bending stress:} \quad \sigma_{bi} \le [\sigma_{bi}]_{al} \quad (8)$$

$$\text{Crushing stress:} \quad \sigma_{ci} \le [\sigma_{ci}]_{al} \quad (9)$$

where σ_b is the bending stress, σ_c is the crushing stress, $[\sigma_b]_{al}$ is the allowable bending stress and $[\sigma_c]_{al}$ is the allowable crushing stress.

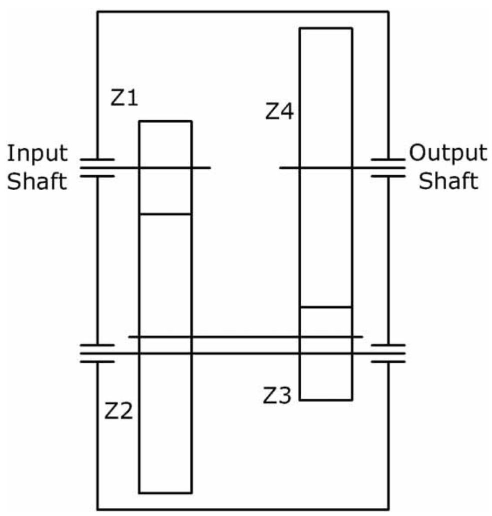
The bending and crushing stresses developed in the ith gear pair are calculated by equations (10) and (11).

Bending stress developed in the ith gear pair is:

$$\sigma_{bi} = \frac{97500\ PK_C K_d (r_i+1)}{w_i\, a_i\, b_i\, m\, r_i\, y_i \cos \Phi} \quad (10)$$

Crushing stress developed in the ith gear pair is:

$$\sigma_{ci} = \frac{0{\cdot}59(r_i+1)}{r_i a_i} \cdot \sqrt{\frac{97500\, PK_C K_d (r_i+1)E}{w_i b_i \sin 2\Phi}} \quad (11)$$



1 Layout of two-stage gear reducer

The transmission ratio is defined as the ratio of number of teeth Z_{wi} in wheel to the number of teeth Z_{pi} in pinion, and it is expressed by equation (12).

$$r_i = \frac{Z_{wi}}{Z_{pi}} \quad (12)$$

The centre distance for the corresponding gear pair is found by equation (13).

$$a_i = \frac{m(Z_{wi} + Z_{pi})}{2} \quad (13)$$

$$y_i = 0{\cdot}52\left(1 + \frac{20}{Z_{wi}}\right) \quad (14)$$

The thickness value should lie between lower and upper limit values, and these constraints are shown by the equation (15)

$$b^{(L)} \le b_i \le b^{(U)} \quad \text{for } i = 1,\ 2 \quad (15)$$

With the number of teeth in gears are also kept as decision variables, the resulting problem must involve additional constraints considering the following aspects.

The maximum gear ratio r^{max} in any gear pair must not exceed a limit. equation

$$\frac{Z_{wi}}{Z_{pi}} \le r^{max}, \quad \text{for } i = 1,\ 2 \quad (16)$$

The number of teeth in each gear pair should be integers and the value must be greater than its lower limit.

$$Z_{wi},\ Z_{pi} \ge Z^{(L)} \quad \text{for } i = 1,\ 2 \quad (17)$$

Since multiple criteria are on different scales, to reflect their actual contribution to the multiple-criterion objective function their values have to be normalised to the same scale. Hence, the combined objective function (COF) is adopted as

$$\text{COF} = \left[\left(\frac{\text{power}}{\text{max.power}} \times \text{NW}_1\right) + \left(\frac{\text{min.weight}}{\text{weight}} \times \text{NW}_2\right) + \left(\frac{\text{efficiency}}{\text{max.efficiency}} \times \text{NW}_3\right) + \left(\frac{\text{min.cent.dist}}{\text{cent.dist}} \times \text{NW}_4\right)\right] \quad (18)$$

where NW_1, NW_2, NW_3 and $\text{NW}_4 = 0{\cdot}25$.

There have been some efforts to solve this complex gearbox design problem with multiple objectives in the past. The classical optimisation technique was found to be difficult in solving more than one objective. The emergence of evolutionary computation techniques provides an easy way to arrive at optimal solutions. In this work, SBA has been employed and the results are compared with the existing model.

Selective breeding algorithm

Selective breeding is the procedure of breeding plants and animals for exacting character normally, strains that are selectively bred are cultivated and the breeding is sometimes done by an expert breeder. Bred animals are known as breeds, while bred plants are known as varieties, cultigens or cultivars. The cross of animals result is called a crossbreed, and crossbred plants are called hybrids. The term selective breeding is synonymous with artificial selection.[18] Sriramya *et al.*[15] proposed an algorithm named selective breeding algorithm (SBA), where the solutions are made to breed, mutate, sort and multiple better solutions are formed when the fitness conditions are input and best solution is generated. In this way, we can extract the best solution among them. The new SBA is described below.

Create an initial population of 'N' haploids (N is the size of population);

$C = 0$; (C is the number of cycles);

For each generation do;

$C = C + 1$;

Find the Objective function for each haploid;

Sort haploids based on objective function;

Divide the haploids into two equal sets based on sorting;

(first set named Dominant set H1 and remaining set named Recessive set h1)

Form the diploid set that contains one dominant and one recessive in order (H1h1, H2h2…etc.);

Do breeding process for all the combination diploid sets;

Perform fusion process for breed diploid set by interchanging genes between each diploid set;

Separate diploid set into haploids;

Do "In Breeding Depression" into haploids (add "B" percentage of new haploids);

Sort the haploids based on objective function and take 'N' number of best haploids for the next generation;

Place best one haploid from each generation in M (separate storage);

Do the above process up to C iterations;

Check the design constraints for each haploid in M;

Verified haploid with best objective function is an output;

The above algorithm performs as follows for a gear reducer problem:

(i) Initialise the required number of population, N (say 20) randomly which contains a string of design variables, such as power, module, gear thickness and number of teeth.
(ii) Find the COF for each string in the population and sort them based on COF.
(iii) Divide the population into two equal groups as Dominant (H) and remaining as Recessive (h).
(iv) Form the diploid set that contains one dominant and one recessive H1h1, H1h2, …, H3h4, …, H5h1, …, H5h5) and do breeding process for all the combinations of diploid sets (i.e. H1h1 × H1h2, …, H4h4 × H5h5), resulting in 190 diploid sets.
(v) Do the fusion process for the diploid sets, that is, selecting two design variables randomly for each generation and swap them between each diploid set. At the end of fusion process separate all diploid sets (190) into individual strings (380).
(vi) Newly generate 10% population and add it to the string pool. This is called as in-breeding depression. Hence, string pool length increased to 418 (380 + 38 = 418).
(vii) Find the COF for entire population and sort them based on COF. Take best strings of 'N' size (20) for next generation.
(viii) Store one best string into separate storage for each generation. Also, repeat the entire process till the required number of generations.
(ix) Verify the design constraints for the best strings stored from each generation and display the best one as optimal solution.

The above algorithm is developed using Microsoft Visual C#, using design parameters (P, m, b and Z_1) with boundary values as inputs. This program is designed to solve user-defined gear ratio, driver speed, pressure angle and gear material properties. For the two-stage gear reducer problem, SBA evaluated with a population size of 20 for 100 generations.

Results and discussion

The optimum values for all the decision variables such as the number of gear teeth, thickness, module and power are to be determined taking into consideration two different gear materials. This would increase the efficiency and power and also reduce the volume and centre distance of gear reducer. The test problem is solved by using alloy steel (40Ni 2Cr1 Mo28) and 45C8 as gear materials with SBA technique. Different gear materials have a vital impact on the gear reducer's weight as proposed by Buiga and Popa.[12] The test problem is solved in two cases. In Case I, module is kept constant and other variables like power, thickness and number of teeth are varied. While in Case II, all variables are varied to get the optimised results.

Constant module (Case I)

In this case the module is kept constant (module = 6 mm). The variables are power, gear thickness, number of teeth and centre distance. The optimised results are shown in Table 1.

Table 1 shows random feasible solutions (obtained from approximately 42 000 randomly created solutions by SBA) in the objective space for Case I. Selective

Table 1 Optimum results for constant module (Case I)

Design tool	No. of teeth		Gear thickness (mm)	Power (kW)	Volume (mm^3)	Eff. (%)	Centre dist. (mm)
Existing design method	$Z_1 = 18$	$Z_2 = 51$	$b_{1,2} = 60$	10	9 924 283	98·44	207
	$Z_3 = 18$	$Z_4 = 51$	$b_{3,4} = 60$				
SBA (C45)	$Z_1 = 18$	$Z_2 = 51$	$b_{1,2} = 49{\cdot}91$	10·16	8 255 349	98·44	207
	$Z_3 = 18$	$Z_4 = 51$	$b_{3,4} = 49{\cdot}91$				
SBA (Alloy steel)	$Z_1 = 17$	$Z_2 = 48$	$b_{1,2} = 41{\cdot}59$	10·52	6 879 182	98·36	195
	$Z_3 = 17$	$Z_4 = 48$	$b_{3,4} = 41{\cdot}59$				

Table 2 Optimum results by SBA for 45C8 and 40Ni 2Cr1Mo28 material

Gear material	Design tool	No. of teeth		Gear thickness (mm)	Module (mm)	Power (kW)	Volume (mm^3)	Eff. (%)	Centre dist. (mm)
45C8	Existing design method	$Z_1 = 18$	$Z_2 = 51$	$b_{1,2} = 60$	6	10	9 924 283	98·44	207
		$Z_3 = 18$	$Z_4 = 51$	$b_{3,4} = 60$					
	SBA	$Z_1 = 18$	$Z_2 = 51$	$b_{1,2} = 48·03$	5·5	10·23	6 675 493	98·44	189·75
		$Z_3 = 18$	$Z_4 = 51$	$b_{3,4} = 48·03$					
Alloy steel	Existing design method	$Z_1 = 18$	$Z_2 = 51$	$b_{1,2} = 50$	5	10	5 743 219	98·44	172·5
		$Z_3 = 18$	$Z_4 = 51$	$b_{3,4} = 50$					
	SBA	$Z_1 = 17$	$Z_2 = 48$	$b_{1,2} = 44·90$	4·75	10·37	4 126 250	98·36	154·375
		$Z_3 = 17$	$Z_4 = 48$	$b_{3,4} = 44·90$					

breeding algorithm runs with a population size of 20 for 100 generations. The independent runs have found very similar results.

In this case, module is kept constant. Power, gear thickness and number of teeth are varied. From the obtained optimum results, it is found that the number of gear teeth values does not vary much from one solution to another. Since Alloy steel has higher stress values when compared to 45C8, by changing the gear material from 45C8 to Alloy steel provides more room to reduce the gear thickness and number of teeth. This solution provides a wide difference when compared with the existing trial design method.

It is clearly understood from Table 1 that SBA shows around 16·8 and 30·6% volume reduction when compared with the existing method for 45C8 and alloy steel, respectively. Also, SBA yields a maximum of 1·6% power gain for 45C8 and 5·2% for alloy steel.

Varying all the considered design variables (Case II)

This problem can be made more flexible by allowing some additional parameters as decision variable. This would yield a better insight into the gear reducer design. In this case all the parameters such as power, thickness, number of teeth and module are considered as variables. Table 2 shows optimal solutions (obtained from approximately 42 000 randomly created solutions by SBA) in the objective space for Case-II. Selective breeding algorithm runs with a population size of 20 for 100 generations.

In this case, no variable is kept constant. Power, module, gear thickness, number of teeth and centre distance are varied. From Table 2, it is observed that the number of teeth does not vary much from one solution to another. When compared with the trial method, SBA shows further reduction in the module and the number of teeth with a slight increase in the power transmission. Also, when compared with Case I, Case II does not show a noticeable variation in the gear thickness because of gear module reduction.

By varying all design parameters such as power, thickness, number of teeth and module, it is obvious from Table 2 (Case II) that SBA performs well and shows a huge reduction in the gear volume when compared to the trial method. 45C8 and alloy steel volume is reduced to around 32 and 28%, respectively. Also, SBA yields a maximum of 2·3% power gain for 45C8 and 3·7% for alloy steel when compared to the existing method.

In both the cases, alloy steel showed better results among the two materials. The results show more than 50% volume reduction when compared with 45C8. It also shows a 5% increase in power and 25% reduction in centre distance when compared with the existing design. Since alloy steel has higher design values, it allows a wider selection range for the design variables in the population and also satisfies the design constraints.

Conclusion

Majority of mechanical engineering design involves extensive calculations and a number of non-linear, non-differentiable and multi-variable objective functions. Evolutionary algorithms like selective breeding algorithm can be effectively applied to arrive at the best solutions in an engineering design. In this work, a two-stage gear reducer was taken with two different gear materials. Optimised results were obtained with two different cases using selective breeding algorithm.

(i) In Case I, by keeping as constant module, selective breeding algorithm shows around 16·8 and 30·6% gear volume reduction when compared with the existing method for 45C8 and alloy steel, respectively. Also, selective breeding algorithm yields a maximum of 1·6% power gain for 45C8 and 5·2% for alloy steel.

(ii) By varying all design variables in Case II, selective breeding algorithm shows a huge reduction in the gear volume when compared to the trial method. 45C8 and alloy steel volume is reduced to around 32 and 28%, respectively. Also, selective breeding algorithm yields a maximum of 2·3% power gain for 45C8 and 3·7% for alloy steel.

(iii) Alloy steel shows better results among the two materials. The results show more than 50% volume reduction when compared with 45C8. It also shows a 5% increase in power and 25% reduction in centre distance when compared with the existing design.

Finally, it is seen that volume of gear reducer is decreased by 50% for alloy steel and 32% for 45C8. This is important as volume directly influences the cost of gear reducer. It also shows considerable increase in power when compared with the conventional design. Application of the evolutionary algorithm in the design of machinery and automobile gearbox helps gear manufacturers to strongly achieve sufficient environment for producing at a sensible cost. Selective breeding algorithm can used in a wide range of engineering design applications like multispeed gearbox, epicyclic gear train, springs and bearings etc.

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