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Research Article

Analyzation of Maximum Matching and Maximal Matching in Various Graphs

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ABSTRACT
This paper presents the finding of Maximum Matching and Maximal Matching Cardinality in various Graphs G(V,E) where V is the set of Vertices and E is the set of Edges. Maximum Matching is the collection of Maximum non-adjacent edges. Maximal Matching is the collection of minimum possible collection of non-adjacent edges. Maximum Matching Cardinality implies the Maximum possible number of non-adjacent edges in the Graph. Maximal Matching Cardinality implies the minimum possible number of non-adjacent edges. Here it analyses which Graph proceeds the same value for Maximum Matching and Maximal Matching. It also tells about the relation between the number of edges as well as the vertices and Maximum Matching Cardinality and Maximal Matching Cardinality by using the different Graphs.

Keywords: Graph, Matching, Maximum Matching, Vertex, Edge, Maximal Matching, Cardinality.
AMS Classification Key: 05C, 05C70, 91B68, 05C07, 54A25.

INTRODUCTION
A Graph G(V,E) consists of the set of all vertices and the set of all edges[1]. Vertices can be represented by dot or small circle or nodes and edges are represented by the straight line or bended line between the two vertices. The graphs are categorized according to their Characteristics and Structure [2]. Some of the Graph has well defined Structures. It should be reflected in nature. A graph can be classified according to the number of edges and vertices. There are many types of Graphs that can be found in nature and it can be classified by Graph Theory [12]. It plays a vital role in Computer Architecture, Medical Science, Anatomy, Bio Medical and in various Engineering Fields [4].

MATCHING
A Matching [7] is the collection of the non-touching edges. It means that it contains the edges those that does not share a common vertex between them. If the vertex is matched or impregnated, it is a terminal point of the edges in the matching set and it should be placed at once in the matching set. Other vertices are called as unmatched. It is used in Various Engineering Fields such as Organ Transplantation, Marriage Proposal, Horoscope, Blood cells Matching and so on.

Maximum Matching
Maximum Matching is the largest possible collection of non-touching edges in the Graph. If the collection of edges has all the vertices of the given Graph, it is called as Perfect Matching [9].

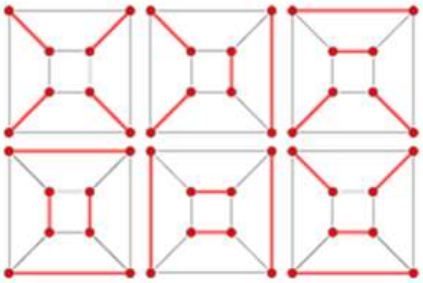


Fig.1: Maximum Matching

Maximal Matching
Maximal Matching is the super set of matching that cannot allow more than one edge in the matching set [10]. It consists of the minimum number of possible edges for matching to cover the non-touching edges in the whole of the

Graph. It is very useful tool for coloring and domination in Graph Theory.

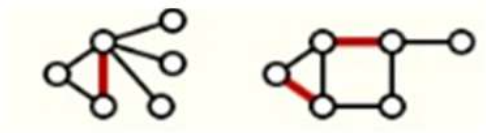


Fig.2: Example of Maximal Matching

Matching Cardinality or Matching Number

Matching Cardinality is the number of collection of edges in the Matching. It is denoted by M(G). Its value always less than the number of edges in the Graph. Maximum Matching Cardinality is denoted by M(G) and Maximal Matching Cardinality is denoted by M'(G). Approximately it gives half of the number of edges in the Graph except a single edge path.

MAXIMAL MATCHING AND MAXIMUM MATCHING

Here to find Maximal Matching and Maximum Matching for the various Graph. The concept of Matching analysed in the different types of the Graph. Some of the following Graph like Null Graph, Regular Graph, Complete Graph,Star Graph and Wheel Graph attains same value for Maximum Matching Cardinaltiy and Maximal Matching Cardinaltiy.

Null Graph, Trivial Graph

Null Graph consists only vertices without edges. Here Maximal Matching Cardinality and Maximum Matching Cardinality both value exists as zero.

Fig.3: Null Graph

Trivial Graph consists only single vertex without edges. This Graph also has samevalue for Maximal Matching Cardinality and Maximum Matching Cardinality as zero.

Fig.4: Trivial Graph

Regular Graph

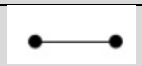
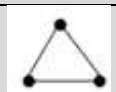
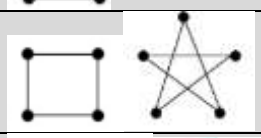
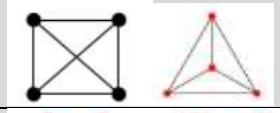
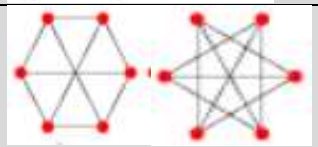
This type of Graph has same degree for all nodes. If the degree of the nodes is *m*, it is called as *m*-regular graph[11]. Here Maximal Matching Cardinality and Maximum Matching Cardinality exceeds the same value.

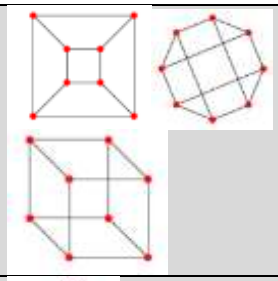
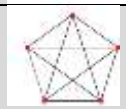
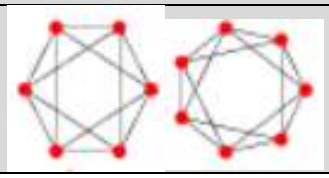
The following Formula can be derived for all Regular Graph by using the number of vertices. This formula can be tested for the various types of the Regular Graph. It is shown as Table 1.

$$\textit{Maximum Matching Cardinality} = \textit{Maximal Matching Cardinality} = \left|\frac{\textit{Number of vertices}}{2}\right|$$

Here Modulus contains whole number only.

Table 1: Regular Graph

S NO	DEGREE OF THE GRAPH(m)	m - REGULAR GRAPH	MAXIMUM MATCHING	MAXIMAL MATCHING
1	1		1	1
2	2		1	1
			2	2
3	3		2	2
4	3		3	3

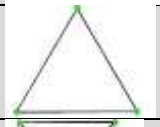
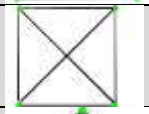
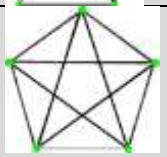
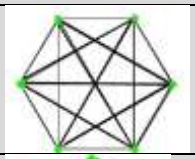
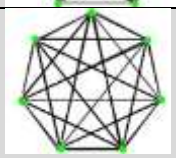
	3		4	4
4	4		2	2
	4		3	3

Complete Graph

In this graph, Every vertices are adjacent to each other vertices by an edge. It is denoted K_n where n is the number of vertices in the graph. It is closed and strongly connected Graph. Each vertices has degree n-1. Here this graph follows a common rule between the number of vertices and Maximal Matching Cardinality and Maximum Matching Cardinality. The formula can be derived by using the number of Vertices in the Graph. It is shown as Table 2. The Formula is,

Maximum Matching Cardinality= Maximal Matching Cardinality $= \left| \frac{Number\ of\ vertices}{2} \right|$

Table 2: Complete Graph

SNO	NUMBER OF VERTICES (K)	K COMPLETE GRAPH	MAXIMUM MATCHING	MAXIMAL MATCHING
1	2		1	1
2	3		1	1
3	4		2	2
4	5		2	2
5	6		3	3
6	7		3	3

Star Graph

It is the one kind of complete bipartite Graph. Here all the nodes are joined to a single node by edges. It is denoted by $K_{1,n}$ where n>2, n is the number of edges in the given Graph. It consists of n+1 vertices and n edges. It looks like a path or tree. Here single edge of the graph is considered as Maximal Matching Cardinality and Maximum Matching Cardinality.

All Star Graph has 1 Matching Cardinality

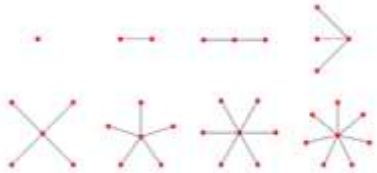


Fig.5: Star Graph

Cycle Graph

It is the closed path Graph. It consists of n vertices and n edges. All the vertices can be joined by an edge in order-wise. It is denoted by C_n where $n \geq 3$. Here Maximal Matching Cardinality and Maximum Matching Cardinality attain the same value for all cycle graphs by the following formula. It attains by using the number of vertices.

Maximum Matching Cardinality= Maximal Matching Cardinality $= \left| \frac{Number\ of\ vertices}{2} \right|$

Here modulus attains the whole number only.

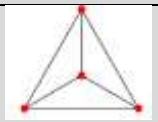
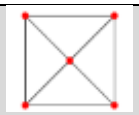
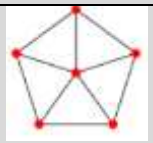
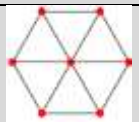
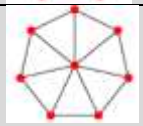
Note: *Number of Vertices = Number of Edges*

Fig.6: Cycle Graph

Wheel Graph

It consists of closed loop. It consists of n+1 edges and n vertices. All the vertices can be connected to a single vertex through an edge as well as they can be joined themselves by an edge in order wise forming a boundary and closed loop. It is shown as Table 3. In this type of the graph also gives the same value for Maximal Matching Cardinality and Maximum Matching Cardinality.

Table 3: Wheel Graph

No	Number of vertices	Wheel graph	Maximum matching	Maximal matching
1	4		2	2
2	5		2	2
3	6		3	3
4	7		3	3
5	8		3	3

CONCLUSION

This paper analyses the Concept of Cardinality for the various Graph which is well defined and similar Structure Graph. Here it is found out which of the Graph attains the same Cardinality value and it is classified in order wise. Here to find the relation between the number of vertices as well as the number of edges and Matching Cardinality. It can be implemented by formulae. It plays a vital role in real life world. Some of them can be exampled such as injecting a drug into injured cells which has Matching Cardinality which is used to give the drug to maximum number of cells at a time, fitting a plumping work which is used to consumption of raw materials and expenses, delivering a message and connecting the people by Social Media like as Twitter, Facebook, Whatsapp, Instagram through the person who has Maximum Matching Cardinality which is easily to connect majority of the people at a short period.

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