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O_f-IDEALS AND O_f-FILTERS IN GENERALIZED ALMOST DISTRIBUTIVE FUZZY LATTICES

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ABSTRACT. In this article, the concepts of O_f-ideals and O_f-filters of Generalized Almost Distributive Fuzzy Lattices (GADFL) is introduced. Further, some characterization of O_f-ideals, O_f-filters, maximal O_f-ideals and O_f-filters are discussed in GADFL. Also, homomorphism on O_f-ideals and O_f-filters is proved.

1. INTRODUCTION

The concept of Generalized Almost Distributive Lattices (GADFL) was introduced by G.C. Rao, Ravi KumarBandaru and N. Rafi [5] as a generalization of an Almost Distributive Lattices (ADLs) [6] which was a common abstraction of almost all the existing ring theoretic generalization of a Boolean algebra on one hand and distributive lattices on the other. On the other hand, L.A. Zadeh [7] in 1965 introduced the notion of fuzzy set. Again in 1971, L.A. Zadeh [8] defined a fuzzy ordering as a generalization of the concept of ordering, that is, a fuzzy

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ordering is a fuzzy relation that is transitive. In particular, a fuzzy partial ordering is a fuzzy ordering that is reflexive and anti symmetric. In 1994, N. Ajmal and K.V. Thomas [1] defined a fuzzy lattice as a fuzzy algebra and characterized fuzzy sub lattices. In 2009, I. Chon [4], considering the notion of fuzzy order of Zadeh, introduced a new notion of fuzzy lattices and studied the level sets of fuzzy lattices. He also introduced the notion of distributive and modular fuzzy lattices and considered some basic properties of fuzzy lattices.

In 2017, Assaye et al, [2] introduce the concept of an Almost Distributive Fuzzy Lattices (ADFLs) as a generalization of Distributive Fuzzy Lattices and Characterized some properties of an ADL using the fuzzy partial order relations and fuzzy lattices defined by I.Chon. Later on Assaye [3] introduce the concept of Generalized Almost Distributive Fuzzy Lattices (GADFLs) as a generalization of ADFLs.

In this article, the concepts of O_f-ideals and O_f-filters of Generalized Almost Distributive Fuzzy Lattices (GADFL) is introduced. Further, Some Characterization of O_f-ideals, O_f-filters, maximal O_f-ideals and O_f-filters are discussed in GADFL. Also, homomorphism on O_f- ideals and O_f-filters is proved.

2. Preliminaries

Some basic definitions are discussed.

Definition 2.1. [10] *Let $(R, \vee, \wedge)$ be an algebra type $(2, 2)$ and (R, A) be a fuzzy poset. Then we call (R, A) is a Generalized Almost Distributive Fuzzy Lattice if it satisfies the following axioms.*

1. $A\big((a \wedge b) \wedge c, a \wedge (b \wedge c)\big) = A\big(a \wedge (b \wedge c), (a \wedge b) \wedge c\big) = 1$
2. $A\big(a \wedge (b \vee c), (a \wedge b) \vee (a \wedge c)\big) = A\big((a \wedge b) \vee (a \wedge c), a \wedge (b \vee c)\big) = 1$
3. $A\big(a \vee (b \wedge c), (a \vee b) \wedge (a \vee c)\big) = A\big((a \vee b) \wedge (a \vee c), a \vee (b \wedge c)\big) = 1$
4. $A\big(a \wedge (a \vee b), a\big) = A\big(a, a \wedge (a \vee b)\big) = 1$
5. $A\big((a \vee b) \wedge a, a\big) = A\big((a, (a \vee b) \wedge a\big) = 1$
6. $A\big((a \wedge b) \vee b, b\big) = A\big(b, (a \wedge b) \vee b\big) = 1$ *for all* $a, b, c \in R$.

Example 1. *Let $R = \{a, b, c\}$. Define two binary operations $\vee$ and $\wedge$ on R as follows:*

$\vee$	a	b	c
a	a	b	a
b	b	b	b
c	c	c	c

$\wedge$	a	b	c
a	a	a	a
b	a	b	c
c	a	a	c

Define a fuzzy relation $A : R \times R \to [0, 1]$ *as follows:*
$A(a, a) = A(b, b) = A(c, c) = 1$,
$A(b, a) = A(b, c) = A(c, a) = A(c, b) = 0$,
$A(a, b) = 0.2$ *and* $A(a, c) = 0.4$.
Clearly (R, A) *is a fuzzy poset. Here* (R, A) *is a GADFL since it satisfies the above six axioms of a GADFL.*

Definition 2.2. [9] *Let* (R, A) *be a GADFL. A non empty subset* I *of* R *is said to be an ideal of* (R, A)*, if it satisfies the following conditions:*

1. *If* $x \in R,\ y \in I$ *and* $A(x, y) > 0$*, then* $x \in I$.
2. *If* $x, y \in I$ *then* $x \vee y \in I$.

Definition 2.3. [9] *Let* (R, A) *be a GADFL. A non empty subset* F *of* R *is said to be a filter of* (R, A)*, if it satisfies the following conditions:*

1. *If* $x \in R,\ y \in F$ *and* $A(y, x) > 0$*, then* $x \in F$.
2. *If* $x, y \in F$ *then* $x \wedge y \in F$.

Definition 2.4. *An Ideal* I *of* (R, A) *is called proper if* $I \neq R$. *A filter* F *of* (R, A) *is called proper if* $F \neq R$. *A proper ideal (filter)* P *of* R *is said to be prime, if for any* $x, y \in R$*,* $x \wedge y \in P(x \vee y \in P) \implies x \in P$ *or* $y \in P$. *It is clear that a subset* P *of* R *is a prime ideal iff* R_P *is prime filter.*

Definition 2.5. *A proper ideal* M *of a* (R, A) *is said to be maximal if it is not properly contained in any proper ideal of* (R, A). *For any ideal* I *of* (R, A)*,*
$M \subseteq I \implies either\ M = I\ (or)\ I = R$.

Definition 2.6. *A proper filter of a* (R, A) *is called a maximal filter of* R *if it is not properly contained in any proper filter of* R. *That is a proper filter* M *of* R *is called maximal filter, if for any filter* F *of* R*,* $M \subseteq F \implies M = F\ (or)\ F = R$.

3. O_f-IDEALS AND O_f-FILTERS IN GADFL

In this section, the concept of O_f-ideals and O_f-filters are introduced in a GADFL and some theorems of these O_f-ideals and O_f-filters are studied. Also deals with the theorems of maximal O_f-ideals and O_f-filters.

Definition 3.1. *Let I be an ideal in GADFL of $L(R,A)$ is called an O_f-ideal if $I = O_f(F) = \{x \in R | A(x \wedge s, 0) > 0 \ \forall \ s \in F\} = U_{x\in F}(x]^*$ for some filter of $L(R,A)$.*

Definition 3.2. *Let F be a filter in GADFL of $L(R,A)$ is called an O_f-filter if $F = O_f(I) = \{x \in R | A(x \vee s, 0) > 0 \ \forall \ s \in I\} = \cup_{x\in I}[x)^*$ for some ideal of $L(R,A)$.*

Example 2. *Let $L(R,A) = \{0,a,b,c,1\}$ be a GADFL whose Hasse diagram is given in the following Figure 1.*

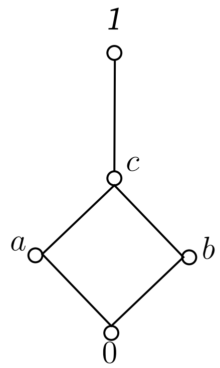

FIGURE 1. Hasse diagram of the GADFL $L(R,A) = \{0,a,b,c,1\}$

Consider $I = \{0,a\}$ and $I_1 = \{0,a,c,1\}$. Clearly I and I_1 are ideals of $L(R,A)$ also $F = \{0,b\}$ and $F_1 = \{0,b,c,1\}$ are filters of $L(R,A)$. therefore $O_f(F) = \cup_{x\in F}[x)^* = \{0,a\} = I = A(I, O_f(F)) > 0$ and $O_f(I) = \cup_{x\in I}[x)^* = \{0,b\} = F = A(F, O_f(I)) > 0$. Clearly $L(R,A)$ is a fuzzy poset, and if it satisfies the GADFL conditions. Therefore, I and F are O_f- ideal and O_f-filter in GADFL.

Theorem 3.1. *Let $L(R,A)$ be a GADFL and F be a filter of $L(R,A)$. Then the set $F^* = \{x \in R | A(x \wedge s, 0) > 0 \ \forall \ s \in F\}$ is an O_f-ideal of $L(R,A)$.*

Proof. Let $L(R,A)$ be a GADFL and F be a filterof $L(R,A)$. By the definition of filter, subsequent conditions.

$x, y \in F \implies x \wedge y \in F$ and $x \in F$, $y \in R \implies x \vee y \in F$. The Set F^* is defined by $F^* = \{x \in R | A(x \wedge s, 0) > 0 \; \forall \; s \in F\}$. Therefore we get $A(x \wedge s, 0) > 0$ (x is a dense element)

$\implies A(0 \wedge s, 0) > 0$

$\implies A(0, 0) > 0$. Clearly $0 \in F^*$.

Therefore F^* is non - empty.

Let $x, y \in F^*$ then we get $A(x \wedge s, 0) > 0$ and $(y \wedge 0, 0) > 0 \; \forall \; s \in F$. Therefore every $x_{-i} \in F$ such that $\{A((\vee_{i=1}^{n} x_i) \wedge s, 0) > 0 \; \forall \; s \in F\}$

$\implies F^* = \cup_{x \in F}\{A((\vee_{i=1}^{n} x_i) \wedge s, 0) > 0 \; \forall \; s \in F\}$. Hence $F^* = \cup_{x \in F}(x]^*$ is an O_f-ideal of $L(R, A)$ be a GADFL. □

Theorem 3.2. *Let $L(R, A)$ be a GADFL and I be an ideal of $L(R, A)$. Then the set $I^* = \{x \in R | A(x \vee s, 0) > 0 \; \forall \; s \in I\}$ is an O_f- filter of $L(R, A)$.*

Proof. Let $L(R, A)$ be a GADFL I be an ideal of $L(R, A)$. By the definition of filter, subsequent conditions. $x, y \in I \implies x \vee y \in I$ and $x \in I, y \in R \implies x \wedge y \in I$. The Set I^* is defined by $I^* = \{x \in R | A(x \vee s, 0) > 0 \; \forall \; s \in I\}$. Therefore we get $A(x \vee s, 0) > 0 \implies A(0 \vee s, 0) > 0 \implies A(0, 0) > 0$. Clearly $0 \in I^*$ is non - empty.

Let $x, y \in I^*$. Then we get $A(x \vee s, 0) > 0$ and $(y \vee s, 0) > 0 \; \forall \; s \in I$. Therefore every $x_i \in I$ such that $\{A((\wedge_{i=1}^{n} x_i) \vee s, 0) > 0, \; \forall \; s \in I\}$. This implies $I^* = \cup_{x \in I}\{A((\wedge_{i=1}^{n} x_i) \vee s, 0) > 0 \; \forall \; s \in I\}$. Hence $I^* = \cup_{x \in I}[x)^*$ is an O_f-filter of $L(R, A)$ be a GADFL. □

Lemma 3.1. *For any filter F of a GADFL $L(R, A)$, $O_f(F)$ is an O_f- ideal in $L(R, A)$.*

Proof. Clearly $0 \in O_f(F)$. Let $x, y \in O_f(F)$. Then $A((a \wedge s), 0) = A((b \wedge u), 0) > 0$ for some $s, u \in F$. Now

$$A((a \vee b) \wedge (s \wedge u), 0) > 0 \implies A\Big(\big((a \wedge s \wedge u) \vee (b \wedge s \wedge u)\big), 0\Big) > 0$$

$$\implies A(((0 \wedge u) \vee (b \wedge u \wedge s)), 0) > 0 \text{ (since a} \wedge \text{s} = \text{b} \wedge \text{u} = 0)$$

$$\implies A\big((0 \vee (s \wedge 0)), 0\big) > 0 \implies A((0 \vee 0), 0) > 0 \implies A(0, 0) > 0$$

Hence $A((a \vee b), 0) \in O_f(F)$. Again, Let $a \in O_f(F)$ and $x \in R$. Then $A((a \wedge s), 0) > 0$ for some $s \in F$. Now

$$A((a \wedge x) \wedge s, 0) > 0 \implies A\big((x \wedge a \wedge s), 0\big) > 0$$

$$\implies A\big((x \wedge 0), 0\big) > 0 \implies A(0, 0) > 0.$$

So, $A\big((a \wedge x), 0\big) \in O_f(F)$. Thus $O_f(F)$ is an O_f-ideal in $L(R, A)$. □

Definition 3.3. *An O_f-ideal $M \subseteq L$ is said to be a maximal O_f-ideal there exist any O_f-ideal I in $L(R, A)$ such that $M \not\subset I$. Every proper O_f-ideal of $L(R, A)$ is contained in the maximal O_f-ideal.*

Theorem 3.3. *If $L(R, A)$ is a GADFL contained the maximal O_f-ideal. Then every proper O_f-ideal of R is contained in a maximal O_f-ideal of R.*

Proof. Let $L(R, A)$ is a GADFL contained the maximal O_f-ideal. Therefore, there exist any O_f-ideal I in $L(R, A)$ such that $M \not\subset I$ not smaller than any other element in R. Let M of R is a proper O_f-ideal it contained a maximal element. This implies M of R does not contained in any proper O_f-ideal of R. Therefore, M is a maximal of O_f-ideal of GADFL. Hence every properO_f-ideal of R is contained in a maximal ideal M. □

Definition 3.4. *An O_f- filter $M \subseteq F$ is said to be a maximal O_f-filter, there exist any O_f-filter F in $L(R, A)$ such that $M \subseteq F$, if it is not properly contained in any proper O_f-filter of $L(R, A)$.*

Theorem 3.4. *Let $L(R, A)$ is a GADFL. Then every proper O_f- filter of $L(R, A)$ is contained in a maximal O_f- filter.*

Proof. Let $L(R, A)$ is a GADFL contained in the maximal O_f-filter. Therefore, there exist any O_f-filter F in $L(R, A)$ such that $M \subseteq F$, greater than any other element in R. Let M of R is a proper O_f-filter it not contained a maximal element. This implies M of R is contained any proper O_f-filter of R, M is a maximal O_f-filter of GADFL. Hence every proper O_f-filter of R is contained in a maximal O_f- filter M. □

Definition 3.5. *For any filter F is a GADFL $L(R, A)$, the set $F^* = \{x \in R | A(x \wedge s, 0) > 0 \ \forall \ s \in F\}$ is an O_f-ideal of $L(R, A)$. It is called an annihilator O_f-ideal of $L(R, A)$.*

In the following theorems we proved the characterization of O_f-ideals and O_f-filters in GADFL.

Theorem 3.5. *Let $L(R, A)$ be a GADFL. Then the following are equivalent.*

(1) $L(R, A)$ is a GADFL.
(2) Every ideal is an O_f-ideal.
(3) Every annihilator ideal is an O_f-ideal.

(4) For $x \in L(R, A)$, I is an O_f-ideal.

Proof.

$(1) \implies (2)$: Assume that $L(R, A)$ is a GADFL. Let I be an ideal of $L(R, A)$. Consider the set $I^* = \{x \in R | A(x \wedge s, 0) > 0 \; \forall \; s \in I\}$. We first prove that I^* is a filter of $L(R, A)$. Since I^* is a filter. Clearly $\phi = F \subseteq I^*$. Let $x, y \in I^*$ then $A(x \wedge s, 0) > 0$ and $A(y \wedge s, 0) > 0 \; \forall \; s \in I$. Since $L(R, A)$ is a GADFL.

$\therefore$ We get $A(0, x \wedge s) > 0$ and $A(0, y \wedge s) > 0 \; \forall \; s \in I$.

This implies $x \wedge s = 0$ and $y \wedge s = 0 \; \forall \; s \in I$

$$A((x \vee y) \wedge s, 0) = A((x \wedge s) \vee (y \wedge s), 0) = A((0 \vee 0), 0) = A(0, 0) > 0$$

$\therefore$ We get $x \vee y \in I^*$.

Again let $x \in I^*$ and $r \in R$. Then we get $A(x \wedge s, 0) > 0$ and $A(r \wedge s, 0) > 0$

$$\implies A((x \vee r) \wedge s, 0) = A((x \wedge s) \vee (r \wedge s), 0) = A((0 \vee 0), 0) \; = A(0, 0) > 0$$

$\therefore$We get $x \vee r \in I^*$. $\therefore$ I^* is a filter of $L(R, A)$. We now show that $I = O_f(I^*)$. Let $x \in O_f(I^*)$. Then $x \wedge f = 0$ for some $f \in I^*$. Hence $x \in (f^*]$.

Now $f \in I^* \implies A(x \wedge f, 0) > 0$ for some $x \in I \implies A(0, 0) > 0 \; \forall \; x \in I$.

$\therefore$ We get $x \wedge f \in I$. Therefore

$$O_f(I^*) \subseteq I. \tag{3.1}$$

Conversely, let $x \in I$ since $L(R, A)$ be a GADFL there exist $y \in R$ such that $\{y \in R | A(y \wedge x, 0) > 0 \; \forall \; x \in I\}$. Since $x \in I$, we get $y \in I^*$. Also $x \in I^* = y$. Hence $x \in O_f(I^*)$. Thus

$$I \subseteq O_f(I^*). \tag{3.2}$$

Therefore from (3.1) and (3.2) we get $I^* = I$. Thus I is an O_f-ideal. Hence every ideal is an O_f-ideal.

$(2) \implies (3)$: Since every annihilator ideal is an ideal, it is clear (by the Definition 3.5).

$(3) \implies (4)$: Since I is an annihilator ideal, it is obvious.

$(4) \implies (1)$: Let $x \in R$, I is an O_f-ideal. This implies $I = O_f(F)$ for some filter F of $L(R, A)$. Let $x, y \in R$ since $A(x \wedge y, (x \wedge y) \wedge y) = A((x \wedge y) \wedge y, x \wedge y) = 1$, then $(x, x \wedge y) \in I$. Also $A(y \wedge y, y \wedge y) = 1$. Hence $(y, y) \in I$. Since I is an O_f-ideal on $L(R, A)(x \vee y, (x \wedge y) \vee y) \in I$. Hence $A((x \vee y) \wedge y, [(x \wedge y) \vee y] \wedge y) = A([(x \wedge y) \vee y] \wedge y, (x \vee y) \wedge y)) = 1$

$$\implies A((x \vee y) \wedge y, y \wedge y) = A(y \vee y, (x \vee y) \wedge y) = 1$$

$$\implies A((x \vee y) \wedge y, y) > 0 \text{ and } A(y, (x \vee y) \wedge y) > 0$$

$\therefore$ $L(R,A)$ is an almost distributive fuzzy lattice. Clearly, it satisfies all the conditions of GADFL. Hence $L(R,A)$ is a GADFL. Hence proved. $\square$

Theorem 3.6. *The following conditions are equivalent in GADFL $L(R,A)$*

(1) $L(R,A)$ *is GADFL.*
(2) *For any two filters F_1 & F_2 of $L(R,A)$, $A(f_1 \vee F_2, O_f(F_1) \vee O_f(F_2)) > 0$*
(3) *For any two filters F_1 & F_2 of $L(R,A)$,$A(O_f(F_1) \vee O_f(F_2), O_f(F_1 \vee F_2) > 0$*
(4) *$I_0(L)$ is a fuzzy sub lattice of $I(L)$*

Proof.

$(1) \implies (2)$: Assume that $L(R,A)$ is a GADFL. Let F_1, F_2 be two filters of $L(R,A)$ such that $F_1 \vee F_2$ in $L(R,A)$. Let $f_1 \in F_1$ and $f_2 \in F_2 \implies f_1 \wedge s = 0$ and $f_2 \wedge s = 0$, we get $A(f_1 \wedge s, 0) > 0$ and $A(f_2 \wedge s, 0) > 0$ for some $s \in F$.

$\therefore A((f_1 \vee f_2) \wedge s, 0) = A((f_1 \wedge s) \vee (f_2 \wedge s)), 0)) = A((0 \vee 0), 0) = A(0,0) > 0$

$\therefore f_1 \vee f_2 \in F_1 \vee F_2$ also $f_1^* \in O_f(F_1)$ and $f_2^* \in O_f(F_2)$

$\implies A(f_1^* \wedge s, 0) > 0$ and $A(f_2^* \wedge s, 0) > 0 \ \ \forall s \in F$.

Now $A(f_1^* \wedge f_2^*) \wedge s, 0) = A(f_1^* \wedge s) \vee (f_2^* \wedge s), 0) = A((0,0),0) = A(0,0) > 0$. Thus $f_1^* \vee f_2^* \in O_f(F_1) \vee O_f(F_2)$. Hence $A(F_1 \vee F_2,\ O_f(F_1) \vee O_f(F_2)) > 0$.

$(2) \implies (3)$: Assume that, let F_1, F_2 be two filters of $L(R,A)$ we have always

$$O_f(F_1) \vee O_f(F_2) \subseteq O_f(F_1 \vee F_2).$$

Let $x \in O_f(F_1 \vee F_2)$. Then $x \wedge a = 0$ for some $a \in F_1 \vee F_2$.

Now $a \in F_1 \vee F_2 \implies A(x \wedge (f_1 \vee f_2), 0) > 0$ where $f_1 \in F_1$ and $f_2 \in F_2$

$\implies A((x \wedge f_1) \vee (x \wedge f_2)), 0) > 0$

$\implies A(x \wedge f_1, 0) > 0$ and $A(x \wedge f_2, 0) > 0$

$\implies A(0,0) > 0$ and $A(0,0) > 0$. $\therefore x \in O_f(F_1) \vee O_f(F_2)$. Hence we get

$$O_f(F_1 \vee F_2) \subseteq O_f(F_1) \vee O_f(F_2).$$

Therefore (1) and (2) we get $O_f(F_1 \vee F_2) = O_f(F_1) \vee O_f(F_2)$ by antisymmetric property of A. Therefore $A(O_f(F_1 \vee F_2) = O_f(F_1) \vee O_f(F_2)) > 0$.

$(3) \implies (4)$: It is obvious.

$(4) \implies (1)$: Assume that $I_0(L)$ is a fuzzy sub lattice of $I(L)$. Let $x, y \in L$ be such that $x \wedge y = 0$ suppose $(x]^* \vee (y]^* \neq L(R,A)$. Since $(x]^*, (y]^*$ are O_f-ideals, by hypothesis. We get that $(x]^* \vee (y]^*$ is a proper O_f-ideal. Let $x, y \in L(R,A)$ with least element 0. Now for any $x \in R, A(x \wedge y, 0) > 0 \implies A(0,0) > 0$. Therefore L is a fuzzy sub lattice of $I_0(L)$ and it is an almost distributive fuzzy lattice. Since $O_f(L) = L$. Therefore, $(L, \vee, \wedge, 0)$ is an almost distributive fuzzy lattices with least element 0. Clearly, it satisfies the conditions of GADFL. There $L(R, A)$ is a GADFL. Hence proved. □

4. HOMOMORPHISM OF O_f-IDEALS AND O_f-FILTERS

In this section, we define homomorphism of GADFLs as follows and we also prove the homomorphism of O_f-ideals and O_f-filters.

Definition 4.1. *Let $L = (R, A)$ and $K = (M, B)$ be two GADFLs and let f be a map from $L \to K$. Then f is said to be homomorphism from a GADFL to an GADFL K if the following axioms hold true:*

(1) $f(x \wedge_R y) = f(x) \wedge_M f(y)$ for all $x, y \in R$;
(2) $f(x \vee_R y) = f(x) \vee_M f(y)$ for all $x, y \in R$;
(3) $f(0_R) = 0_M$ where 0_R and 0_M are the zero of R and M respectively.

A homomorphism f from L to K is called epimorphism, if f is an on-to map from L to K.

Definition 4.2. *Let $L = (R, A)$ and $K = (M, B)$ be two GADFLs and let f be a homomorphism from L to K. The kernel of f is defined as follows $Kerf = \{x \in R | A(f(x), 0_M) > 0\}$.*

Theorem 4.1. *Let L and K be two GADFLs and $f : L \to K$ be an epimorphism. If $Kerf = \{0\}$, then f is a homomorphism.*

Proof. Assume that f is an onto and $Kerf = \{x \in R | A(f(x), 0_M) > 0\}$. Let F be a filter, then $A(f(x), 0) > 0$ and it follows that $A(x \wedge (a \vee x), 0) > 0$. Then either $A(x, 0) > 0$ or $A(a \vee x, 0) > 0$. Let $x \in f(F) \subseteq K$, since f is onto, there exist $y \in L$ such that $f(y) = x$,. Now $f(y) \in f(F) \implies A(f(y) \wedge f(s), 0) > 0'$ for some $s \in F$.

$\implies A(f(y \wedge s), 0) > 0' \implies y \wedge s \in Kerf(0)$
$\implies y \in F \implies x = f(y) \in f(F)$.

Hence f is a homomorphism. □

Theorem 4.2. *Let $L(R_1, A)$ and $L(R_2, A)$ be two GADF's and $f : R_1 \to R_2$ is a homomorphism between the GADFs.*

(1) If I is a maximal O_f-ideal then I^C is a maximal O_f-ideal of R_2 where $I^C = f^{(-1)}(I) = \{a \in R_1 | f(a) \in I \subset R_2\}$.
(2) $I \subseteq I_1 \implies I^C \subseteq I_1^C$.
(3) If $f : R_1 \to R_2$ is bijective homomorphism, for every $I \in I(A)$, then $I = (I^C)^C$.

Proof. Let $L(R_1, A)$ and $L(R_2, B)$ be two GADFL's and $f : A \to B$ is a homomorphism between the GADFLs.

1) If I is a maximal O_f-ideal of R_1 and $I^C = f^{(-1)}(I) = \{a \in R_1 | f(a) \in I \subseteq R_2\}$. Let I is an O_f-ideal every $a, b \in I$. Now

$$a, b \in I \implies A(a \wedge b, 0) > 0 \implies A(0, 0) > 0 \ (\because a \wedge b = 0) \implies a \wedge b \in I$$

$\therefore$ we get $f(a \wedge b) \in I^C \implies I^C = \cup_{a \in I}(a^*]$. Therefore $(a \wedge b) \in f^{(-1)}(I)$. If $a \in f^{(-1)}(I)$ and $x \in R_1 \implies f(a) \in I \subseteq R_2$ and $f(x) \in R_2$. Hence we get $(a \wedge x) = f(a) \wedge f(x) \in I^C$, Since I is a maximal O_f-ideal. Let $a \in f^{(-1)}(I), x \in R_1 \implies A(a \wedge s, 0) > 0$ and $A(x \wedge s, 0) > 0 \implies A(0, 0) > 0$. Therefore we get $(a \wedge x) \in f^{(-1)}(I) = I^C$. Hence $f^{(-1)}(I) = I^C$ is a maximal O_f-ideal of R_2.

2) $I \subseteq I_1$, then $f(I) \subseteq f(I_1)$ this implies the smallest O_f- ideal of $f(I)$ contained smallest O_f-ideal in $f(I_1)$. Therefore we get $I^C \subseteq I_1^C$.

3) Let $I^C = f^{(-1)}(I) = \{a \in R_1 | f(a) \in I \subset R_2\}$ this implies $f(I) \subseteq I^C$, we get

$$I \subseteq (I^C)^C \tag{4.1}$$

Assume $f : R_1 \to R_2$ is one to one homomorphism and $a \in (I^C)^C$. Then $f(a)$ belongs to smallest O_f-ideal of $f(I)$. This implies $f(a) = f(b) \wedge c$ for $a, b \in I$ and $x \in R_2$. Let f is onto there exists $x \in R_2$ such that $c = f(x)$ for some $c \in I$. Therefore $f(a) = f(b) \wedge f(x) = f(b \wedge x)$. Let f is one to one $a = b \wedge x$ for $b \in I$ (since $a \in I$), therefore we get,

$$(I^C)^C \subseteq I \tag{4.2}$$

From (4.1) and (4.2) we get $I = (I^C)^C$. Hence Proved. □

Theorem 4.3. *Let $L(R_1, A)$ and $L(R_2, B)$ be two GADFL's and $f : R_1 \to R_2$ is a homomorphism between the GADFLs.*

(1) If F is a maximal O_f-filter of A, then F^C is a maximal O_f-filter of R_2 where $F^C = f^{(-1)}(F) = \{a \in R_1 | f(a) \in F \subset R_2\}$.
(2) $F \subseteq F_1 \implies F^C \subseteq F_1^C$.

(3) If $f : R_1 \to R_2$ is bijective homomorphism, for every $F \in F(A)$, then $F = (F^C)^C$.

Proof. Let $L(R_1, A)$ and $L(R_2, B)$ be two GADFL's and $f : A \to B$ is a homomorphism between the GADFLs.

1. If F is a maximal O_f-filter of R_1 and $F^C = \{a \in R_1 | f(a) \in F \subseteq R_2\}$. Let F is an O_f-filter every $a, b \in F \implies (a \vee b) \in F$. $\therefore$ we get $f(a \vee b) \in F^C$ this implies $F^C = \cup_{a \in F}(a^*]$.
Therefore $(a \vee b) \in f^{(-1)}(F)$.

If $a \in f^{(-1)}(F)$ & $x \in R_1 \implies f(a) \in F \subseteq R_2$ & $f(x) \in R_2$. Hence $(a \vee x) = f(a) \vee f(x) \in F^C$, Since F is a maximal O_f-filter and f is homomorphism.Therefore we get $(a \vee x) \in f^{(-1)}(F) = F^C$. Hence $f^{(-1)}(F) = F^C$ is a maximal O_f-filter of R_2.

2. Let $F \subseteq F_1$, then $f(F) \subseteq f(F_1)$ this implies the smallest O_f-filter of $f(F)$ contained in smallest O_f-filter in $f(F_1)$ Since f is homomorphism. $\therefore$ we get $F^C \subseteq F_1^C$.

3. Let $F^C = f^{(-1)}(F) = \{a \in R_1 | f(a) \in F \subset R_2\}$ this implies $f(F) \subseteq F^C$, we get

$$F \subseteq (F^C)^C. \tag{4.3}$$

Assume $f : R_1 \to R_2$ is one to one homomorphism and $a \in (F^C)^C$. Then $f(a)$ belongs to smallest O_f-filter of $f(F)$. This implies $f(a) = f(b) \vee c$ for $a, b \in F$ and $x \in R_2$. Let f is onto there exists $x \in R_2$ such that $c = f(x)$ for some $c \in F$. Therefore $f(a) = f(b) \vee f(x) = f(b \vee x)$. Let f is one to one $a = b \vee x$ for $b \in F$ (since $a \in F$). $\therefore$ we get

$$(F^C)^C \subseteq F. \tag{4.4}$$

From (4.3) and (4.4) we get $F = (F^C)^C$. Hence Proved. □

5. Conclusion

In this article, we proved several properties of O_f- ideals and O_f- filters in GADFL. We also investigated the characterization of O_f-ideals and O_f-filters in GADFL. Finally, we conclude that our restults of a homomorphism of O_f-ideals and O_f-filters provide a bridge between GADL $\to$ GADFL. Future work, Congruence of Ideals and Filters in GADFL and also isomorphism of GADFL.

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