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A Study on $\delta - B$ Open Sets in Intuitionistic Topological Space

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ABSTRACT

In this paper, some related generalized sets of B namely $\delta - B$ open sets in intuitionistic topological space, Intuitionistic δAB- open set, Intuitionistic δC open set are introduced. The relationships between these sets are investigated and some of the properties are also studied.

INTRODUCTION

Topology is the mathematical study of the properties that are preserved through deformations, twistings and stretchings of objects. The introduction of intuitionistic fuzzy sets is due to K.T.Atanassov (1983) who also defined various topological operators for these sets.

PRELIMINERIES

DEFINITION 1:

A set A is said to be $\beta - open, \alpha - open, \delta - open$ if

i)$A \subseteq cl(int(cl(A)))$,

ii) $A \subseteq int(cl(int(A)))$

iii)$A = int(A)$ respectively where A is a subset of X.

Compliment of these open sets are closed.

DEFINITION 2:

Let $< X, T >$ be an intuitionistic topological space. An intuitionistic set $A =< x, A^1, A^2 >$ in X is said to be

i) intuitionistic t-open if $Iint(A) = Iint(Icl(A))$.

ii) The closure and interior of A are defined as

$$cl(A) = \cap \{K: K is\ an\ intuitionistic\ closed\ set\ in\ X\ and\ A \subseteq K\}$$

$$int(A) = \cup \{G: G\ is\ an\ intuitionistic\ open\ set\ in\ X\ and\ G \subseteq A\}$$

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$\delta - B$ OPEN SETS

DEFINITION 1:

Let $< X, T >$ be an intuitionistic topological space. An intuitionistic set $A = < x, A^1, A^2 >$ in X is said to be

i) intuitionistic $\delta - B$ open if $A = U \cap V$ where $U = < x, U^1, U^2 >$ is δ open and $V = < x, V^1, V^2 >$ is an $It - open\ set$.

ii) intuitionistic $\delta B\ interior$ of A if

$$I\delta Bint(A) = \cup \{G = < x, G^1, G^2 >: G\ is\ intuitionistic\ \delta B\ open\ and\ G \subseteq A\}$$

iii) intuitionistic $\delta B\ closure$ of A if

$$I\delta Bcl(A) = \cap \{K = < x, K^1, K^2 >: K\ is\ intuitionistic\ \delta B\ closed\ and\ A \subseteq K\}$$

REMARK 1:

Let $< X, T >$ be an intuitionistic topological space and $A = < x, A^1, A^2 >$ be an intuitionistic set in X.

Then

i) $I\delta Bint(A) \subseteq A \subseteq I\delta Bcl(A)$.

ii) A is an intuitionistic $\delta B - closed\ set$ iff $A = I\delta Bcl(A)$.

iii) A is an intuitionistic $\delta B - open\ set$ iff $A = I\delta Bint(A)$.

PROPOSITION 1:

Let $< X, T >$ be an intuitionistic topological space and $A = < x, A^1, A^2 >$,

$B = < x, B^1, B^2 >$ and $G = < x, G^1, G^2 >$ be any three intuitionistic sets. Then the following conditions hold:

i) If $A \subseteq B$, then $I\delta Bint(A) \subseteq I\delta Bint(B)$.

ii) $I\delta Bint(I\delta Bint(A)) = I\delta Bint(A)$.

iii) $I\delta Bint(A \cap B) = I\delta Bint(A) \cap I\delta Bint(B)$.

iv) $I\delta Bint(X_{\sim}) = X_{\sim}$.

PROOF:

i) $I\delta Bint(A) = \cup \{G: G\ is\ intuitionistic\ \delta B\ open\ and\ G \subseteq A\}$

$\subseteq \cup \{G: G\ is\ intuitionistic\ \delta B\ open\ and\ G \subseteq B\}$

$\subseteq I\delta Bint(B)$

$\therefore\ I\delta Bint(A) \subseteq I\delta Bint(B)$

ii) $I\delta Bint(I\delta Bint(A)) = \cup \{G: G\ is\ intuitionistic\ \delta B\ open\ and\ G \subseteq I\delta Bint(A)\}$

$\because I\delta Bint(A)$ is an intuitionistic δB open set,

$I\delta Bint\big(I\delta Bint(A)\big) = I\delta Bint(A)$.

iii) $I\delta Bint(A \cap B) = \cup \{G: G \text{ is intuitionistic } \delta B \text{ open and } G \subseteq (A \cap B)\}$

$= (\cup \{G: G \text{ is intuitionistic } \delta B \text{ open and } G \subseteq A\})$

$\cap (\cup \{G: G \text{ is intuitionistic } \delta B \text{ open and } G \subseteq B\})$

$= I\delta Bint(A) \cap I\delta Bint(B)$.

$\therefore\ I\delta Bint(A \cap B) = I\delta Bint(A) \cap I\delta Bint(B)$.

iv) $\because X_{\sim}$ is an intuitionistic $\delta B - open\ set$, $I\delta Bint(X_{\sim}) = X_{\sim}$.

PROPOSITION 2:

Let $< X, T >$ be an intuitionistic topological space and $A =< x, A^1, A^2 >$, $B =< x, B^1, B^2 >$ and $G =< x, G^1, G^2 >$ be any three intuitionistic sets. Then the following conditions hold:

i) If A⊂B, then I$\delta Bcl(A) \subseteq I\delta Bcl(B)$

ii) I$\delta Bcl\big(I\delta Bcl(A)\big) = I\delta Bcl(A)$

iii) I$\delta Bcl(A) \cup I\delta Bcl(B) = I\delta Bcl(A \cup B)$

iv) $I\delta Bcl(\varphi_{\sim}) = \varphi_{\sim}$

PROOF:

i) I$\delta Bcl(A)$=$\cap \{G: G is\ intuitionistic\ \delta B - closed\ and\ A \subseteq G\}$

$\subseteq \cap \{G: g\ is\ intuitionistic\ \delta B - closed\ and\ B \subseteq G\}$

$\subseteq I\delta Bcl(B)$

$\therefore$ I$\delta Bcl(A) \subseteq I\delta Bcl(B)$

ii) I$\delta Bcl\big(I\delta Bcl(A)\big) = \cap \{G: G\ is\ an\ intuitionistic\ \delta B - closed\ set\ and\ I\delta Bcl(A) \subseteq G\}$

$\because I\delta Bcl(A)$ is an intuitionistic $\delta B -$ closed set,

I$\delta Bcl\big(I\delta Bcl(A)\big) = I\delta Bcl(A)$.

iii) $I\delta Bcl(A \cup B) = \cap \{G: G\ is\ an\ intuitionistic\ \delta B - closed\ and\ (A \cup B) \subseteq G$

$= (\cap \{G: G\ is\ an\ intuitionistic\ \delta B - closed\ and\ A \subseteq G\})$

$\cup (\cap \{G: G\ is\ an\ intuiyionistic\ \delta B - closed\ and\ B \subseteq G\})$

$= \text{I}\delta Bcl(A) \cup I\delta Bcl(B)$

$\therefore \text{I}\delta Bcl(A) \cup I\delta Bcl(B) = I\delta Bcl(A \cup B)$

iv) $\because \varphi_\sim$ is an intuitionistic $\delta B - open\ set, I\delta Bint(\varphi_\sim) = \varphi_\sim$.

PROPOSITION 3:

Let <X,T> be an intuitionistic topological spaces.

i) Any finite intersection of intuitionistic δB open sets is an intuitionistic δB open set.

ii) Any finite union of intuitionistic δB closed sets is an intuitionistic δB closed set.

PROOF:

i) Let $\{A_i =< x, A^1, A^2 >\}_{i=1}^n$ be the finite collection of intuitionistic δB open sets. Then for each i, $A_i = U_i \cap V_i$ where $U_i =< x, U_i^1, U_i^2 >$ is an intuitionistic δ open set and $V_i =< x, V_i^1, V_i^2 >$ is an intuitionistic t-open set.

Now, $\bigcap_{i=1}^n A_i = \bigcap_{i=1}^n (U_i \cap V_i)$

$= (\bigcap_{i=1}^n U_i) \cap (\bigcap_{i=1}^n V_i)$

Since any finite intersection of intuitionistic open sets is an intuitionistic open set and by proposition (3),

i) $\bigcap_{i=1}^n (A_i)$ is an intuitionistic δB open set.

Hence finite intersection of intuitionistic δB open sets is an intuitionistic δB open set.

ii) Proof is similar to the proof of (i).

REMARK 2:

The union of any two intuitionistic δB open sets need not be an intuitionistic δB open sets as shown in the following example.

EXAMPLE:

Let $X = \{a, b, c\}$ be a non-empty set. Let $T = \{\varphi_\sim, X_\sim, A, B\}$ where $A =< x, \{a, b\}, \{c\} >$ and $B =< x, \{a\}, \{c\} >$. Then T is an intuitionistic topology on X. Let $C =< x, \{c\}, \{a, b\} >$ and $D =< x, \{a\}, \{c\} >$. Now C and D are intuitionistic δB open sets in $< X, T >$. $C \cup D$ is not an intuitionistic δB open set.

PROPOSITION 4:

Let $< X, T >$ be an intuitionistic topological space. Every intuitionistic semi regular set is an intuitionistic δAB open set.

PROOF:

Let $< X, T >$ be an intuitionistic topological space. let $A =< x, A^1, A^2 >$ be any intuitionistic semi regular set. Now $A = X_\sim \cap A$.

$\because X_\sim$ is an intuitionistic open set and A is an intuitionistic semi regular set, A is an intuitionistic δB open set.

REMARK 3:

The converse of proposition 4 need not be true as shown in the following example.

EXAMPLE:

Let $X = \{a, b, c\}$ be a non-empty set. Let $T = \{\varphi_{\sim}, X_{\sim}, A, B, C\}$ where

$A =< x, \{a\}, \{b, c\} >$, $B =< x, \{b\}, \{a\} >$ and $c =< x, \{a, b\}, \{\varphi\} >$. Then T is an intuitionistic topology on X. Let $D =< x, \{\varphi\}, \{a, b\} >$. Now, D is an intuitionistic δAB open set but it is not an intuitionistic semi regular set.

PROPOSITION 5:

Let $< X, T >$ be an intuitionistic topological space. every intuitionistic semi regular set is an intuitionistic t-open set.

PROOF:

Let $A =< x, A^1, A^2 >$ be any intuitionistic semi regular set.

∴ A is both intuitionistic semi open and intuitionistic semi closed.

$\because A \subseteq Icl(A)$

$Iint(A) \subseteq Iint(Icl(A))$

$\because Iint\big(Icl(A)\big) \subseteq A, Iint\big(Icl(A)\big) \subseteq Iint(A).$

$\therefore Iint(A) = Iint\big(Icl(A)\big).$

Hence, A is an intuitionistic t open set.

REMARK 4:

The converse of proposition 5 is false as shown in the following example.

EXAMPLE:

Let $X = \{a, b, c\}$ be a non-empty set. Let $T = \{\varphi_{\sim}, X_{\sim}, A, B, C\}$ where $A =< x, \{a, b\}, \{\varphi\} >$, $B =< x, \{a, b\}, \{c\} >$ and $C =< x, \{c\}, \{a, b\} >$.

Then T is an intuitionistic topology on X. let $D =< x, \{\varphi\}, \{a, b\} >$. Now, D is an intuitionistic t open set but it is not an intuitionistic semi regular set.

PROPOSITION 6:

Let $< X, T >$ be an intuitionistic topological space. Every intuitionistic δAB-open set is an intuitionistic δB-open set.

PROOF:

Let $A =< x, A^1, A^2 >$ be any intuitionistic δAB open set. Therefore, $A = U \cup V$ where $U =< x, U^1, U^2 >$ is an intuitionistic δ open set and $V =< x, V^1, V^2 >$ is an intuitionistic semi regular set.

∵ Every intuitionistic semi regular set is an intuitionistic t-open set, v is an intuitionistic t-open set. Hence, A is an intuitionistic δB open set.

REMARK 5:

The converse of proposition 6 is false as shown in the following example.

EXAMPLE:

Let $X = \{a, b, c\}$ be a non-empty set. Let $T = \{\varphi_{\sim}, X_{\sim}, A, B, C, D\}$ where

$A =< x, \{a\}, \{b, c\} >$, $B =< x, \{c\}, \{a, b\} >$, $C =< x, \{a, b\}, \{c\} >$ and $D =< x, \{a, c\}, \{b\} >$. Then T is an intuitionistic topology on X. Let $E =< x, \{b, c\}, \{a\} >$. Now, E is an intuitionistic δB open set but it is not an intuitionistic δAB open set.

PROPOSITION 7:

Let $< X, T >$ be an intuitionistic topological space, Every intuitionistic t-open set is an intuitionistic α^*-open set.

PROOF:

Let $A =< x, A^1, A^2 >$ be an intuitionistic t-open set.

Now,

$$\overline{Iint(Icl(Iint(A)))} = Icl(\overline{Icl(Iint(A))})$$

$$= Icl(Iint(\overline{Iint(A)}))$$

$$= Icl(Iint(Icl(\bar{A})))$$

$$\subseteq Icl(Icl(\bar{A}))$$

$$= Icl(\bar{A})$$

$$= \overline{Iint(A)}$$

$$Iint(A) \subseteq Iint(Icl(Iint(A))) \qquad \text{-------------- (1)}$$

Now from hypothesis,

$$Iint(A) \supseteq Iint(Icl(A)) \supseteq Iint(Icl(Iint(A))) \qquad \text{--------------- (2)}$$

∴ From (1) and (2), A is an intuitionistic α^* open set.

REMARK 6:

The converse of proposition 7 need not be true as shown in the following example.

EXAMPLE:

Let $X = \{a, b, c, d\}$ be a non-empty set. let $T = \{\varphi_{\sim}, X_{\sim}, A\}$ where $A =< x, \{a, d\}, \{b, c\} >$, then T is an intuitionistic topology on X. Let $B =< x, \{a, b\}, \{d\} >$. Now B is an intuitionistic α^* open set but it is not an intuitionistic t-open set.

PROPOSITION 8:

Let $< X, T >$ be an intuitionistic topological space. Every intuitionistic δB open set is an intuitionistic δC open set.

PROOF:

Let $A =< x, A^1, A^2 >$ be any intuitionistic δB open set.

$\therefore A = U \cap V$ where $U =< x, U^1, U^2 >$ is an intuitionistic δ open set and V is an intuitionistic t-open set.

$\because$ Every intuitionistic t-open set is an intuitionistic α^*open set, V is an intuitionistic α^*open set. Hence, A is an intuitionistic δC open set.

REMARK 7:

The converse of proposition 8 need not true as shown in the following example.

EXAMPLE:

Let $X = \{p, q, r, s\}$ be a non-empty set. Let $T = \{\varphi_{\sim}, X_{\sim}, A, B, C, D, E, F, G\}$ where $A =< x, \{p\}, \{q, r, s\} >$, $B =< x, \{p, q\}, \{r, s\} >$, $C =< x, \{r\}, \{p, q, s\} >$, $D =< x, \{r, s\}, \{p, q\} >$, $E =< x, \{p, r\}, \{q, s\} >$, $F =< \{x, \{p, r, s\}, \{q\} >$ and $G =< x, \{q, r\}, \{p\} >$. Now, H is an intuitionistic δC open set but it is not an intuitionistic δB open set.

REMARK 8:

Clearly the following diagram holds.

Intuitionistic δAB- open set	$\rightarrow$ $\nleftarrow$	Intuitionistic δB open set	$\rightarrow$ $\nleftarrow$	Intuitionistic δC open set

CONCLUSION

In this paper a new set called $\delta - B$ open set in intuitionistic topological space has been introduced and results has been discussed. This paper can be extended to any topological space like ideal topological space.

REFERENCES

1) Introduction to topology by James R. Munkres.

2)Decomposition of Intuitionistic B-Open Sets, International Journal of Pure and Engineering Mathematics, 2(2014), 121-131.

3) N. Bourbaki: Elements of mathematics, General topology, Springer, Berlin, (1989).

4) D. Coker: An introduction to intuitionistic topological spaces, BUSEFAL, 81(2000), 51–56.

5) M. Caldas, S. Jafari and T. Noiri : Notions via g-open sets, Kochi. J. Math., 2(2007), 43–50.

6) D. Coker: A note on intuitionistic sets and intuitionistic points, Turkish J. Math., 20(1996), 343–351.