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Refinements of Nash equilibrium in a Pentagonal Fuzzy Bimatrix Game

C.Karthi[1] and K.Selvakumari[2*]

[1]Research Scholar, Department of Mathematics,

[2]Professor, Department of Mathematics,

[1,2]Vels Institute of Science, Technology and Advanced Studies, Chennai, Tamilnadu, India.

karthiananya.r@gmail.com[1], selvafeb6@gmail.com[2]

ABSTRACT--In this paper, we introduce the concept of symmetric pentagonal fuzzy bimatrix and constant pentagonal fuzzy matrix for which we define the existence of Nash equilibrium in both pure and mixed strategies along with some numerical illustrations. By applying Ranking to the pay-offs, we convert the fuzzy valued game problem to crisp valued game problem. Therefore, Solution concept of perfect equilibrium for normal form game is reviewed and a new concept of proper equilibrium of pentagonal fuzzy bimatrix game is discussed.
Keywords-Bimatrix game, Nash equilibrium Approximation, fuzzy numbers, pentagonal fuzzy number, expected payoff, symmetric pentagonal fuzzy bimatrix, constant pentagonal fuzzy matrix, Defuzzification.

1. Introduction

In real world, people were overcoming lot of uncertainty in the day to day life. Fuzzy environment has a potential to solve such kind of uncertainty. Fuzzy set theory was introduced by L.A.Zadeh in the year 1965[13] which plays an vital role in predicting the solution for the problems, it involves in many fields namely medicine, engineering etc.

Depending on the nature of the impreciseness and problems in various applications, we are using different fuzzy numbers and interval valued fuzzy number. Pentagonal fuzzy number for the first time was used by Raj Kumar and T.Pathinathan [7].Also they developed the generalised concepts of pentagonal fuzzy number in 2015 along with the set theoretic operations. The concept of Equilibrium, as defined by John Nash [5] is one of the most important and elegant idea in game theory. Unfortunately a game can have many Nash Equilibria and some of the Equilibria may be inconsistent with our intuitive notions about what should be the outcome of a game.

Neumann and Morgenstern [12] invented the mathematical theory of games. Bimatrix games with PFN as payoffs have been discussed in many articles. Ranking of fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making .we use ranking functions to defuzzified the fuzzy numbers

In this paper, we consider the bimatrix game under pentagonal fuzzy environment and we focus on the solution method of Equilibria for the bimatrix games with PFN as payoffs and then the result achieved.

2. Preliminaries

A. Bimatrix game

A two person finite game in a strategic form which is defined as the matrix of ordered pairs is called a Bimatrix game. A **Bimatrix game** is a 2 player regular game where

Player 1 with a finite set of strategy $S = \{s_1, s_2, \ldots\ldots s_m\}$

Player 2 with a finite set of strategy$T = \{t_1, t_2, \ldots\ldots t_n\}$

When the pair of strategies (s_i, t_j) is choosen, the first player's payoff is a_{ij}=$u_1(s_i, t_j)$ and the second player's payoff is b_{ij}=$u_2(s_i, t_j)$ such that u_1, u_2 are called the **payoff functions**.

The outcomes of payoff values can be represented by a Bimatrix

Player 2

Player 1

Strategy	**t$_1$**	**t$_2$**	**…**	**t$_n$**
s$_1$	(a_{11}, b_{11})	(a_{12}, b_{12})	…	(a_{1n}, b_{1n})
s$_2$	(a_{21}, b_{21})	(a_{22}, b_{22})	…	(a_{2n}, b_{2n})
…	…	…	…	…
s$_m$	(a_{m1}, b_{m1})	(a_{m2}, b_{m2})	…	(a_{mn}, b_{mn})

The payoff matrix of Player 1 and Player 2 is

$$A = \begin{pmatrix} a_{11} & a_{12}\cdots & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{mn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12}\cdots & b_{1n} \\ b_{21} & b_{22} & b_{2n} \\ \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & b_{mn} \end{pmatrix}$$

Player 1's payoff representation is the first component of the ordered pairs and the player 2's payoff representation is the second component of the ordered pairs.

B. Nash Equilibrium

A Nash equilibrium also called strategic equilibrium, is a list of strategies one for each player, which has the property that no player can unilaterally change his strategy to get a better payoff. A Nash equilibrium for a game $\urcorner = (\tilde{x}, \tilde{y})$ is a Nash equilibrium for a Bimatrix game $\urcorner = (A, B)$ if

(i) For every mixed strategy x of the row player $x^T Ay \leq \tilde{x}^T A\tilde{y}$and
(ii) For every mixed strategy y of the column player$x^T By \leq \tilde{x}^T B\tilde{y}$

C. 2.3 Expected payoff

For a mixed strategy Nash equilibrium the **expected payoff** for that player is given by multiplying each probability in each cell by his/her respective payoff in that cell.

Therefore, The Expected payoffs are defined by the relations

Player 1: $\pi_1(p, q) = \Sigma\,\Sigma p_i q_j a_{ij}$where $i = 1\ tom$ and $j = 1ton$

Player 2: $\pi_2(p, q) = \Sigma\,\Sigma p_i q_j b_{ij}$where $i = 1tom$ and $j = 1ton$

D. Strictly dominating strategy

A strategy $a_i \in A_i$ is strictly dominated by $a_i \in A_i$if $U_i(a_i, a_{-i}) < U_i(a_i, a_{-i})$ ∀ $a_{-i} \in A_{-i}$set

E. Fuzzy set(13)

A fuzzy set Ã is defined by $\mu_{\tilde{A}}(x)$: R $\mu \rightarrow$[0, 1]Ã = $\{(x, \mu_A(x)): x \in A,\ \mu_A(x) \in [0,1]\}$

In the pair$(x, \mu_A(x))$, the first element x belong to the classical set A, the second element$\mu_A(x)$, belong to the interval [0,1] called Membership function.

F. Fuzzy number(13)

A fuzzy subset $\tilde{A}$ defined on R, is said to be a fuzzy number if its membership function $\mu_{\tilde{A}}$(x) satisfies the following conditions

- There exist at least one $x_0 \in R$, $\mu_{\tilde{A}}(x_0)$ =1
- $\mu_{\tilde{A}}$(x) is piecewise continuous
- $\tilde{A}$ must be normal and convex

G. Symmetric bimatrix games (12)

A 2-player Strategic game is said to be symmetric, if the pure strategy of the players are the same and the Player's payoff functions U_1 and U_2 are such that$U_1(S_1, S_2) = U_2(S_2, S_1)$

(i.e.) a symmetric game does not change when the players change roles.

Using the notation of bimatrix gmaes, an m x n Bimatrix game ┐ = (A, B) is symmetric if

m = n and $a_{ij} = b_{ji}$ for all j ∈ {1, 2,……n} or equivalently B = A^T

3. Pentagonal fuzzy number (1,7)

A Pentagonal Fuzzy number (PFN) of a fuzzy set A is defined as $A_P = \{a, b, c, d, e\}$, and its membership function is given by,

$$\mu_{A_P}(x) = \begin{cases} 0 \ for\ x < a, \\ \frac{(x-a)}{(b-a)} \ for\ a \le x \le b \\ \frac{(x-b)}{(c-b)} \ for\ b \le x \le c \\ 1 \quad x = c \\ \frac{(d-x)}{(d-c)} \ for\ c \le x \le d \\ \frac{(e-x)}{(e-d)} \ for\ d \le x \le e \\ 0 \quad x > e \end{cases}$$

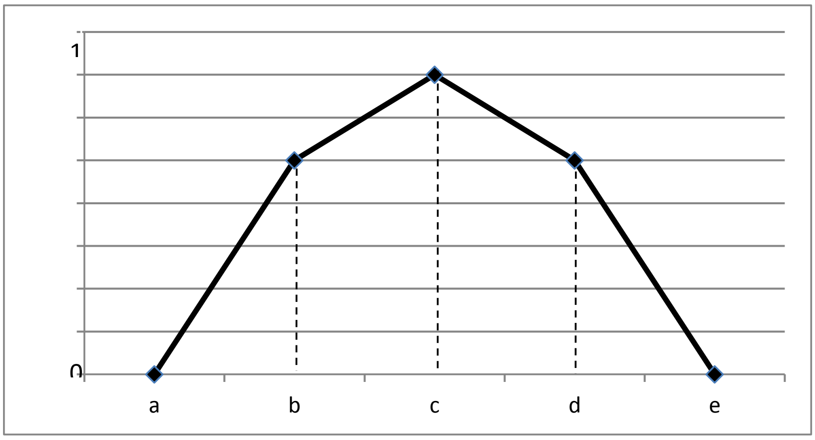

Fig. 1: Graphical representation of Pentagonal Fuzzy Number (PFN)

A. Constant pentagonal fuzzy matrix (CPFM)

A square PFM A =(a_{ij}) of order n x n is called a constant PFM if all the rows are equal to each other, i.e.,

$$(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}, a_{5ij}) = (a_{1rj}, a_{2rj}, a_{3rj}, a_{4rj}, a_{5rj}) \ \forall \text{ i, r, j}$$

4. Defuzzification

Defuzzification is an operation of translating an output fuzzy variable into a unique number (crisp)

Ranking function of a pentagonal number A (a, b, c, d, e) be a PFN then,

$$R\,(A) = \frac{2a+3b+2c+3d+2e}{4}$$

5. Numerical example: 1

Let us consider two students Suvetha and Anu participated in the dance competition. They are the top both contestants of this competition. Choreographers will judge based on the criteria listed below and will receive judges brief comments immediately following their performance.

The Bimatrix represent the scores in the form of PFN.

Table 1: Criteria

S.No	Criteria	Possible points
1.	Presentation skills(Body expression, Eye contact)	10
2.	Audience response to performance(Are people kidding or ignoring the performance)	10

Table 2: Anu

Suvetha	Anu		
		E_1	E_2
	M_1	P((1,3,6,7,9)(3,6,7,8,10))	P((4,5,7,8,9)(5,6,7,8,9))
	M_2	P((3,5,6,8,10)(2,3,7,8,8))	P((1,2,6,6,7)(2,4,5,7,8))

After defuzzification

Suvetha	Anu		
		E_1	E_2
	M_1	(14,20.5)	(19.75,21)
	M_2	(19,25,16.7)	(13,15.75)

By best reply response method,

Best reply of Player 1 to the strategic T of Player 2 is defined as the set

$$R_1(t) = \{s^* \in S;\ u_1(s^*,t) \geq u_1(s,t), \forall\, s \in S$$

Similarly Best reply of Player 2 to the strategy S of Player 1 is defined as

$$R_2\,(S) = \{t^* \in T;\ u_2(s,t^*) \geq u_2(s,t), \forall\, t \in T\}$$

(M_1, E_2) and (M_2,E_1) are the two pure Nash equilibrium points by best reply response method for the given pentagonal fuzzy bimatrix game.

Numerical Example: 2

Nowadays there are numerous of online marketing houses, namely Amazon, flipkart, snap deal, stub borne etc...Among these we consider two companies whose targeted aims are to increase their market shares under increasing demands of product in market. They are following the two categories strategy 1: COD/ CASH ON DELIVERY, Strategy 2: Net banking payment or Debit card payment. We are considering two companies as player 1 and 2 respectively. Here we use PFN as payoffs to represent such ambiguity of the data character.

Table 3: Company 2

Company 1	Company 2		
		T_1	T_2
	S_1	P((24,6,8,10),(1,3,6,9,11))	(0.5,0.3)
	S_2	P((1,3,5,7,9)(0,2,5,8,10))	(0.6,0.1)

Table 4: Solution

Company 1	Company 2		
		T_1	T_2
	S_1	(18,18)	(0.5,0.3)

	S_2	(15,15)	(0.6,0.1)

Therefore we have (18, 18) as the dominate strategy pure Nash equilibrium point.

Numerical Example: 3 Consider a pentagonal fuzzy bimatrix game and computing the equilibrium solution strategies.

Table 5: Player 2

		Player 2	
		b_1	b_2
Player 1	a_1	P((1,2,3,4,5)(0,1.5,3,4.5,5.5))	(0.7,0.2)
	a_2	P((1,2,2.8,4,5)(0,1.5,2.8,4.5,5.5))	(0.7,0.2)

After defuzzification, we obtain the bimatrix as

		Player 2		
		b_1	b_2	
Player1	a_1	(9,7.625)	(0.7,0.2)	p_1
	a_2	(8.9,8.65)	(0.7,0.2)	p_2
		q_1	q_2	

By Expected payoff method,

Expected payoff of a_1 and a_2 given by

$$E(a_1) = 9q_1 + 0.7q_2$$
$$E(a_2) = 8.9q_1 + 0.7q_2$$

$E(a_1) = E(a_2)$On simplification,

q_1 =0 Such that $q_1 + q_2 = 1$

- ⇨ $q_2 = 1$
- ⇨ Therefore (0,1) is a mixed Nash equilibrium point.

Similarly, (9, 7.625) is the only pure Nash equilibrium point.

Numerical Example: 4

Consider a 4×4 pentagonal fuzzy bimatrix game and computing Equilibria strategies.

		Player 2			
		b_1	b_2	b_3	b_4
Player 1	a_1	(P (7,8,9,10,11), (1,2,3,4,5))	(P(1,2,,3,4,5) ,(4,6,8,10,12))	P(4,5,6,7,8) (0,2,4,6,8))	(P(0,2,4,6,8) ,(1,2,3,4,5))
	a_2	(P(1,3,5,7,9)	(P(0,2,4,6,8)	P(0,1,2,3,4)	P((4,5,6,7,8)

		(1,2,3,4,5)	(1,3,5,7,9))	(0,2,4,6,8))	(3,5,7,8,9))
	a_3	P(0,1,2,3,4) (2,4,6,7,8))	P(1,2,4,5,6) (4,5,6,7,8))	P(1,2,3,5,6) (1,3,5,7,9))	P(1,2,3,4,5) (2,3,4,5,6))
	a_4	P(1,2,3,5,7) (2,4,6,7,8))	P(1,3,5,7,9) (2,3,4,5,6))	P(0,1,2,3,4) (0,2,4,6,8))	P(1,3,5,6,7) (1,2,3,4,5))

		Player 2			
		b_1	b_2	b_3	b_4
Player 1	a_1	(27,9)	(9,24)	(18,2)	(12,9)
	a_2	(15,9)	(12,15)	(6,12)	(18,19.25)
	a_3	(6,!6)	(10.5,18)	(10.25,15)	(9,12)
	a_4	(10.75,16)	(15,12)	(6,12)	(13.25,9)

By iterative elimination of strictly dominance property,

Since b_3 is strictly dominated by b_2 , therefore eliminating b_3 we have

		Player 2		
		b_1	b_2	b_4
Player1	a_1	(27,9)	(9,24)	(12,9)
	a_2	(15,9)	(12,15)	(18,19.25)
	a_3	(6,16)	(10.5,18)	(9,12)
	a_4	(10.75,16)	(15,12)	(13.25,9)

Since a_3 is strictly dominated by a_4 , therefore eliminating a_3 we have

		Player 2		
		b_1	b_2	b_4
Player1	a_1	(27,9)	(9,24)	(12,9)
	a_2	(15,9)	(12,15)	(18,19.25)
	a_4	(10.75,16)	(15,12)	(13.25,9)

Since b_1 is strictly dominated by b_2 , therefore eliminating b_1 we have

		Player 2	
		b_2	b_4

Player 1	a_1	(9,24)	(12,9)
	a_2	(12,15)	(18,19.25)
	a_4	(15,12)	(13.25,9)

Since a_1 is strictly dominated by a_2 , therefore eliminating a_1 we have

		Player 2	
		b_2	b_4
Player 1	a_2	(12,15)	(18,19.25)
	a_4	(15,12)	(13.25,9)

Therefore we have (a_2, b_4) and (a_4, b_2) are the two Nash equilibrium points for the given pentagonal fuzzy game problem.

Numerical Example: 5

Consider a constant symmetric pentagonal fuzzy matrix given by

$$A=\begin{pmatrix} (-1,0,1,2,4) & (0,1,2,4,5) & (1,2,3,4,5) \\ (-1,0,1,2,4) & (0,1,2,4,5) & (1,2,3,4,5) \\ (-1,0,1,2,4) & (0,1,2,4,5) & (1,2,3,4,5) \end{pmatrix}$$

After defuzzification, the matrix A is given by

$$A=\begin{pmatrix} (3.5) & (5) & (9) \\ (3.5) & (5) & (9) \\ (3.5) & (5) & (9) \end{pmatrix}$$

Since it is a symmetric PFM we have A=B^T

$$\text{Therefore, } B^T=\begin{pmatrix} (3.5) & (5) & (9) \\ (3.5) & (5) & (9) \\ (3.5) & (5) & (9) \end{pmatrix}$$

$$B=\begin{pmatrix} (3.5) & (5) & (9) \\ (3.5) & (5) & (9) \\ (3.5) & (5) & (9) \end{pmatrix}$$

Hence the obtained bimatrix is

$$(A, B)=\begin{pmatrix} (3.5,3.5) & (5,3.5) & (9,3.5) \\ (3.5,5) & (5,5) & (9,5) \\ (3.5,9) & (5,9) & (9,9) \end{pmatrix}$$

		Player 2		
		b_1	b_2	b_3
Player1	a_1	(3.5,3.5)	(5,3.5)	(9,3.5)
	a_2	(3.5,5)	(5,5)	(9,5)
	a_3	(3.5,9)	(5,9)	(9,9)

By best reply response method, all the points are pure Nash equilibrium points.

6. Result

For a constant symmetric pentagonal fuzzy bimatrix, all the components are only pure Nash equilibrium points, there doesn't exist mixed Nash equilibrium point for Constant symmetric pentagonal fuzzy bimatrix game.

7. Conclusion

In this paper, we have analysed the Equilibria of the Bimatrix game in both pure and mixed strategies under the pentagonal fuzzy environment. We have used Defuzzification technique to the payoff matrix to obtain crisp values of the game. Few numerical illustrations have been solved for the proposed method.

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