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Bi-conditional Cordial Labeling for Extended Duplicate Graph of Certain Classes of Graphs

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Bi-conditional Cordial Labeling for Extended Duplicate Graph of Certain Classes of Graphs

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Abstract –In this article we admit Bi- Conditional for Duplicate graph of Quadrilateral Snake $EDG(QS_m)m \geq 2$, ***Double Quadrilateral SnakeGraphs*** $EDG(DQS_m)m \geq 2$, ***and Triangular ladder graph.*** $EDG(TQL_m)m \geq 2$.

AMS Subject Classification: 05C78
Keywords: graph labeling, Extended Duplicate, labeling, Bi conditional labeling.

1. Introduction:

Murali, Thirusangu and Madura Meenakshi introduced new graph labeling called Bi-conditional cordial labeling [3]. Let G (N, L) be a graph. If there exist a mapping $g: N \rightarrow \{0,1\}$ such that the induced function $g^*(L) = \{0,1\}$ given by $g^*(uv) = \begin{cases} 1: \text{if } g(u) = g(v) \\ 0: \text{if } g(u) \neq g(v) \end{cases}$ for every $uv \in L(G)$, if the number of nodes labeled '0' and the number of nodes labeled '1' differ by at most '1' and also the number of lines labeled '0' and the number of lines labeled '1' differ by at most 1. Murali, Thirusangu, Balamurugan proved Combination Cordial Labeling for Flower and Corona Graphs [7]. Selvam, Thirusangu, they are proved Z_3-vertex and Z_3-edge magic total labeling for the duplicate graph of quadrilateral snake graph. [4]. Nandagopal, Maheswari and Vijayakumar existence on Signed product, sum difference, and Sum Divisor Cordial labeling for the Duplicate graphs of Quadrilateral snake Graph [5]. Ulaganathan, Selvam, and Vijayakumar proved the duplicate of Bistar, Double star and Triangular ladder graphs existence on Signed Product labeling [8].

Definition 1.1

Let G (N, L) be a simple graph. A duplicate graph of G is DG = (N_1, L_1), where the nodes set $N_1 = N \cup N'$ and $N \cap N' = \emptyset$ and g: $N \rightarrow N'$ is bijective (for $n \in N$, we write $g(n) = n'$ for convenience) and the lines set L_1 of DG is defined as: The line xy is in L if and only if both xy′ and x′y are lines in L_1.

Definition: 1.2

Let $DG = (N_1, L_1)$ be a duplicate graph of the path graph $G(N, L)$. we add an line between any one nodes from N to any other node in 'N'. For convenience, let us take $n_2 \in N$ and $n_2' \in N'$ and thus line(n_2, n'_2) is formed. This graph is called the Extended Duplicate Graph of the path P_k and it's denoted by $EDG(p_k)$.

Remarks:

1.3 The Extended Duplicate graph of Quadrilateral snake graph is denoted by $EDG(QS_m)m \geq 2$, and it has 6m+2 nodes and 8m lines. Where m is the connected path of the Quadrilateral graphs

1.4 The Extended Duplicate graph of Double Quadrilateral snake graph is denoted by

$EDG(DQS_m) m \geq 2$, and it has 10m+2 nodes and 14m lines. Where m is the connected path of the Double Quadrilateral graphs.

1.5 The Extended Duplicate graph of Triangular ladder graph is denoted by $EDG(TL_m) m \geq 2$, and it has 4m+4 nodes and 8m+2 lines. Where m is the number of rungs in the ladder.

2. Bi-conditional cordial labeling $EDG(QS_m)$

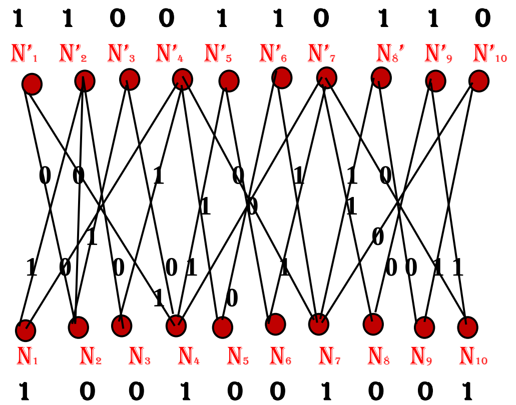
Algorithm 2.1: Allocation of labeled nodes

Let $N \to \{n_1, n_2 \dots \dots n_{3m+1}. n'_1, n'_2, \dots \dots . n'_{3m+1}\}$
$L \to \{l_1, l_2, l_3 \dots \dots, l_{4m}. l'_1, l'_2, l'_3 \dots \dots \dots . l'_{4m}\}$
For $1 \leq k \leq m$
$n_{3k-2} \leftarrow 1$ $n_{3k-1} \leftarrow 0$. $n_{3k} \leftarrow 0$
$n'_{3k+1} \leftarrow 0, n'_{3k-1} \leftarrow 1$
For $k = 3m + 1$
$n_k \leftarrow 1$
For $1 \leq k < m$
$n'_{3k+3} \leftarrow 1$
Fix $n'_1 \leftarrow 1, n'_3 \leftarrow 0$

Theorem 2.2: The extended duplicate graph of Quadrilateral Snake Graph $EDG(QS_m)$ $m \geq 2$, Bi-conditional cordial labeling admits.

Proof:
Let $N \to \{n_1, n_2, \dots \dots, n_{3m+1}. n'_1, n'_2, \dots \dots . n'_{3m+1}\}$, $L \to \{l_1, l_2, l_3 \dots \dots, l_{4m}. l'_1, l'_2, l'_3 \dots \dots \dots . l'_{4m}\}$ be the set of nodes and lines of the duplicate graph of Quadrilateral graph $EDG(QS_m)$. using the algorithm 2.1, each of the $3m + 1$nodes receive label 0 and 1 respectively. Using the induced function g^* defined by $g^*(uv) = \begin{cases} 1: \text{if } g(u) = g(v) \\ 0: \text{if } g(u) \neq g(v) \end{cases}$, the m lines namely $l_1, l_5, l_9, l_{13} \dots \dots l_{4m-3}$ receive label 1, the m lines namely $l_4, l_8, l_{12}, l_{16} \dots \dots l_{4m}$ label 1, the 1 line namely l_3 receive label 1. $m - 1$ lines namely $l'_5, l'_9, l'_{13}, l'_{17}, \dots \dots \dots \dots l'_{4m-3}$. receive label 1. $m - 1$ lines namely $l'_8, l'_{12}, l'_{16}, \dots \dots \dots \dots \dots \dots l'_{4m}$.receive label 1. One line namely l'_2 receive label 1.
Thus 4m lines receive label 1 and the m lines namely $l_2, l_6, l_{10}, l_{14}, \dots \dots \dots \dots \dots l_{4m-2}$. receive label 0, the $m - 1$ lines namely $l_7, l_{11}, l_{15}, l_{19}, , \dots \dots \dots \dots \dots l_{4m-1}$ receive label 0, $m - 1$ lines namely $l'_6, l'_{10}, l'_{14}, \dots \dots \dots \dots \dots \dots l'_{4m-2}$ receive label 0, m lines namely $l'_3, l'_7, l'_{11}, \dots \dots \dots \dots l'_{4m-1}$. receive label 0. 2 lines namely l'_1 and l'_4 receive label 0. Thus the $4m$ lines receive label0. Hence the duplicate graph of the Quadrilateral snake $EDG(QS_m)$ $m \geq 2$, Bi-Conditional Cordial labeling are admits.

2.3. Illustration:



Bi-conditional cordial labeling $EDG(QS_3)$

3. Bi-conditional cordial labeling EDG(DQS $_m$)

Algorithm 3.1: Allocation of labeled nodes

Case1: When m is Even
$For 1 \leq k \leq m$
$fix\, n_1 \rightarrow 1, \ \ n_1' \rightarrow 0$
do
{
$if\ 1 \leq k \leq \frac{m}{2}$
{
$g(n_{10k-8}) = 0.g\,(n_{10k-5}) = 1\ g(n_{10k-7}) = 0.g(n_{10k-4}) = 1\ g(n_{10k-6}) = 0.g(n_{10k-1}) = 1$
$g(n_{10k-3}) = 0.g(n_{10k}) = \ 1\ g(n_{10k-2}) = 0.$
}
End the process.
do
{
$if\ 1 \leq k < \frac{m}{2}$
{
$g(n'_{10k-8}) = 1g(n'_{10k-5}) = 0\ g(n'_{10k-7}) = 0g(n'_{10k-4}) = 0\ g(n'_{10k-6}) = 1g(n'_{10k-1}) = 1\ g(n'_{10k-3}) = 1g(n'_{10k}) = \ 0$
$g(n'_{10k-2}) = 1g(n'_{10k-5}) = \ 0$
}
End the process.
$For\ k = 5m + 1$
$n_k \rightarrow 1. n_k' \rightarrow 0$

Case2: When m is Odd
$For\ \ 1 \leq k \leq m$
do
{
$if\ 1 \leq k \leq \frac{m+1}{2}$
{
$g(n_{10k-8}) = 1.g(n_{10k-7}) = 0\ \ g(n_{10k-5}) = 1.g(n_{10k-6}) = 0\ \ g(n_{10k-9}) = \ 0.g(n_{10k-4}) = 0$
$g(n'_{10k-8}) = 1.g(n'_{10k-7}) = 1\ g(n'_{10k-5}) = 0.g(n'_{10k-6}) = 1\ g(n'_{10k-9}) = \ 0.g(n'_{10k-4}) = 1$
}
End the process.
do.
{
$if\ 1 \leq k \ < \frac{m+1}{2}$
{
$g(n_{10k-3}) = 1g(n_{10-2}) = 0\ \ g(n_{10k-1}) = 0\,g(n_{10k}) = 1\ \ g(n'_{10k-3}) = 1.g(n'_{10-2}) = 0\ g(n'_{10k-1}) = 1g(n'_{10k}) = 0$
}
End the process.

Theorem 3.2: The extended duplicate graph of Double Quadrilateral Snake Graph $EDG(DQS_m)$ $m \geq 2$, Bi-conditional cordial labeling admits.

Proof:
When m is Even
Let $N \to \{n_1, n_2 \dots\dots n_{5m+1}.\, n'_1, n'_2 \dots\dots. n'_{5m+1}\}$ $E \to \{l_1, l_2 \dots\dots, l_{7m}. l'_1, l'_2, \dots\dots. l'_{7m}\}$ be the set of nodes and lines of the duplicate graph of Double Quadrilateral graph $EDG(DQS_m)$. Using **case1**, each of the 7m lines receive label 1 and 0 respectively. Using the algorithm 3.1, the induced function

$$g^*(uv) = \begin{cases} 1: \text{if } g(u) = g(v) \\ 0: \text{if } g(u) \neq g(v) \end{cases}$$

The 7m lines receive label 1 as below
For i = 2 to m
{
do
{
$g^*(l_{12}) = g^*(l_{26}) = \dots = g^*(l_{7m-2}) \to 1/*\ upto \frac{m}{2} lines */$
$g^*(l_1) = g^*(l_{15}) = \dots = g^*(l_{7m-1}) \to 1/*\ upto \frac{m}{2} lines */$
$g^*(l_8), g^*(l_9) = g^*(l_{22}), g^*(l_{23}) = \dots = g^*(l_{7m-6}), g^*(l_{7m-5}) \to 1/*\ upto\ m\ lines */$
$g^*(l_5), g^*(l_6) = g^*(l_{19}), g^*(l_{20}) = \dots = g^*(l_{7m-9}), g^*(l_{7m-3}) \to 1/*\ upto\ m\ lines */$
$g^*(l'_1), g^*(l'_2) = g^*(l'_{15}), g^*(l'_{16}) = \dots = g^*(l_{7m-13}), g^*(l_{7m-12}) \to 1/*\ upto\ m\ lines */$
$g^*(l'_8), g^*(l'_9) = g^*(l'_{22}), g^*(l'_{23}) = \dots = g^*(l_{7m-6}), g^*(l_{7m-5}) \to 1/*\ upto\ m\ lines */$
$g^*(l'_{11}), g^*(l'_{12}) = g^*(l'_{25}), g^*(l'_{26}) = \dots = g^*(l_{7m-3}), g^*(l_{7m-2}) \to 1/*\ upto\ m\ lines */$
$g^*(l'_{13}) = g^*(l'_{27}) = \dots = g^*(l_{7m-1}) \to 1/*\ upto\ \frac{m}{2}\ lines */$
$g^*(l'_6) = g^*(l'_{20}) = \dots = g^*(l_{7m-3}) \to 1/*\ upto\ \frac{m}{2}\ lines */$
}
End the process.
Hence **7m lines received label 1,**
Similarly, the remaining 7m lines label 0 as below
{
do
{
$g^*(l_7) = g^*(l_{21}) = \dots = g^*(l_{7m-7}) \to 0/*\ upto \frac{m}{2} lines */$
$g^*(l_2), g^*(l_3), g^*(l_4), = g^*(l_{16}), g^*(l_{17}) g^*(l_{18}), \dots =$
$g^*(l_{7m-12}), g^*(l_{7m-11}) g^*(l_{7m-10}), \to 0/*\ upto\ \frac{3m}{2}\ lines */$
$g^*(l_{10}), g^*(l_{11}) = g^*(l_{24}), g^*(l_{25}) = \dots = g^*(l_{7m-4}), g^*(l_{7m-3}) \to 0/*\ upto\ m\ lines */$
$g^*(l_{13}), g^*(l_{14}) = g^*(l_{27}), g^*(l_{28}) = \dots = g^*(l_{7m-6}), g^*(l_{7m-7}) \to 0/*\ upto\ m\ lines */$
$g^*(l'_3), g^*(l'_4), g^*(l'_5) = g^*(l'_{17}), g^*(l'_{18}), g^*(l'_{19}) = \cdots$
$= g^*(l_{7m-11}), g^*(l_{7m-10}), g^*(l_{7m-9}) \quad \to 0/*\ upto\ \frac{3m}{2}\ lines */$
$g^*(l'_7) = g^*(l'_{14}) = \dots = g^*(l_{7m-7}) \to 0/*\ upto\ \frac{m}{2}\ lines */$
$g^*(l'_{10}), g^*(l'_{14}) = g^*(l'_{24}), g^*(l'_{28}) = \dots = g^*(l_{7m-4}), g^*(l_{7m}) \to 0/*\ upto\ m\ lines */$
}
End the process.

Hence 7mlines received label 0.

When m is Odd

Let $N \to \{n_1, n_2 \dots \dots n_{5m+1}, n'_1, n'_2, \dots \dots . n'_{5m+1}\} E \to \{l_1, l_2 \dots \dots, l_{7m}. l'_1, l'_2, \dots \dots . l'_{7m}\}$ be the set of nodes and lines of the duplicate graph of Double Quadrilateral graph $EDG(QS_m)$. Using **case2,** each of the 7m lines receive label 1 and 0 respectively. Using the induced function g^* defined by

$$g^*(uv) = \begin{cases} 1: \text{if } g(u) = g(v) \\ 0: \text{if } g(u) \neq g(v) \end{cases}$$

For i = 3 to m

{

do

{

$g^*(l_1), g^*(l_2) = g^*(l_{15}), g^*(l_{16}) =, \dots = g^*(l_{7m-6}), g^*(l_{7m-5}) \to 0 /*\ upto\ m+1\ lines\ */$

$g^*(l_3), g^*(l_6) = g^*(l_{17}), g^*(l_{20}) =, \dots = g^*(l_{7m-4}), g^*(l_{7m-1}) \to 0 /*\ upto\ m+1\ lines\ */$

$g^*(l_8) = g^*(l_{22}) = \dots = g^*(l_{7m-13}) \to 0 /*\ upto \frac{m-1}{2} lines\ */$

$g^*(l_{14}) = g^*(l_{28}) = \dots = g^*(l_{7m-7}) \to 0 /*\ upto \frac{m-1}{2} lines\ */$

$g^*(l'_1), g^*(l'_{15}) = g^*(l'_{29}) = \dots = g^*(l'_{7m-6}) \to 0 /*\ upto\ \frac{m+1}{2}\ lines\ */$

$g^*(l'_4), g^*(l'_{18}) = g^*(l'_{32}) = \dots = g^*(l'_{7m-3}) \to 0 /*\ upto\ \frac{m+1}{2}\ lines\ */$

$g^*(l'_6), g^*(l'_{20}) = g^*(l'_{34}) = \dots = g^*(l'_{7m-1}) \to 0 /*\ upto\ \frac{m+1}{2}\ lines\ */$

$g^*(l'_9), g^*(l'_{10}) = g^*(l'_{23}), g^*(l'_{24}) = \dots = g^*(l'_{7m-12}), g^*(l'_{7m-11}) \to 0 /*\ upto\ m-1\ lines\ */$

$g^*(l'_{13}) = g^*(l'_{27}) = \dots = g^*(l'_{7m-8}) \to 0 /*\ upto\ \frac{m-1}{2}\ lines\ */$

$g^*(l'_{11}), g^*(l'_{12}) = g^*(l'_{25}), g^*(l'_{26}) = \dots = g^*(l'_{7m-10}), g^*(l'_{7m-11}) \to 0 /*\ upto\ m-1\ lines\ */$

}

End the process.

Hence 7mlines receive label 0,

Similarly, the remaining 7m lines label 1 as below

{

do

{

$g^*(l_4), g^*(l_5) = g^*(l_{18}), g^*(l_{19}) =, \dots = g^*(l_{7m-3}), g^*(l_{7m-2}) \to 1 /*\ upto\ m+1\ lines\ */$

$g^*(l_9), g^*(l_{10}) = g^*(l_{23}), g^*(l_{24}) =, \dots = g^*(l_{7m-12}), g^*(l_{7m-11}) \to 1 /*\ upto\ m-1\ lines\ */$

$g^*(l_7) = g^*(l_{21}) = \dots = g^*(l_{7m}) \to 1 /*\ upto \frac{m+1}{2} lines\ */$

$g^*(l_{11}) = g^*(l_{25}) = \dots = g^*(l_{7m-10}) \to 1 /*\ upto \frac{m-1}{2} lines\ */$

$g^*(l_{12}), g^*(l_{13}) = g^*(l_{26}), g^*(l_{27}) =, \dots = g^*(l_{7m-9}), g^*(l_{7m-8}) \to 1 /*\ upto\ m-1\ lines\ */$

$g^*(l'_2), g^*(l'_3) = g^*(l'_{16}), g^*(l'_{17}) = \dots = g^*(l'_{7m-5}), g^*(l'_{7m-4}) \to 1 /*\ upto\ m+1\ lines\ */$

$g^*(l'_8) = g^*(l'_{22}) = \cdots = g^*(l'_{7m-13}) \to \frac{1}{*\ upto} \frac{m-1}{2}\ lines\ */$

$g^*(l'_7), g^*(l'_{21}) = g^*(l'_{35}) = \dots = g^*(l'_{7m}) \to 1 /*\ upto\ \frac{m+1}{2}\ lines\ */$

$g^*(l_5'), g^*(l_{19}') = g^*(l_{33}') = \ldots = g^*(l'_{7m-2}) \rightarrow 1 /* \, upto \, \frac{m+1}{2} \, lines */$

$g^*(l_{14}'), g^*(l_{28}') = g^*(l_{42}') = \ldots = g^*(l'_{7m-7}) \rightarrow 1 /* \, upto \, \frac{m-1}{2} \, lines */$

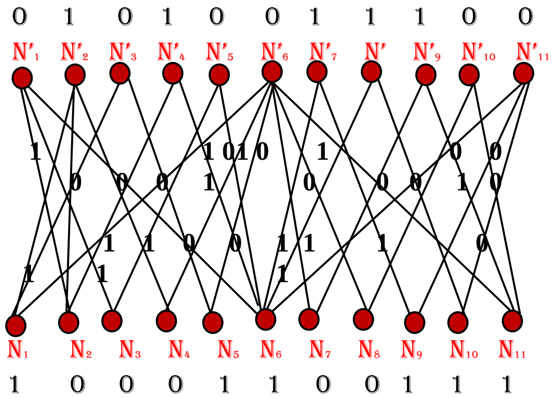
}

End the process.

Thus 7m lines receive label 1.

Hence the duplicate graph of the Double Quadrilateral snake $EDG(\,QS_m\,)\, m \geq 2$, Bi-Conditional Cordial labeling admits.

3.3. Illustration:



Bi-conditional cordial labeling $\boldsymbol{EDG(\,DQS_2)}$

4. Bi-conditional cordial labeling $\boldsymbol{DG(TL_m)}$

Algorithm 4.1: Allocation of labeled nodes

Let $N \rightarrow \{n_1, n_2 \ldots\ldots, n_{2m+2}. n_1', n_2', n_3', \ldots\ldots. n_{2m+2}'\}$
$L \rightarrow \{l_1, l_2, l_3 \ldots\ldots, l_{4m+1}. l_1', l_2', l_3' \ldots\ldots. l_{4m+1}'\}$
For $1 \leq k \leq m+1$
$n_{2k-1} \leftarrow 1 \; n_{2k} \leftarrow 0.$
For $1 \leq k \leq m$
Fix $n'_1 \leftarrow 1. n'_2 \leftarrow 0.$
$n'_{2k+1} \leftarrow 0, n'_{2k+2} \leftarrow 1.$

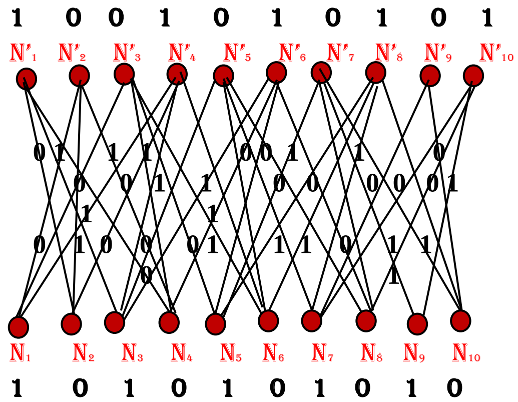
Theorem 4.2: The extended duplicate graph of Triangular ladder Graph $EDG(TQL_m) m \geq 2$. **Bi-conditional cordial labeling admits.**

Proof:

Let $N \rightarrow \{n_1, n_2 \ldots\ldots, n_{2m+2}. n_1', n_2', n_3', \ldots\ldots. n_{2m+2}'\}\; L \rightarrow \{l_1, l_2, l_3, \ldots\ldots, l_{4m+1}. l_1', l_2', l_3', \ldots\ldots. l_{4m+1}'\}$ be the set of nodes and lines of the duplicate graph of Triangular ladder graph $DG(\,TLS_m\,)$. using the algorithm 4.1, each of the $2m+2$ nodes receive label 0 and 1 respectively. Using the induced function g^* defined by $g^*(uv) = \begin{cases} 1: \text{if } g(u) = g(v) \\ 0: \text{if } g(u) \neq g(v) \end{cases}$ the m lines namely l_2, l_6, l_{10} , ……. l_{4m-2}. receive label 0, the m lines namely l_4, l_8, l_{12} , …… . l_{4m}. receive label 0, the 1 line namely l_1 receive label 0. $m-1$ lines namely $l'_6, l'_{10}, l'_{14}, \ldots\ldots\ldots\ldots\ldots l'_{4m-2.}$ receive label 0. $m-1$ lines namely $l'_8, l'_{12}, l'_{16}, \ldots\ldots\ldots\ldots\ldots l'_{4m}$. receive label 0, and 2 lines namely l'_1 and l'_3receive label 0.

Thus $4m + 1$ lines receive label 0. and the m lines namely $l'_7, l'_{11}, \ldots \ldots \ldots \ldots . \ldots l'_{4m-1}$. receive label 1, the m lines namely $l_5, l_9, l_{13}, \ldots \ldots \ldots \ldots . \ldots . l_{4m+1}$ receive label 1, $m-1$ lines namely $l_7, l_{11}, l_{15}, \ldots \ldots \ldots \ldots . \ldots \ldots l_{4m-1}$. receive label 1, m lines namely $l'_5, l'_9, l'_{13}, \ldots \ldots \ldots \ldots . \ldots l'_{4m+1}$. receive label 1. 2 lines namely l'_2 and l'_4 receive label 1. **Thus the $4m + 1$ lines receive label 1.**
Hence the duplicate graph of the Triangular ladder Snake Graph EDG(TL_m)$m \geq 2$, is Bi-Conditional Cordial labeling admits.

4.3. Illustration:



Bi-conditional cordial labeling $EDG(\ TL_4)$

5. Conclusion

We have proved the existence of the Bi-Conditional cordial labeling in Extended Duplicate graph of the Quadrilateral Snake $EDG(QS_m)m \geq 2$, Double Quadrilateral $EDG(DQS_m)m \geq 2$, and Triangular Ladder Snake Graph. $EDG(TL_m)m \geq 2$.

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