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V. Saranya; M. Srividya; Dr. D. Vidhya

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On Some Applications Of β B – Open Sets On Intuitionistic Topological Space

V.Saranya[1,a)], M.Srividya[2] and Dr.D.Vidhya[3]

[1,2]*Assistant Professor, Department of Mathematics, Shri Sakthikailassh Women's College, Salem, Tamilnadu, India.*
[3]*Assistant professor, Department of Mathematics, Vels Institute of Science, Technology and Advanced studies (VISTAS), Pallavaram, Chennai, Tamilnadu, India.*
[a)] saran.math87@gmail.com

Abstract. In this paper, the author has brought and studied the standards of intuitionistic β B – open sets, β B – interior, β B – closure, β AB – open sets, β C – open sets. Some interesting properties, characterizations and interrelations some of the ideas are established.

INTRODUCTION

The standards of intuitionistic sets and intuitionistic topological spaces were introduced by D. Coker [3]. The principles of intuitionistic B-open sets were delivered and developed by P. Saranya [1]. In this connection, intuitionistic β B-open sets, intuitionistic β AB-open sets and intuitionistic β C-open sets are brought and studied. In this paper a few interrelations are discussed with suitable counter examples.

PRELIMINERIES

DEFINITION 1.1

Let $< X,T >$be an intuitionistic topological space. An intuitionistic set $A =< x, A^1, A^2 >$ in X is said to be

(i) $\beta - open$ if $A \subseteq cl(int(cl(A)))$
(ii) $\alpha - open$ if $A \subseteq int(cl(int(A)))$
(iii) intuitionistic t-open if $Iint(A) = Iint(Icl(A))$
(iv) Intuitionistic semi open set if $A \subseteq Icl(Iint(A))$
(v) intuitionistic semi closed set if $Iint(Icl(A)) \subseteq A$
(vi) intuitionistic semi regular set if it is both intuitionistic semi open set and intuitionistic semi closed set

DEFINITION 1.2

The intuitionistic interior and closure are defined by

(i) (interior of A) $int(A) = \cup \{G: G\ is\ an\ I\ open\ set\ in\ X\ and\ G \subseteq A\}$
(ii) (closure of A) $cl(A) = \cap \{K: K is\ an\ I\ closed\ set\ in\ X\ and\ A \subseteq K\}$

$\boldsymbol{\beta - B}$ OPEN SETS

DEFINITION 2.1

Let $< X,T >$be an intuitionistic topological space. An intuitionistic set $A =< x, A^1, A^2 >$ in X is said to be intuitionistic $\beta - B$ open set if $A = U \cap V$ where $U =< x, U^1, U^2 >$ is β- open set and $V =< x, V^1, V^2 >$ is an intuitionistic $t - open\ set$.

DEFINITION 2.2

Let $< X,T >$ be an intuitionistic topological space and $A =< x, A^1, A^2 >$ be an intuitionistic set in X. Then the intuitionistic $\beta\ B\ interior$ and $\beta\ B\ closure$ are denoted and defined as

$$I\beta Bint(A) = \cup \{G =< x, G^1, G^2 >: G\ is\ intuitionistic\ \beta B\ open\ and\ G \subseteq A\}$$

$$I\beta Bcl(A) = \cap \{K =< x, K^1, K^2 >: K\ is\ intuitionistic\ \beta B\ closed\ and\ A \subseteq K\}$$

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REMARK 2.1

Let $< X, T >$ be an intuitionistic topological space and $A =< x, A^1, A^2 >$ be an intuitionistic set in X. Then

i) $I\beta Bint(A) \subseteq A \subseteq I\beta Bcl(A)$.

ii) A is an intuitionistic $\beta B - closed\ set$ iff $A = I\beta Bcl(A)$.

iii) A is an intuitionistic $\beta B - open\ set$ iff $A = I\beta Bint(A)$.

PROPOSITION 2.1

Let $< X, T >$ be an intuitionistic topological space and $A =< x, A^1, A^2 >$, $B =< x, B^1, B^2 >$ and $G =< x, G^1, G^2 >$be any three intuitionistic sets. Then the following conditions hold:

i) If $A \subseteq B$,then $I\beta Bint(A) \subseteq I\beta Bint(B)$

ii) $I\beta Bint\big(I\beta Bint(A)\big) = I\beta Bint(A)$.

iii) $I\beta Bint(A \cap B) = I\beta Bint(A) \cap I\beta Bint(B)$.

iv) $I\beta Bint(X_\sim) = X_\sim$.

PROOF:

i) $I\beta Bint(A) = \cup \{G: G\ is\ intuitionistic\ \beta B\ open\ and\ G \subseteq A\}$

$\subseteq \cup \{G: G\ is\ intuitionistic\ \beta B\ open\ and\ G \subseteq B\}$

$\subseteq I\beta Bint(B)$

$\therefore\ I\beta Bint(A) \subseteq I\beta Bint(B)$

ii) $I\beta Bint\big(I\beta Bint(A)\big) = \cup \{G: G\ is\ intuitionistic\ \beta B\ open\ and\ G \subseteq I\beta Bint(A)\}$

$\because\ I\beta Bint(A)$is an intuitionistic βB open set,

$$I\beta Bint\big(I\beta Bint(A)\big) = I\beta Bint(A).$$

iii) $I\beta Bint(A \cap B) = \cup \{G: G\ is\ intuitionistic\ \beta B\ open\ and\ G \subseteq (A \cap B)\}$

$= (\cup \{G: G\ is\ intuitionistic\ \beta B\ open\ and\ G \subseteq A\})$

$\cap (\cup \{G: G\ is\ intuitionistic\ \beta B\ open\ and\ G \subseteq B\})$

$= I\beta Bint(A) \cap I\beta Bint(B)$.

$\therefore\ I\beta Bint(A \cap B) = I\beta Bint(A) \cap I\beta Bint(B)$.

iv) $\because X_\sim$ is an intuitionistic $\beta B - open\ set$, $I\beta Bint(X_\sim) = X_\sim$.

PROPOSITION 2.2

Let $< X, T >$ be an intuitionistic topological space and $A =< x, A^1, A^2 >$, $B =< x, B^1, B^2 >$ and $G =< x, G^1, G^2 >$be any three intuitionistic sets. Then the following conditions hold:

i) If $A \subset B$, then $I\beta Bcl(A) \subseteq I\beta Bcl(B)$

ii) I$\beta Bcl\big(I\beta Bcl(A)\big) = I\beta Bcl(A)$

iii) I$\beta Bcl(A) \cup I\beta Bcl(B) = I\beta Bcl(A \cup B)$

iv)$I\beta Bcl(\varphi_{\sim}) = \varphi_{\sim}$

PROOF:

i) I$\beta Bcl(A)$=$\cap \{G: G is\ intuitionistic\ \beta B - closed\ and\ A \subseteq G\}$

$$\subseteq \cap \{G: G\ is\ intuitionistic\ \beta B - closed\ and\ B \subseteq G\}$$

$$\subseteq I\beta Bcl(B)$$

$\therefore$I$\beta Bcl(A) \subseteq I\beta Bcl(B)$

ii)I$\beta Bcl\big(I\beta Bcl(A)\big)$ =$\cap \{G: G\ is\ an\ intuitionistic\ \beta B - closed\ set\ and\ I\beta Bcl(A) \subseteq G\}$

$\because$ $I\beta Bcl(A)$is an intuitionistic $\beta B -$ closed set,

I$\beta Bcl\big(I\beta Bcl(A)\big) = I\beta Bcl(A)$.

iii) $I\beta Bcl(A \cup B)$ =$\cap \{G: G\ is\ an\ intuitionistic\ \beta B - closed\ and\ (A \cup B) \subseteq G$

$$= (\cap \{G: G\ is\ an\ intuitionistic\ \beta B - closed\ and\ A \subseteq G\})$$

$$\cup (\cap \{G: G\ is\ an\ intuiyionistic\ \beta B - closed\ and\ B \subseteq G\})$$

=I$\beta Bcl(A) \cup I\beta Bcl(B)$

$\therefore$ I$\beta Bcl(A) \cup I\beta Bcl(B) = I\beta Bcl(A \cup B)$

iv) $\because$ $\varphi_{\sim}$ is an intuitionistic $\beta B - open\ set$, $I\beta Bint(\varphi_{\sim}) = \varphi_{\sim}$.

REMARK 2.2

Let $< X, T >$ be an intuitionistic topological space.

(i) Any finite intersection of intuitionistic t – open sets is an intuitionistic t – open set.
(ii) Any finite union of intuitionistic t – closed sets is an intuitionistic t – closed set.

PROPOSITION 2.3

Let <X,T> be an intuitionistic topological spaces.

i) Any finite intersection of intuitionistic βB open sets is an intuitionistic βB open set.

ii) Any finite union of intuitionistic βB closed sets is an intuitionistic βB closed set.

PROOF:

i) Let $\{A_i =< x, A^1, A^2 >\}_{i=1}^{n}$ be the finite collection of intuitionistic βB open sets. Then for every i, $A_i = U_i \cap V_i$ where $U_i =< x, U_i^1, U_i^2 >$ is an intuitionistic β open set and $V_i =< x, V_i^1, V_i^2 >$ is an intuitionistic t-open set.

Now, $\bigcap_{i=1}^{n} A_i = \bigcap_{i=1}^{n}(U_i \cap V_i)$

$$= (\bigcap_{i=1}^{n} U_i) \cap (\bigcap_{i=1}^{n} V_i)$$

Since any finite intersection of intuitionistic open sets is an intuitionistic open set and by Remark (2.2),

i) $\bigcap_{i=1}^{n}(A_i)$is an intuitionistic $\beta - B$ open set.

Hence finite intersection of intuitionistic $\beta - B$ open sets is an intuitionistic $\beta - B$ open set.

ii) Proof is analogous to the proof of (i).

REMARK 2.3

The union of any two intuitionistic βB open sets need not be an intuitionistic $\beta\ B$ open sets as shown in the following example.

EXAMPLE 2.1

Let $X = \{p, q, r\}$ be a non-empty set. Let $T = \{\varphi_\sim, X_\sim, A, B\}$ where $A =< x, \{p, q\}, \{r\} >$ and $B =< x, \{p\}, \{r\} >$. Then T is an intuitionistic topology on X. Let $C =< x, \{r\}, \{p, q\} >$ and $D =< x, \{p\}, \{r\} >$. Now C and D are intuitionistic βB open sets in $< X, T >$. But $C \cup D$is not an intuitionistic βB open set.

DEFINITION 2.3

Let $< X, T >$ be an intuitionistic topological space and $A =< x, A^1, A^2 >, U =< x, U^1, U^2 >$ and $V =< x, V^1, V^2 >$ be any three intuitionistic sets. Then A is said to be

(i) An intuitionistic βAB open set if $A = U \cap V$, where U is an β −open set and V is an intuitionistic semi regular set.
(ii) An intuitionistic α^*- open set if $Iint(A) = Iint\ (Icl(Iint(A)))$
(iii) An intuitionistic βC - open set if $A = U \cap V$, where U is an β - open set and V is an intuitionistic α^*- open set

PROPOSITION 2.4

Let $< X, T >$ be an intuitionistic topological space. Every intuitionistic semi regular set is an intuitionistic βAB open set.

PROOF:

Let $< X, T >$ be an intuitionistic topological space. let $A =< x, A^1, A^2 >$ be any intuitionistic semi regular set. Now $A = X_\sim \cap A$.

$\because X_\sim$is an intuitionistic open set and A is an intuitionistic semi regular set, A is an intuitionistic βB open set.

REMARK 2.4

The converse of proposition 2.4 need not be true as shown in the following example.

EXAMPLE 2.2

Let $X = \{p, q, r\}$ be a non-empty set. Let $T = \{\varphi_\sim, X_\sim, A, B, C\}$ where

$A =< x, \{p\}, \{q, r\} >$, $B =< x, \{q\}, \{p\} >$ and $C =< x, \{p, q\}, \{\varphi\} >$. Then T is an intuitionistic topology on X. Let $D =< x, \{\varphi\}, \{p, q\} >$. Now, D is an intuitionistic βAB open set but it is not an intuitionistic semi regular set.

PROPOSITION 2.5

Let $< X, T >$ be an intuitionistic topological space. Every intuitionistic semi regular set is an intuitionistic t-open set.

PROOF:

Let $A =< x, A^1, A^2 >$ be any intuitionistic semi regular set.

$\therefore$ A is both intuitionistic semi open and intuitionistic semi closed.

$$\because A \subseteq Icl(A)$$

$$Iint(A) \subseteq Iint(Icl(A))$$

$$\because Iint\big(Icl(A)\big) \subseteq A, Iint\big(Icl(A)\big) \subseteq Iint(A).$$

$$\therefore Iint(A) = Iint\big(Icl(A)\big).$$

Hence, A is an intuitionistic t open set.

REMARK 2.5

The converse of proposition 2.5 need not be true as shown in the following example.

EXAMPLE 2.3

Let $X = \{p, q, r\}$ be a non-empty set. Let $T = \{\varphi_{\sim}, X_{\sim}, A, B, C\}$ where $A =< x, \{p, q\}, \{\varphi\} >$, $B =< x, \{p, q\}, \{r\} >$ and $C =< x, \{r\}, \{p, q\} >$.

Then T is an intuitionistic topology on X. let $D =< x, \{\varphi\}, \{p, q\} >$. Now, D is an intuitionistic t - open set but it is not an intuitionistic semi regular set.

PROPOSITION 2.6

Let $< X, T >$ be an intuitionistic topological space. Every intuitionistic βAB-open set is an intuitionistic βB-open set.

PROOF:

Let $A =< x, A^1, A^2 >$ be any intuitionistic βAB open set. Therefore, $A = U \cup V$ where $U =< x, U^1, U^2 >$ is an intuitionistic β open set and $V =< x, V^1, V^2 >$ is an intuitionistic semi regular set.

$\because$ Every intuitionistic semi regular set is an intuitionistic t-open set, v is an intuitionistic t-open set. Hence, A is an intuitionistic βB open set.

REMARK 2.6

The converse of proposition 2.6 needn't be true as shown in the following example.

EXAMPLE 2.4

Let $X = \{p, q, r\}$ be a non-empty set. Let $T = \{\varphi_{\sim}, X_{\sim}, A, B, C, D\}$ where

$A =< x, \{p\}, \{q, r\} >$, $B =< x, \{r\}, \{p, q\} >, C =< x, \{p, q\}, \{r\} >$ and $D =< x, \{p, r\}, \{q\} >$. Then T is an intuitionistic topology on X. Let $E =< x, \{q, r\}, \{p\} >$. Now, E is an intuitionistic βB open set but not an intuitionistic βAB open set.

PROPOSITION 2.7

Let $< X, T >$ be an intuitionistic topological space. Every intuitionistic βB open set is an intuitionistic βC open set.

PROOF:

Let $A =< x, A^1, A^2 >$ be any intuitionistic βB open set.

$\therefore A = U \cap V$ where $U =< x, U^1, U^2 >$ is an intuitionistic β open set and V is an intuitionistic t-open set.

$\because$ Every intuitionistic t-open set is an intuitionistic α^*open set, V is an intuitionistic α^*open set. Hence, A is an intuitionistic βC open set.

REMARK 2.7

The converse of proposition 2.7 needn't true as shown within the following example.

EXAMPLE 2.5

Let $X = \{p, q, r, s\}$ be a non-empty set. Let $T = \{\varphi_{\sim}, X_{\sim}, A, B, C, D, E, F, G\}$ where $A =< x, \{p\}, \{q, r, s\} >$, $B =< x, \{p, q\}, \{r, s\} >$, $C =< x, \{r\}, \{p, q, s\} >$, $D =< x, \{r, s\}, \{p, q\} >$,

$E =< x, \{p, r\}, \{q, s\} >$,$F =< \{x, \{p, r, s\}, \{q\} >$ and $G =< x, \{p, q, r\}, \{s\} >$. Now, $H =< x, \{q, r\}, \{p\} >$ is an intuitionistic βC open set but it's not an intuitionistic β B open set.

REMARK 2.8

Clearly the subsequent diagram holds.

Intuitionistic βAB- open set	$\rightarrow$ $\not\leftarrow$	Intuitionistic βB open set	$\rightarrow$ $\not\leftarrow$	Intuitionistic βC open set

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