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On Direct Sum of Two Picture Fuzzy Graph

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Abstract. This paper investigates the concept of direct sum $G_1 \oplus G_2$of two picture fuzzy graphs G_1 and G_2 are defined and when two picture fuzzy graphs are effective then their direct sum need not be effective is proved. The degree of vertices in the direct sum $G_1 \oplus G_2$of picture fuzzy graph is obtained in terms of degrees of vertices in the two picture fuzzy graphs G_1 and G_2.

Keywords. Direct sum of picture fuzzy graph, Effective picture fuzzy graph, and degrees of direct sum of picture fuzzy graph.

INTRODUCTION

Fuzzy graph theory was introduced by AzrielRosenfled in 1975. The properties of intuitionistic fuzzy graphs have been studied by S. Sheik Dhavudh and R. Srinivasan [9]. Some operation of picture fuzzy graph was introduced by Cen Zun, Anitha pal, ArindamDey [2]. The direct sum of two fuzzy graph is defined by K Radha and S. Arumugam [8]. Throughout this paper, the concept of direct sum of two picture fuzzy graphs G_1 and G_2.are defined, The degree of vertices of $G_1 \oplus G_2$ of picture fuzzygraph is obtained in terms of degrees of vertices of two picture fuzzy graphs G_1 and G_2. This has been illustrating through some examples.

PRELIMINARIES

Definition 1: [2] Let A be a picture fuzzy set, A in X is defined by $A = \{x, \mu_A(x), \eta_A(x), \gamma_A(x) / x \in X\}$ Where $\mu_A(x), \eta_A(x) and \gamma_A(x)$ follow the condition $0 \le \mu_A(x) + \eta_A(x) + \gamma_A(x) \le 1$. The $\mu_A(x), \eta_A(x), \gamma_A(x) \in [0,1]$, denotes respectively the positive membership degree, neutral membership degree and negative membership degree of the element in the set A.

Definition2: [8] Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ denote two fuzzy graphs with underlying crisp graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. Let $V = V_1 \cup V_2$ and let $E = \{uv / u, v \in V; uv \in E_1$ or $uv \in E_2$ but not both } Define G= (σ, μ) by

$$\sigma(u) = \begin{cases} \sigma_1(u), if u \epsilon V_1 \\ \sigma_2(u), if u \epsilon V_2 \\ \sigma_1(u) \vee \sigma_2(u), if u \epsilon V_1 \cup V_2 \end{cases}$$

$$\mu(uv) = \begin{cases} \mu_1(uv), if uv \in E_1 \\ \mu_2(uv), if uv \in E_2 \end{cases}$$

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Then if $uv \in E_1, \mu(uv) = \mu_1(uv) \le \sigma_1(u) \wedge \sigma_1(v) \le \sigma(u) \wedge \sigma(v)$ and if $uv \in E_2$, $\mu(uv) = \mu_2(uv) \le \sigma_2(u) \wedge \sigma_2(v) \le \sigma(u) \wedge \sigma(v)$. Therefore (σ, μ) defines a fuzzy graph. This is called the direct sum of two fuzzy graph.

Definition 3: [2] Let G^* = (V, E) be a graph. A pair G = (A, B) is called a picture fuzzy graph on G*, where A= $(\mu_A, \eta_A, \gamma_A)$ is a picture fuzzy set on V and B = $(\mu_B, \eta_B, \gamma_B)$ is a picture fuzzy set on $E \subseteq VXV$ such that for each are $uv \in E$, $\mu_B(u,v) \le \min(\mu_A(u), \mu_A(v))$;$\eta_B(uv) \le \min(\eta_A(u), \eta_A(v))$and $\gamma_B(uv) \ge \max(\gamma_A(u), \gamma_A(v))$

DIRECT SUM OF PICTURE FUZZY GRAPH

Definition 4: Let $G_1 = (A_1, B_1)$ where $A_1=(\mu_{A1}, \eta_{A1}, \gamma_{A1})$ and $B_1=(\mu_{B1}, \eta_{B1}, \gamma_{B1})$ and $G_2 = (A_2, B_2)$ where $A_2=(\mu_{A2}, \eta_{A2}, \gamma_{A2})$ and $B_2=(\mu_{B2}, \eta_{B2}, \gamma_{B2})$ are vertices and edge sets of G_1 and G_2 and denote two picture fuzzy graphs with underlying crisp graph G_1^*: (V_1, E_1) and G_2^*: (V_2, E_2) respectively. Let $V = V_1 \cup V_2$ and $E = \{uv / u, v \in V; uv \in E_1$ or $uv \in E_2$ but not both } Define G= $G_1 \oplus G_2$ by

$$(\mu, \eta, \gamma)(u) = \begin{cases} (\mu_{A1}, \eta_{A1}, \gamma_{A1})(u), if u \epsilon V_1 \\ (\mu_{A2}, \eta_{A2}, \gamma_{A2})(u), if u \epsilon V_2 \\ (\mu_{A1} \vee \mu_{A2}, \eta_{A1} \vee \eta_{A2}. \gamma_{A1} \vee \gamma_{A2})(u)\ , if u \epsilon V_1\ \cap V_2 \end{cases}$$

$$(\mu, \eta, \gamma)(uv) = \begin{cases} (\mu_{B1}, \eta_{B1}, \gamma_{B1})(uv), if uv \epsilon E_1 \\ (\mu_{B2}, \eta_{B2}, \gamma_{B2})(uv), if uv \epsilon E_2 \end{cases}$$

Then if $uv \in E_1, \mu(uv) = \mu_{B1}(uv) \le \mu_{A1}(u) \wedge \mu_{A1}(v)$;
$\eta(uv) = \eta_{B1}(uv) \le \eta_{A1}(u) \wedge \eta_{A1}(v)$; $\gamma(uv) = \gamma_{B1}(uv) \ge \gamma_{A1}(u) \vee \gamma_{A1}(v)$ and
if $uv \in E_2$, $\mu(uv) = \mu_{B2}(uv) \le \mu_{A2}(u) \wedge \mu_{A2}(v)$;
$\eta(uv) = \eta_{B2}(uv) \le \eta_{A2}(u) \wedge \eta_{A2}(v)$; $\gamma(uv) = \gamma_{B2}(uv) \ge \gamma_{A2}(u) \vee \gamma_{A2}(v)$.
Therefore, this G defines a picture fuzzy graph. This is called direct sum of picture fuzzy graph.

Example 1: In Figure1 give an example of the direct sum of two picture fuzzy graph in which distinct edge sets.

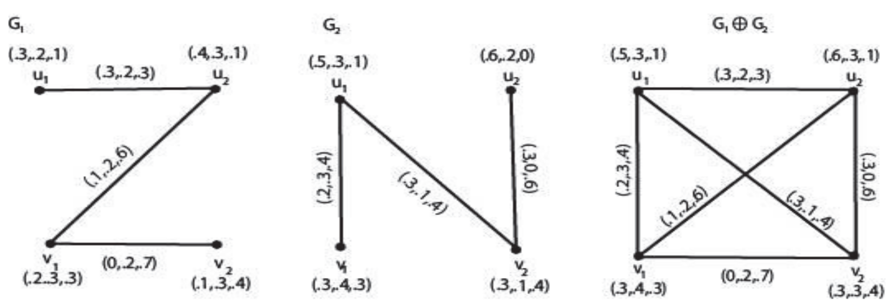

FIGURE 1.

In Figure 2 give an example of the direct sum of two picture fuzzy graphs in which the edge sets are not disjoint

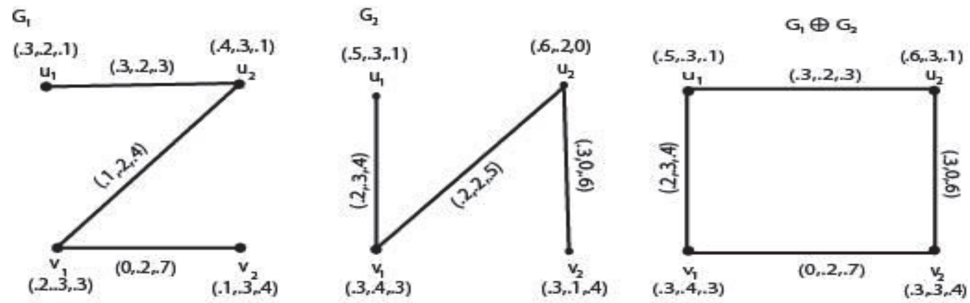

FIGURE 2.

Remark: If G_1 and G_2 are two effective picture fuzzy graphs, their direct sum $G_1 \oplus G_2$ need not be effective picture fuzzy graph in which the following example figure 3

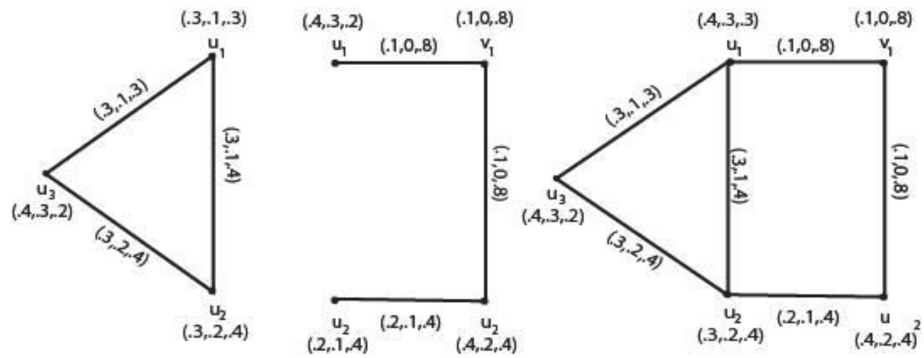

FIGURE 3.

Theorem 1: If G_1 and G_2 are two effective picture fuzzy graphs such that no edge of $G_1 \oplus G_2$ has both ends in $V_1 \cap V_2$ and every edge uv of $G_1 \oplus G_2$ with one end $u \epsilon V_1 \cap V_2$ and uv $\epsilon E_1 (or E_2)$ is such that $\mu_{A1}(u) \geq \mu_{A1}(v)$; $\eta_{A1}(u) \geq \eta_{A1}(v)$;and $\gamma_{A1}(u) \geq \gamma_{A1}(v)$[(or) $\mu_{A2}(u) \geq \mu_{A2}(v)$; $\eta_{A2}(u) \geq \eta_{A2}(v)$;and $\gamma_{A2}(u) \geq \gamma_{A2}(v)$] then $G_1 \oplus G_2$ is an effective picture fuzzy graph.

Proof: Let uv $\epsilon V_1 \cap V_2$ be an edge of $G_1 \oplus G_2$, we have consider two cases,

Case(i)

Given u, v $\notin V_1 \cap V_2$

Then u, v $\epsilon V_1 or V_2$ but not both

Suppose Let u, v ϵV_1, then uv ϵE_1

Therefore, the vertices$\mu(u) = \mu_{A1}(u)$; $\mu(v) = \mu_{A1}(v)$; $\eta(u) = \eta_{A1}(u)$; $\eta(v) = \eta_{A1}(v)$ and $\gamma(u) = \gamma_{A1}(u)$; $\gamma(v) = \gamma_{A1}(v)$

Since G_1 effective picture fuzzy graph.

And the edges $\mu(uv) = \mu_{B1}(uv) = \mu_{A1}(u) \wedge \mu_{A1}(v) = \mu(u) \wedge \mu(v)$;

$\eta(uv) = \eta_{B1}(uv) = \eta_{A1}(u) \wedge \eta_{A1}(v) = \eta(u) \wedge \eta(v)$;

$\gamma(uv) = \gamma_{B1}(uv) = \gamma_{A1}(u) \vee \gamma_{A2}(v) = \gamma(u) \vee \gamma(v)$

The similar proof for u, v ϵV_2

Case(ii)

If u $\epsilon V_1 \cap V_2$, v $\notin V_1 \cap V_2$ (or viceversa)

without loss of generality, assume that v ϵV_1 ,then $\mu(v) = \mu_{A1}(v)$,$\eta(v) = \eta_{A1}(v)$,$\gamma(v) = \gamma_{A1}(v)$

By hypothesis, $\mu_{A1}(u) \geq \mu_{A1}(v)$, $\eta_{A1}(u) \geq \eta_{A1}(v)$, $\gamma_{A1}(u) \geq \gamma_{A1}(v)$

Now $\mu(u)$=$\mu_{A1}(u) \vee \mu_{A2}(u)$ $\eta(u)$=$\eta_{A1}(u) \vee \eta_{A2}(u)$
$\geq \mu_{A1}(u) \geq \mu_{A1}(v) = \mu(v) \geq \eta_{A1}(u) \geq \eta_{A1}(v) = \eta(v)$
So,$\mu(u) \wedge \mu(v)$= $\mu(v)$ So, $\eta(u) \wedge \eta(v)$= $\eta(v)$ and

$\gamma(u)$=$\gamma_{A1}(u) \vee \gamma_{A2}(u)$
$\geq \gamma_1(u) \geq a\gamma_{A1}(v) = \gamma(v)$.
So, $\gamma(u) \vee \gamma(v)$= $\gamma(v)$

Hence, $\mu(uv) = \mu_{A1}(uv)$ $\eta(uv) = \eta_{A1}(uv)$
$= \mu_{A1}(u) \wedge \mu_{A1}(v)$=$\eta_{A1}(u) \wedge \eta_{A1}(v)$
$= \mu_{A1}(v) = \mu(v)$=$\eta_{A1}(v) = \eta(v)$
=$\mu(u) \wedge \mu(v)$ =$\eta(u) \wedge \eta(v)$

and$\gamma(uv) = \gamma_{A1}(uv)$
=$\gamma_{A1}(u) \vee \gamma_{A1}(v)$
$= \gamma_{A1}(v) = \gamma(v)$
=$\gamma(u) \vee \gamma(v)$
Therefore, $G_1 \oplus G_2$ is an effective picture fuzzy graph.

DEGREE OF VERTEX IN THE DIRECT SUM

We can find the degree of vertices in the direct sum $G_1 \oplus G_2$ of two picture fuzzy graph $G_1 and G_2$ in terms of degree of vertices in the picture fuzzy graph. $G_1 and G_2$.

Theorem 2
The degree of a vertex in the direct sum $G_1 \oplus G_2$ in terms of the degree of the vertices $G_1 and G_2$ is given by

$$d_{G_1 \oplus G_2}(u)=\begin{cases} d_{G_1}(u), if\ u\epsilon V_1 - V_2 \\ d_{G_2}(u) if\ u\epsilon V_2 - V_1 \\ d_{G_1}(u) + d_{G_2}(u) if\ u\epsilon\ V_1 \cap V_2, E_1 \cap E_2 = \Phi, \\ d_{G_1}(u) + d_{G_2}(u) - \sum_{uv\in E_1 \cap E_2} \begin{pmatrix} \mu_{B1}(uv) + \mu_{B2}(uv), \eta_{B1}(uv) + \eta_{B2}(uv), \\ \gamma_{B1}(uv) + \gamma_{B2}(uv)\ if u\epsilon\ V_1 \cap V_2\ , E_1 \cap E_2 \neq \Phi \end{pmatrix} \end{cases}$$

Proof:
For any vertex in the direct sum $G_1 \oplus G_2$: (V, E) we have three cases to consider

Case (i)
Either u ϵV_1 or u ϵV_2 but not both. There is no edge incident at u lies in $E_1 \cap E_2$

$$(\mu_{B1} \oplus \mu_{B2})(\mathrm{uv}) = \begin{cases} \mu_{B1}(uv) if \mathrm{u}\ \epsilon V_1\ (i.e)\mathrm{uv}\ \epsilon E_1 \\ \mu_{B2}(uv) if \mathrm{u}\ \epsilon V_2\ (i.e)\mathrm{uv}\ \epsilon E_2 \end{cases}$$

$$(\eta_{B1} \oplus \eta_{B2})(\mathrm{uv}) = \begin{cases} \eta_{B1}(uv) if \mathrm{u}\ \epsilon V_1\ (i.e)\mathrm{uv}\ \epsilon E_1 \\ \eta_{B2}(uv) if \mathrm{u}\ \epsilon V_2\ (i.e)\mathrm{uv}\ \epsilon E_2 \end{cases}$$

$$(\gamma_{B1} \oplus \gamma_{B2})(\mathrm{uv}) = \begin{cases} \gamma_{B1}(uv) if \mathrm{u}\ \epsilon V_1\ (i.e)\mathrm{uv}\ \epsilon E_1 \\ \gamma_{B2}(uv) if \mathrm{u}\ \epsilon V_2\ (i.e)\mathrm{uv}\ \epsilon E_2 \end{cases}$$

Hence if u ϵV_1 , $then d_{G_1 \oplus G_2}(\mathrm{u}) = \sum_{uv\epsilon E_1}(\mu_{B1}(uv), \eta_{B1}(uv), \gamma_{B1}(uv)) = d_{G_1}(u)$

and u ϵV_2 , $then d_{G_1 \oplus G_2}$(u) = $\sum_{uv\epsilon E_2}\left(\mu_{B2}(uv), \eta_{B2}(uv), \gamma_{B2}(uv)\right) = d_{G_2}(u)$

Case (ii)

If u $\epsilon V_1 \cap V_2$ but no edge incident at u lies in $E_1 \cap E_2$. Then any edge incident at u is either in E_1 or in E_2 but not in $E_1 \cap E_2$. Also all these edges are included in $G_1 \oplus G_2 : (V, E)$.
Hence degree of u in $G_1 \oplus G_2$ is given

$d_{G_1 \oplus G_2}(u) = \sum_{uv\epsilon E}\left(\mu_{B1}(uv) \oplus \mu_{B2}(uv), \eta_{B1}(uv) \oplus \eta_{B2}(uv), \gamma_{B1}(uv) \oplus \gamma_{B2}(uv)\right)$

$= \sum_{uv\epsilon E_1}\left(\mu_{B1}(uv), \eta_{B1}(uv), \gamma_{B1}(uv)\right) + \sum_{uv\epsilon E_2}\left(\mu_{B2}(uv), \eta_{B2}(uv), \gamma_{B2}(uv)\right)$

$= d_{G_1}(u) + d_{G_2}(u)$

Case (iii)

If u $\epsilon V_1 \cap V_2$ and some edges incident at u are in $E_1 \cap E_2$. By the definition any edge on $E_1 \cap E_2$. Will not be in $G_1 \oplus G_2$. Then the degree of u in the direct sum $G_1 \oplus G_2$

$$d_{G_1 \oplus G_2}(u) = \sum_{uv \in E}\left(\mu_{B1}(uv) \oplus \mu_{B2}(uv), \eta_{B1}(uv) \oplus \eta_{B2}(uv), \gamma_{B1}(uv) \oplus \gamma_{B2}(uv)\right)$$

$= \sum_{uv\epsilon E_{1-E_2}}\left(\mu_{B1}(uv), \eta_{B1}(uv), \gamma_{B1}(uv)\right) + \sum_{uv\epsilon E_{2-E_1}}\left(\mu_{B2}(uv), \eta_{B2}(uv), \gamma_{B2}(uv)\right)$

$$= \sum_{uv\epsilon E_{1-E_2}}\left(\mu_{B1}(uv), \eta_{B1}(uv), \gamma_{B1}(uv)\right) + \sum_{uv\epsilon E_{2-E_1}}\left(\mu_{B2}(uv), \eta_{B2}(uv), \gamma_{B2}(uv)\right)$$
$$+ \sum_{uv \in E_1 \cap E_2}\left(\mu_{B1}(uv) + \mu_{B2}(uv), \eta_{B1}(uv) + \eta_{B2}(uv), \gamma_{B1}(uv) + \gamma_{B2}(uv)\right)$$
$$- \sum_{uv \in E_1 \cap E_2}\left(\mu_{B1}(uv) + \mu_{B2}(uv), \eta_{B1}(uv) + \eta_{B2}(uv), \gamma_{B1}(uv) + \gamma_{B2}(uv)\right)$$

$= \left[\sum_{uv\epsilon E_{1-E_2}}\left(\mu_{B1}(uv), \eta_{B1}(uv), \gamma_{B1}(uv)\right) + \sum_{uv \in E_1 \cap E_2}\left(\mu_{B1}(uv), \eta_{B1}(uv), \gamma_{B1}(uv)\right)\right]$

$+ \left[\sum_{uv\epsilon E_{2-E_1}}\left(\mu_{B2}(uv), \eta_{B2}(uv), \gamma_{B2}(uv)\right) + \sum_{uv \in E_1 \cap E_2}\left(\mu_{B2}(uv), \eta_{B2}(uv), +\gamma_{B2}(uv)\right)\right]$

$$- \left[\sum_{uv \in E_1 \cap E_2}\left(\mu_{B1}(uv) + \mu_{B2}(uv), \eta_{B1}(uv) + \eta_{B2}(uv), \gamma_{B1}(uv) + \gamma_{B2}(uv)\right)\right] d_{G_1 \oplus G_2}(u)$$

$= d_{G_1}(u) + d_{G_2}(u) - \sum_{uv \in E_1 \cap E_2}\left(\mu_{B1}(uv) + \mu_{B2}(uv), \eta_{B1}(uv) + \eta_{B2}(uv), \gamma_{B1}(uv) + \gamma_{B2}(uv)\right)$

Hence the theorem is proved.

Example 2:

Consider the two picture fuzzy graph $G_1 and G_2$ in which the edge sets are disjoint and their direct sum $G_1 \oplus G_2$ in **figure 4**

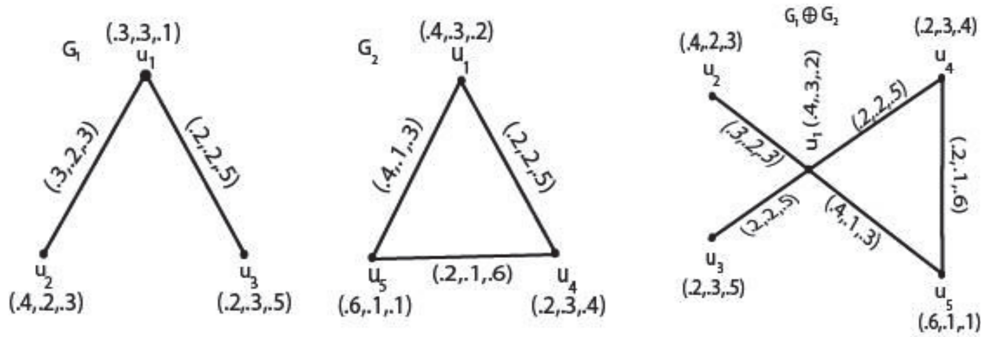

FIGURE 4.

$d_{G_1 \oplus G_2}(u_1)$ = (.2, .2, .5) + (.3, .2, .3) + (.2, .2, .5) + (.4, .1, .3) = (1.1, .7, 1.6)
$d_{G_1}(u_1)$ = (.3, .2, .3) + (.2, .2, .5) = (.5, .4, .8);
$d_{G_2}(u_1)$ = (.4, .1, .3) + (.2, .2, .5) = (.6, .3, .8)$d_{G_1}(u_1) + d_{G_2}(u_1)$ = (1.1, .7, 1.6)

Therefore, $d_{G_1 \oplus G_2}(u_1) = d_{G_1}(u_1) + d_{G_1}(u_2)$

Since there is no edge is common in $G_1 and G_2$. $u_1 \epsilon V_1 \cap V_2$, the degree of $G_1 \oplus G_2$ is the sum of the degree of $G_1 and G_2$. The vertices u_2 and u_3 are in v_1 but not in v_2 and the vertices u_4 and u_5 are in v_2 but not in v_1. Hence degrees of u_2, u_3, u_4 and u_5 in $G_1 \oplus G_2$ is equal to the degree of u_2 and u_3 in G_1 and u_4 and u_5 in G_2.

$d_{G_1 \oplus G_2}(u_2)$ = (.3, .2, .3) ; $d_{G_1 \oplus G_2}(u_3)$ = (.2, .2, .5);
$d_{G_1 \oplus G_2}(u_4)$ = (.2, .2, .5) + (.2, .1, .6) = (.4, .3,1.1)
$d_{G_1 \oplus G_2}(u_5)$= (.2, .1, .6) + (.4, .1, .3) = (.6, .2, .9)

Example 3: Consider the two picture fuzzy graph $G_1 and G_2$ in which the edge sets are not disjoint and their direct sum $G_1 \oplus G_2$ in figure 5

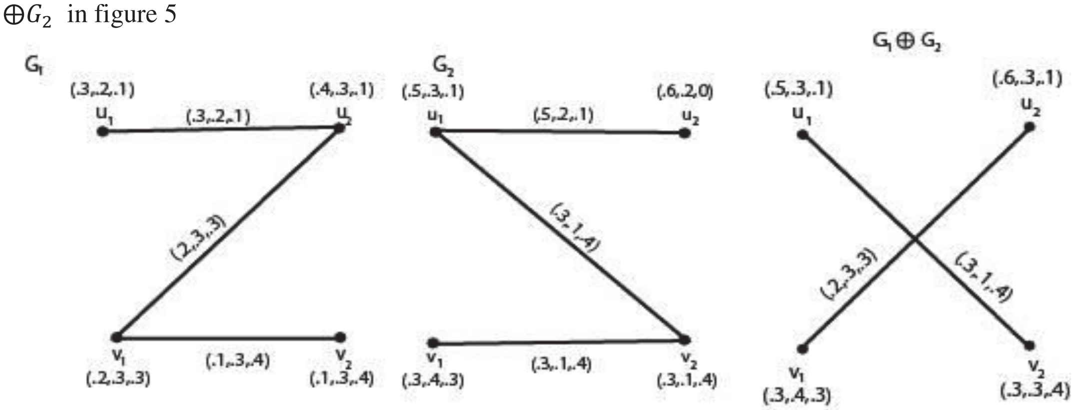

FIGURE 5.

Here $E_1 \cap E_2 = \{u_1u_2, v_1v_2\}$,From the direct sum $G_1 \oplus G_2$, we see that the degrees of $G_1 \oplus G_2$ in terms of the degrees of the vertices in $G_1 and G_2$ are

$$d_{G_1 \oplus G_2}(u_1) = d_{G_1}(u_1) + d_{G_2}(u_1) - \sum_{uv \in E_1 \cap E_2} \begin{pmatrix} \mu_{B1}(u_1u_2) + \mu_{B2}(u_1u_2), \\ \eta_{B1}(u_1u_2) + \eta_{B2}(u_1u_2), \gamma_{B1}(u_1u_2) + \gamma_{B2}(u_1u_2) \end{pmatrix}$$

= (.3, .2, .1) + [(.5, .2, .1) + (.3, .1, .4)] – [(.3, .2, .1) + (.5, .2, .1)]
= (.3, .1, .4)

$$d_{G_1 \oplus G_2}(u_2) = d_{G_1}(u_2) + d_{G_2}(u_2) - \sum_{uv \in E_1 \cap E_2} \begin{pmatrix} \mu_{B1}(u_2u_1) + \mu_{B2}(u_2u_1), \\ \eta_{B1}(u_2u_1) + \eta_{B2}(u_2u_1), \gamma_{B1}(u_2u_1) + \gamma_{B2}(u_2u_1) \end{pmatrix}$$

= [(.3, .2, .1) + (.2, .3, .3)] + (.5, .2, .1) – [(.3, .2, 1) + (.5, .2, .1)]
= (.2, .3, .3)

$$d_{G_1 \oplus G_2}(v_1) = d_{G_1}(v_1) + d_{G_2}(v_1) - \sum_{uv \in E_1 \cap E_2} \begin{pmatrix} \mu_{B1}(v_1v_2) + \mu_{B2}(v_1v_2), \\ \eta_{B1}(v_1v_2) + \eta_{B2}(v_1v_2), \gamma_{B1}(v_1v_2) + \gamma_{B2}(v_1v_2) \end{pmatrix}$$

= [(.1, .3, .4) + (2, .3, .3)] + (.3, .1, .4) - [(.1, .3, .4) + (.3, .1, .4)]
= (.2, .3, .3)

$$d_{G_1 \oplus G_2}(v_2) = d_{G_1}(v_2) + d_{G_2}(v_2) - \sum_{uv \in E_1 \cap E_2} \begin{pmatrix} \mu_{B1}(v_2v_1) + \mu_{B2}(v_2v_1), \\ \eta_{B1}(v_2v_1) + \eta_{B2}(v_2v_1), \gamma_{B1}(v_2v_1) + \gamma_{B2}(v_2v_1) \end{pmatrix}$$

= (.1, .3, .4) + [(.3, .1, .4) + (.3, .1, .4)] – [(.1, .3, .4) + (.3, .1, .4)]
= (.3, .1, .4)

CONCLUSION

We conclude this paper, the direct sum of two picture fuzzy graphs G_1 and G_2.are defined and when two picture fuzzy graph are effective then their direct sum need not be effective is proved. A formula to find the degree of vertices in the direct sum $G_1 \oplus G_2$of two picture fuzzy graph is obtained in terms of degrees of vertices in the picture fuzzy graph G_1 and G_2. To illustrate some examples. A step in that direction is made through this paper.

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