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
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# Fingerprint Matching Based on Fuzzy Atribution

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**Abstract:** Fingerprint matching is used to match the two or more impressions to identify the result match / non-match. Suppose the result match then the system allows the person to use or enter otherwise exit. Fingerprint recognition, Access control and Identity management are automatic applications for matching. This paper explores the comparison of three generation fingerprints from a family by using fuzzy relation.

## INTRODUCTION AND PRELIMINARIES

Author	Theory discussed by author
<sup>[3]</sup> Davide Maltoni, Dario Maio, Anil K. Jain and Salil Prabhakar	Introduction, fingerprint sensing, fingerprint analysis and representation, fingerprint matching, fingerprint classification and indexing, synthetic fingerprint generation, biometric fusion, fingerprint individuality and securing fingerprint systems.
<sup>[5]</sup> Kaufmann. A	Fundamental notions, fuzzy graphs and fuzzy relations, fuzzy logic, the laws of fuzzy composition, generalization of the notion of fuzzy subset.
<sup>[10]</sup> Zimmermann. J. H	Introduction to fuzzy sets, fuzzy mathematics: fuzzy sets-basic definition, extensions, fuzzy measures and measures of fuzziness, the extension principle and applications, fuzzy relations and fuzzy graphs, fuzzy analysis, possibility theory, probability theory and fuzzy set theory, Applications of fuzzy set theory: fuzzy logic and approximate reasoning, fuzzy sets and expert systems, fuzzy control, fuzzy data analysis, decision making in fuzzy environments, fuzzy set models in operations research, empirical research in fuzzy set theory, future perspectives.
<sup>[6]</sup> Klir. J. G and Bo Yuan	Theory: from classical (crisp) sets to fuzzy sets: a grand paradigm shift, fuzzy sets versus crisp sets, operations on fuzzy sets, fuzzy arithmetic, fuzzy relations, fuzzy relation equations, possibility theory, fuzzy logic, uncertainty-based information, Applications: constructing fuzzy sets and operations

on fuzzy sets, approximate reasoning, fuzzy systems, pattern recognition, fuzzy databases and information retrieval systems, fuzzy decision making, engineering applications and miscellaneous applications.

<sup>[9]</sup> Vinita Dutt, Vikas Chaudhry, Imran Khan

Introduction, the definition of pattern recognition, the research of pattern recognition methods, pattern recognition system, applications, related fields, conclusion.

Lotfi. A. Zadeh (1965) published the “Theory of fuzzy subsets”. A fuzzy logic provides a flexible way to observe a human reasoning problem. It only allows the membership values [0, 1] for making a decision in mathematical way. The pattern recognition is unfamiliar to everyone. It is explanation of subject researching object and fuzzy logic is making a comfortable mode to the infinite shape to find the accurate result. Fingerprint matching is complicated process to identify individual and general structure. To reach a satisfied result we choose fuzzy pattern recognition.

Forensic applications (Missing children, Cadaver identification, Sexual abuse, Criminal investigation, Terrorist identification, Parenthood verification), Commercial applications (Computer network logon, Electronic data security, ATM (Automated Teller Machine), Credit card, Cellular phones, Personal details, Medical record management, Electronic banking, Attendance entry, Distance learning, E-commerce (electronic commerce) Internet approach), Government applications (National ID card (aadhar), Correctional facility, Driving license, Social security, Border control, Passport control), other main industries used fingerprint applications (Law enforcement, Health care, Military, Gaming and hospitality (casinos, bars, hotels, etc.), Manufacturing, Retail, High technology and telecommunications, Education).

### Definition 1 [5, 6, 10]

Let  $X = \{x_1, x_2, x_3, \dots, x_n\}$  and  $Y = \{y_1, y_2, y_3, \dots, y_m\}$  are two fuzzy sets. Then  $\tilde{A}$  be a fuzzy subset of  $X \times Y$ . (i. e.)  $\tilde{A} \subset X \times Y$

$$\tilde{A} = \{(x, y), \mu_{\tilde{A}}(x, y) \mid (x, y) \in X \times Y\}$$

is known as fuzzy relation  $\tilde{A}(X, Y)$ .

### Definition 2[5, 6, 10]

If  $X = x_i$  and  $Y = y_j$  be two fuzzy sets, let  $\tilde{A}_1 \subset X \times Y$  and  $\tilde{A}_2 \subset X \times Y$  be any fuzzy relations. Then,

$$\mu_{\tilde{A}_1 \cup \tilde{A}_2}(X, Y) = \text{MAX} [\mu_{\tilde{A}_1}(x_i, y_j), \mu_{\tilde{A}_2}(x_i, y_j)]$$

(or)

$$= \mu_{\tilde{A}_1}(x_i, y_j) \cup \mu_{\tilde{A}_2}(x_i, y_j)$$

is known as union-relation (or) max-relation where  $i = 1, 2, 3, \dots, n$ ;  $j = 1, 2, 3, \dots, m$  and  $\cup$  is also called maximum ( $\vee$ ).

Similarly,

$$\mu_{\tilde{A}_1 \cap \tilde{A}_2}(X, Y) = \text{MIN} [\mu_{\tilde{A}_1}(x_i, y_j), \mu_{\tilde{A}_2}(x_i, y_j)]$$

(or)

$$= \mu_{\tilde{A}_1}(x_i, y_j) \cap \mu_{\tilde{A}_2}(x_i, y_j)$$

is known as intersection-relation (or) min-relation where  $i = 1, 2, 3, \dots, n$ ;  $j = 1, 2, 3, \dots, m$  and  $\cap$  is also called minimum ( $\wedge$ ).

### Definition 3[5, 6, 10]

Let  $\tilde{A}_1$  be any  $x_i \times y_j$  matrix,  $\tilde{A}_2$  be any  $y_j \times z_k$  matrix. Define

$$\text{(i.e.) } \tilde{A}_1 = \{(x, y), \mu_{\tilde{A}_1}(x, y)\} \mid x \in X, y \in Y, \tilde{R} \subset X \times Y$$

and

$$\tilde{A}_2 = \{((y, z), \mu_{\tilde{A}_2}(y, z)) \mid y \in Y, z \in Z, \tilde{R} \subset Y \times Z\}$$

Then, the max-min composition is expressed by

$$\mu_{\tilde{A}_2 \circ \tilde{A}_1}(x, z) = \bigvee_y [\mu_{\tilde{A}_1}(x, y) \wedge \mu_{\tilde{A}_2}(y, z)]$$

(or)

$$\mu_{\tilde{A}_2 \circ \tilde{A}_1}(x, z) = \max [\min (\mu_{\tilde{A}_1}(x, y), \mu_{\tilde{A}_2}(y, z))]$$

where  $i = 1, 2, 3, \dots, n$ ;  $j = 1, 2, 3, \dots, m$ ;  $k = 1, 2, 3, \dots, l$ ,  $\vee$  and  $\wedge$  is also called union ( $\cup$ ) and intersection ( $\cap$ ) (or) maximum and minimum relation.

### Definition 4[5, 6, 10]

An operation of fuzzy relation which is introduced in terms of the max-min composition, over the union and intersection is said to be fuzzy associative, if for any  $x \in X, y \in Y$  and  $z \in Z$ .

- (i)  $(\tilde{A}_3 \circ \tilde{A}_2) \circ \tilde{A}_1 = \tilde{A}_3 \circ (\tilde{A}_2 \circ \tilde{A}_1)$
- (ii)  $\tilde{A}_3 \circ (\tilde{A}_1 \cap \tilde{A}_2) = (\tilde{A}_3 \circ \tilde{A}_1) \cap (\tilde{A}_3 \circ \tilde{A}_2)$
- (iii)  $\tilde{A}_3 \circ (\tilde{A}_1 \cup \tilde{A}_2) = (\tilde{A}_3 \circ \tilde{A}_1) \cup (\tilde{A}_3 \circ \tilde{A}_2)$

### Types Of Fingerprint Patterns[4, 7]

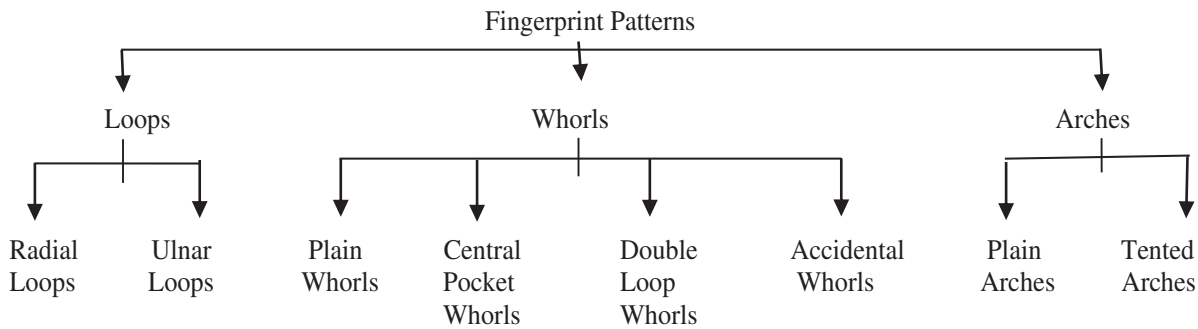


FIGURE 1. Fingerprint pattern-flow chart.

- **Radial Loops** - It begins and flow back on same side of radius bone (bone in the forearm) or thumb. These patterns have one core and one delta. After the delta ridges the impression is converted to plain arch pattern.



FIGURE2. Radial loops (Left hand)

- **Ulnar Loops** - The loop start and end on same side of ulna bone (or) pinky and looks like an opposite of radial loop. This pattern also called as ulnar bone. It has one core and delta.



FIGURE 3. Ulnar loops (Right hand)

- **Plain Whorls** - The plain whorls or concentric circles look like a 360° closed circle or circular shape. It has two deltas and one core. After the delta the patterns change to plain arch.



FIGURE 4. Plain whorls

- **Central Pocket Whorls (Peacock Eye Whorl)** - This print has more than one circular shapes and convert into loop pattern. It has one delta and two cores.



FIGURE 5. Central pocket whorls

- **Double Loops Whorls** - Double loop pattern is a formation of ulnar and radial loops or S-like pattern. These prints have two deltas and two cores.



FIGURE 6. Double loop whorls

- **Accidental Whorls** - This pattern have a combination of two different patterns. It cannot have a regular shape. It has two or more cores and deltas.



**FIGURE 7.** Accidental whorls

- **Plain Arches** - Plain arches start from left side and end on other side, in between it makes up a wave-like shape. It has no delta and core. This pattern cannot change its shapes.



**FIGURE 8.** Plain arches

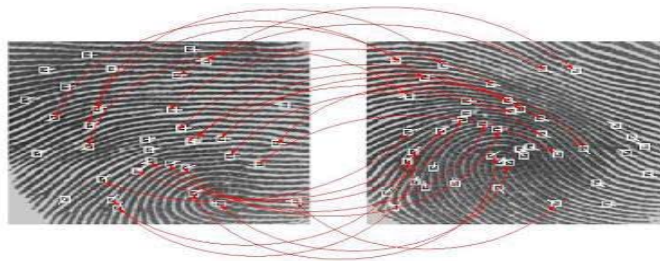
- **Tented Arches** -These prints are same like plain arches but center part creates a little sharp shape. It has one core and one delta.



**FIGURE 9.** Tented arches

### **Fingerprint Matching [3, 1, 8]**

Fingerprint matching compares the two / more impressions to find the match or non-match. It matches every minutiae from a ridge lines then display the result or degree of match. Fingerprint matching is a complicated pattern recognition problem because large number of different impressions on same finger placed on scanning sensor. Here, they use template (T) and input (I) for automatic matching. Template represented as the ridge points are already stored and input represented by given impression ridge points. Fingerprint matching algorithm is similar with verification and identification.



**FIGURE 10.** Fingerprint matching

It has main two sub-domains:

1. Minutiae based matching- Minutiae are selected from two fingerprints and saved as a point of two-dimensional planes. The importance of this matching is to find the arrangement between the template and selected set of points that gives the maximum ridges of minutiae pairing.
2. Non-minutiae feature-based matching - It is difficult to find minutiae between low quality extraction images, but other features of ridge pattern like ridge shape, texture, frequency, etc., can be identified eminently than minutiae even their quality is low result.

### Ridges and Valleys [2]

Ridges and valley lines created the fingerprint pattern. Dark line represented as ridges and white lines represented as valley. Ridge characteristic have more type which is used to identify the individual person. They are ridge ending, bifurcation, hook, island, etc. Ridge characteristic is also known as minutiae.



FIGURE 11. Ridges and valleys

### Ridge Characteristics

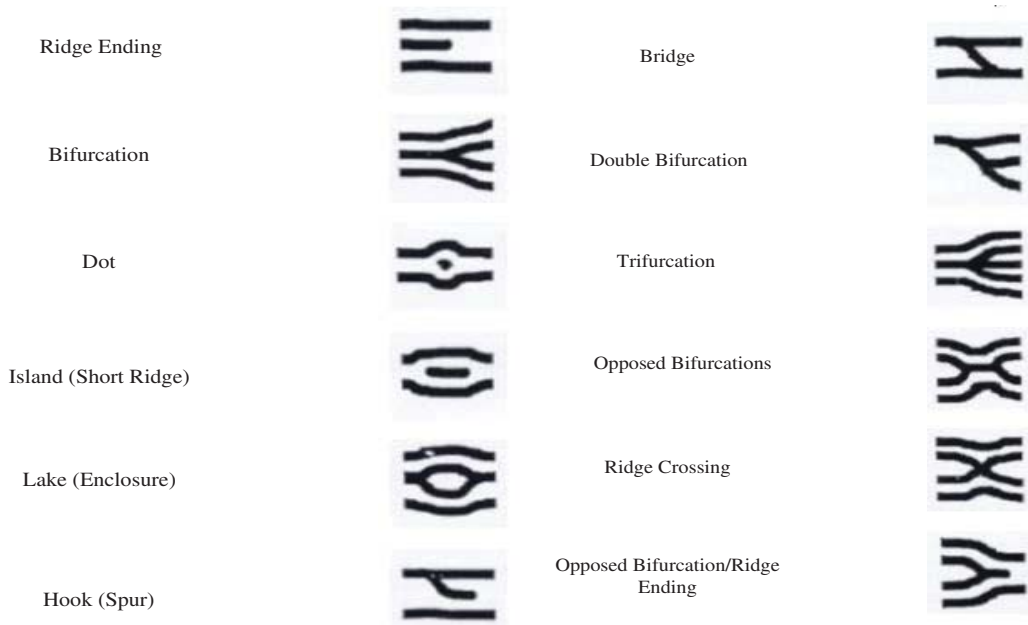


FIGURE 12. Ridge Characteristics (Minutiae)

## EXPERIMENTAL AND RESULTS

Compare the three generation fingerprints from the samples of 52 families. First, we tag for three generations.

- G - Grandparent
- P - Paternal / maternal
- C - Grandchild

Compare two combination of fingerprints one after other. We fix a fingerprint by using gridlines and assuming 3x3 matrices for matching in its pattern area. We create 3x3 matrices table, from each cell select any  $x_i$  and  $y_j$  where  $i = j$  is compared.

In fuzzy logic, we use membership value [0, 1]. So, we construct values from the matching of cells. If both cells are equal then we consider the value 1, if the cells have small different then we consider the value 0.8, if the cells have lot of different then we consider the value 0.3 and if the cells are inequality then we consider the value 0.

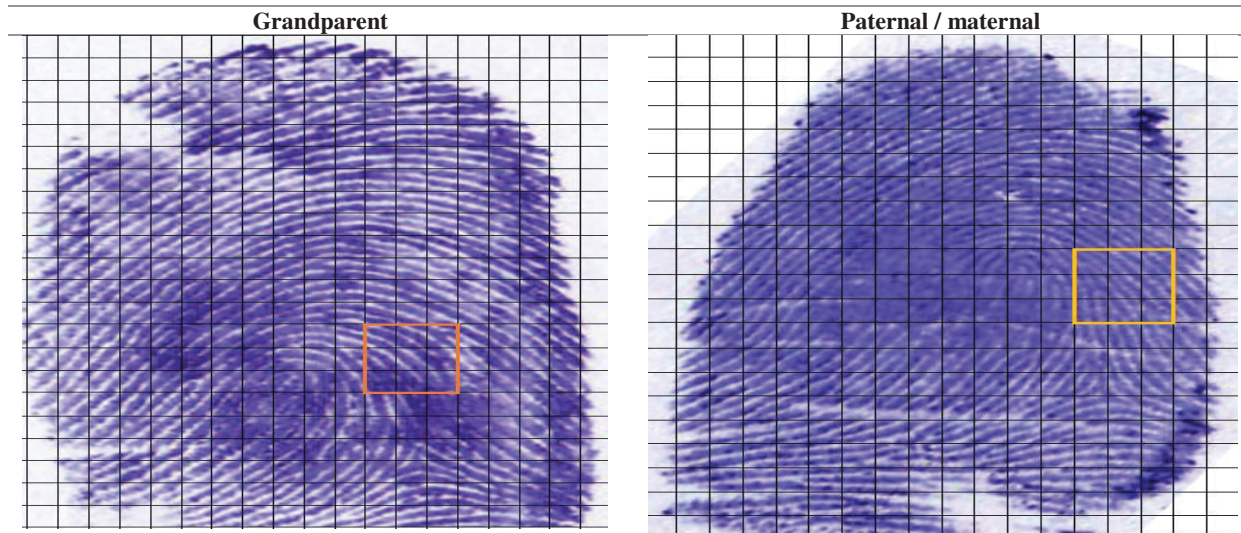
- Inequality = 0
- Minimum equal = 0.3
- Maximum equal = 0.8
- Equal = 1

From these values we create a matrix table. The comparison of first, second and third matrix table values are calculate from the definition of fuzzy astrubutive. From the definition 2.1.4, we have to get equal values on left and right hand side matrices.

Now, we choose a family,

- Devi (49) - Grandparent
- Kasturi (31) - Paternal / maternal
- Mohammed Rafick (10) - Grandchild

**TABLE 1.** Comparison of G and P

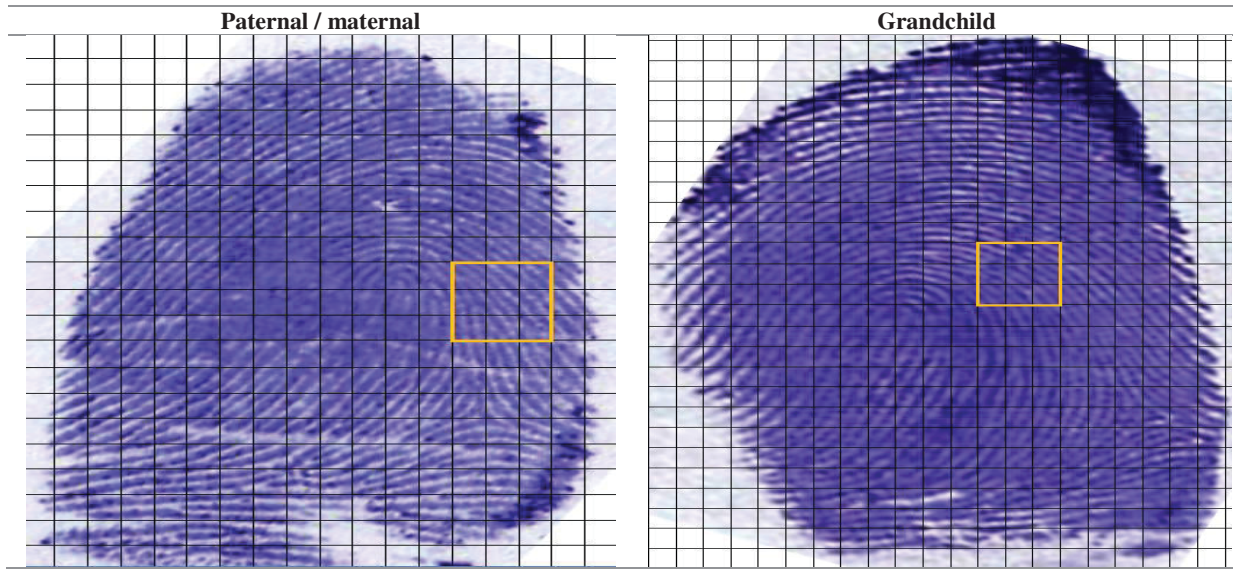


From table 1, we create a 3x3 matrix table.

0.8	0	0
0.8	1	0.8
0	1	1



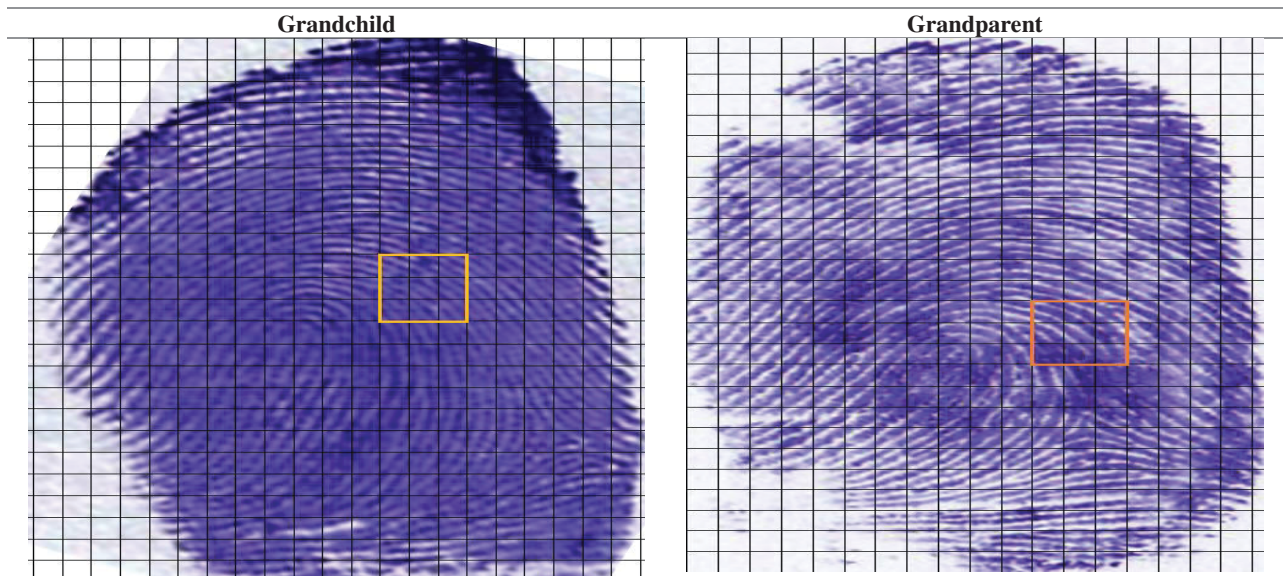
**TABLE 2.** Comparison of P and C



From table 2, we create a 3x3 matrix table.

0.8	0	1
1	1	1
0	0.8	0.8

**TABLE 3.** Comparison of C and G



From table 3, we create a 3x3 matrix table.

0.8	0.8	0.3
0.8	0.8	0.8
0.3	1	0.8

From table 1, 2, 3. We assuming the labels and create the images by using  $\tilde{A}_i$  where  $i = 1, 2, 3$ .

In figure 13,  $\tilde{A}_1$  be a matrix of  $x_i$  rows and  $y_j$  columns where  $i = j = 1, 2, 3$ .

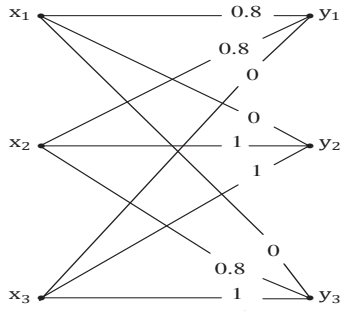


FIGURE 13.  $\tilde{A}_1$

	$y_1$	$y_2$	$y_3$
$x_1$	0.8	0	0
$x_2$	0.8	1	0.8
$x_3$	0	1	1

In figure 14,  $\tilde{A}_2$  be a matrix of  $y_j$  rows and  $z_k$  columns where  $j = k = 1, 2, 3$ .

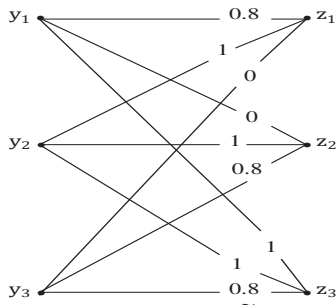


FIGURE 14.  $\tilde{A}_2$

	$z_1$	$z_2$	$z_3$
$y_1$	0.8	0	1
$y_2$	1	1	1
$y_3$	0	0.8	0.8

In figure 15,  $\tilde{A}_3$  be a matrix of  $z_k$  rows and  $t_l$  columns where  $k = l = 1, 2, 3$ .

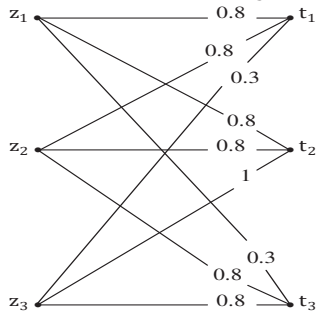


FIGURE 15.  $\tilde{A}_3$

	$t_1$	$t_2$	$t_3$
$z_1$	0.8	0.8	0.3
$z_2$	0.8	0.8	0.8
$z_3$	0.3	1	0.8

Here, we combine table 1, 2, 3 and create an image.

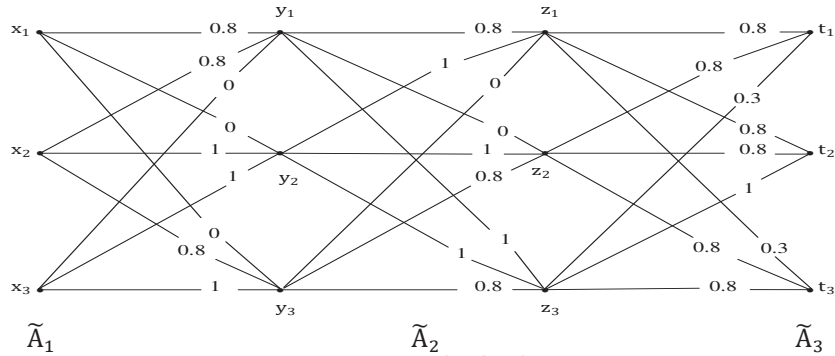


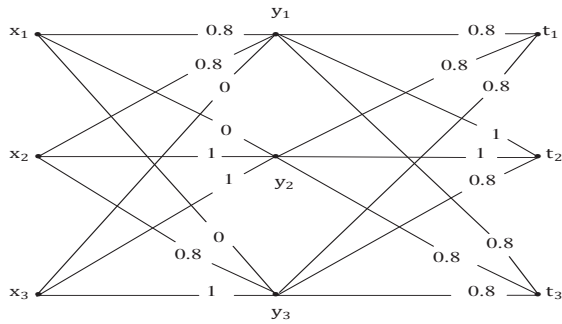
FIGURE 16.  $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3$

Using the definition 4,

$$(i) \quad (\tilde{A}_3 \circ \tilde{A}_2) \circ \tilde{A}_1 = \tilde{A}_3 \circ (\tilde{A}_2 \circ \tilde{A}_1)$$

Left hand side

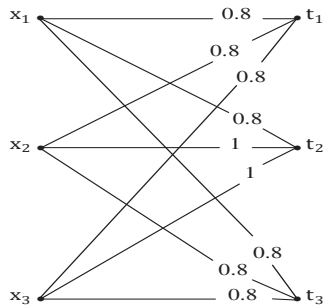
Calculate  $(\tilde{A}_3 \circ \tilde{A}_2) \circ \tilde{A}_1$  by using max-min composition.



**FIGURE 17.**  $\tilde{A}_3 \circ \tilde{A}_2$

$$\tilde{A}_3 \circ \tilde{A}_2$$

	$t_1$	$t_2$	$t_3$
$y_1$	0.8	1	0.8
$y_2$	0.8	1	0.8
$y_3$	0.8	0.8	0.8



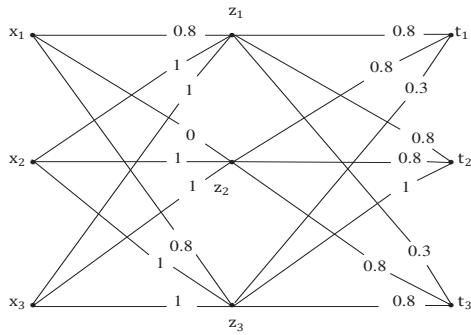
**FIGURE 18.**  $(\tilde{A}_3 \circ \tilde{A}_2) \circ \tilde{A}_1$

$$(\tilde{A}_3 \circ \tilde{A}_2) \circ \tilde{A}_1$$

	$t_1$	$t_2$	$t_3$
$x_1$	0.8	0.8	0.8
$x_2$	0.8	1	0.8
$x_3$	0.8	1	0.8

Right hand side

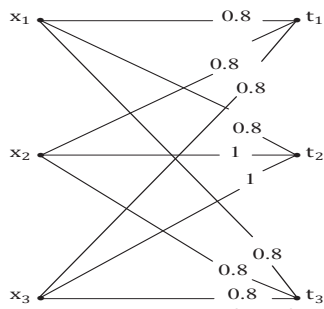
Calculate  $\tilde{A}_3 \circ (\tilde{A}_2 \circ \tilde{A}_1)$  by using max-min composition.



**FIGURE 19.**  $\tilde{A}_2 \circ \tilde{A}_1$

$$\tilde{A}_2 \circ \tilde{A}_1$$

	$z_1$	$z_2$	$z_3$
$x_1$	0.8	0	0.8
$x_2$	1	1	1
$x_3$	1	1	1



**FIGURE 20.**  $\tilde{A}_3 \circ (\tilde{A}_2 \circ \tilde{A}_1)$

$$\tilde{A}_3 \circ (\tilde{A}_2 \circ \tilde{A}_1)$$

	$t_1$	$t_2$	$t_3$
$x_1$	0.8	0.8	0.8
$x_2$	0.8	1	0.8
$x_3$	0.8	1	0.8

∴ Left hand side = Right hand side.

$$(ii) \quad \tilde{A}_3 \circ (\tilde{A}_1 \cap \tilde{A}_2) = (\tilde{A}_3 \circ \tilde{A}_1) \cap (\tilde{A}_3 \circ \tilde{A}_2)$$

Here, we consider  $\tilde{A}_1$  and  $\tilde{A}_2$  be a matrix of  $x_i$  rows and  $y_j$  columns where  $i = j = 1, 2, 3$  and  $\tilde{A}_3$  with  $y_j$  rows and  $z_k$  columns where  $j = k = 1, 2, 3$ .

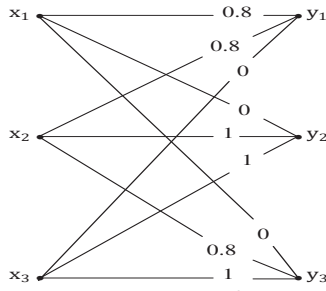


FIGURE 21.  $\tilde{A}_1$

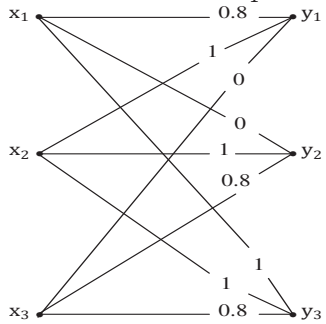


FIGURE 22.  $\tilde{A}_2$

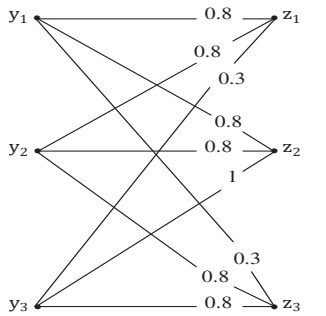


FIGURE 23.  $\tilde{A}_3$

$\tilde{A}_3$	$z_1$	$z_2$	$z_3$
$y_1$	0.8	0.8	0.3
$y_2$	0.8	0.8	0.8
$y_3$	0.3	1	0.8

$\tilde{A}_1$	$y_1$	$y_2$	$y_3$
$x_1$	0.8	0	0
$x_2$	0.8	1	0.8
$x_3$	0	1	1

$\tilde{A}_2$	$y_1$	$y_2$	$y_3$
$x_1$	0.8	0	1
$x_2$	1	1	1
$x_3$	0	0.8	0.8

Left hand side

Calculate  $\tilde{A}_3 \circ (\tilde{A}_1 \cap \tilde{A}_2)$  by using max-min composition and min-relation.

$(\tilde{A}_1 \cap \tilde{A}_2)$	$y_1$	$y_2$	$y_3$
$x_1$	0.8	0	0
$x_2$	0.8	1	0.8
$x_3$	0	0.8	0.8

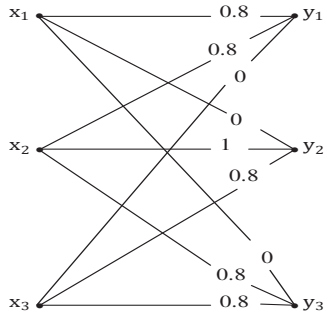


FIGURE 24.  $\tilde{A}_1 \cap \tilde{A}_2$

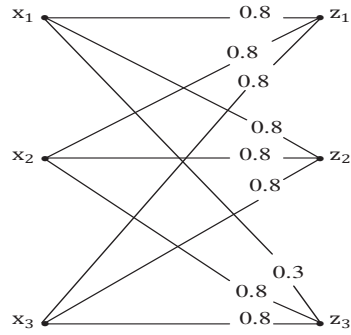


FIGURE 25.  $\tilde{A}_3 \circ (\tilde{A}_1 \cap \tilde{A}_2)$

$$\tilde{A}_3 \circ (\tilde{A}_1 \cap \tilde{A}_2)$$

$x_1$   
 $x_2$   
 $x_3$

	$z_1$	$z_2$	$z_3$
$x_1$	0.8	0.8	0.3
$x_2$	0.8	0.8	0.8
$x_3$	0.8	0.8	0.8

Right hand side

Calculate  $(\tilde{A}_3 \circ \tilde{A}_1) \cap (\tilde{A}_3 \circ \tilde{A}_2)$  by using max-min composition and min-relation.

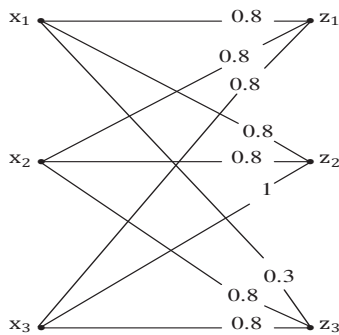


FIGURE 26.  $\tilde{A}_3 \circ \tilde{A}_1$

$$\tilde{A}_3 \circ \tilde{A}_1$$

$x_1$   
 $x_2$   
 $x_3$

	$z_1$	$z_2$	$z_3$
$x_1$	0.8	0.8	0.3
$x_2$	0.8	0.8	0.8
$x_3$	0.8	1	0.8

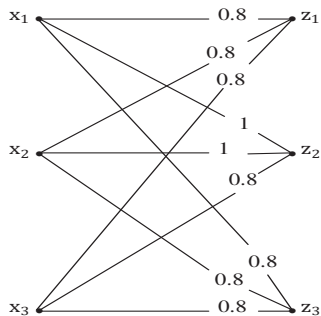


FIGURE 27.  $\tilde{A}_3 \circ \tilde{A}_2$

$$\tilde{A}_3 \circ \tilde{A}_2$$

$x_1$   
 $x_2$   
 $x_3$

	$z_1$	$z_2$	$z_3$
$x_1$	0.8	1	0.8
$x_2$	0.8	1	0.8
$x_3$	0.8	0.8	0.8

$$(\tilde{A}_3 \circ \tilde{A}_1) \cap (\tilde{A}_3 \circ \tilde{A}_2)$$

$z_1$        $z_2$        $z_3$

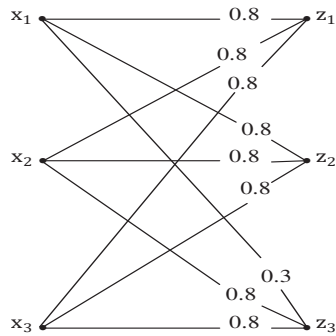


FIGURE 28.  $(\tilde{A}_3 \circ \tilde{A}_1) \cap (\tilde{A}_3 \circ \tilde{A}_2)$

$x_1$   
 $x_2$   
 $x_3$

0.8	0.8	0.3
0.8	0.8	0.8
0.8	0.8	0.8

$\therefore$  Left hand side = Right hand side.

(iii)  $\tilde{A}_3 \circ (\tilde{A}_1 \cup \tilde{A}_2) = (\tilde{A}_3 \circ \tilde{A}_1) \cup (\tilde{A}_3 \circ \tilde{A}_2)$

Left hand side

Calculate  $\tilde{A}_3 \circ (\tilde{A}_1 \cup \tilde{A}_2)$  by using max-min composition and max-relation.

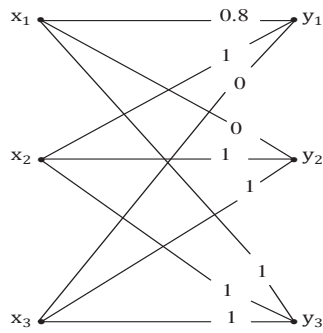


FIGURE 29.  $\tilde{A}_1 \cup \tilde{A}_2$

$\tilde{A}_1 \cup \tilde{A}_2$   
 $x_1$   
 $x_2$   
 $x_3$

	$y_1$	$y_2$	$y_3$
$x_1$	0.8	0	1
$x_2$	1	1	1
$x_3$	0	1	1

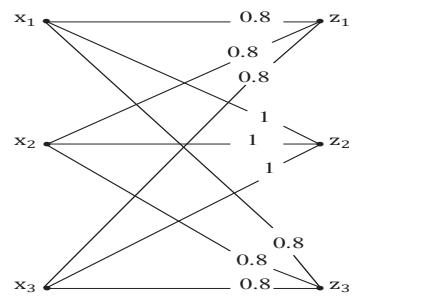


FIGURE 30.  $\tilde{A}_3 \circ (\tilde{A}_1 \cup \tilde{A}_2)$

$\tilde{A}_3 \circ (\tilde{A}_1 \cup \tilde{A}_2)$   
 $x_1$   
 $x_2$   
 $x_3$

	$z_1$	$z_2$	$z_3$
$x_1$	0.8	1	0.8
$x_2$	0.8	1	0.8
$x_3$	0.8	1	0.8

Right hand side

Calculate  $(\tilde{A}_3 \circ \tilde{A}_1) \cup (\tilde{A}_3 \circ \tilde{A}_2)$  by using max-min composition and max-relation.

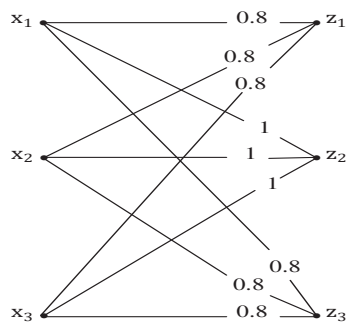


FIGURE 31.  $(\tilde{A}_3 \circ \tilde{A}_1) \cup (\tilde{A}_3 \circ \tilde{A}_2)$

$(\tilde{A}_3 \circ \tilde{A}_1) \cup (\tilde{A}_3 \circ \tilde{A}_2)$   
 $x_1$   
 $x_2$   
 $x_3$

	$z_1$	$z_2$	$z_3$
$x_1$	0.8	1	0.8
$x_2$	0.8	1	0.8
$x_3$	0.8	1	0.8

$\therefore$  Left hand side = Right hand side.

Hence definition 4 satisfied.

## CONCLUSION

Three generation fingerprints are compared by using the fuzzy relation of fuzzy attributive. We finally, concluded that comparison of three generation fingerprints is almost similar.

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