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To cite this article: T Sudha and G Jayalalitha 2020 *J. Phys.: Conf. Ser.* **1597** 012022

Fuzzy triangular numbers in - Sierpinski triangle and right angle triangle

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Abstract--Sierpinski Triangle or Sierpinski Gasket or Sierpinski Sieve is a fractal and attractive fixed set with the overall shape of an equilateral triangle, subdivided recursively into smaller equilateral triangles. In this paper the number theoretical aspect of Fuzzy Triangular Numbers (FTN) in Sierpinski Triangle and Right Angle Triangle have been established and some arithmetic operations of α – Cut , Centroid of the Triangle , Approximation of the Triangular Fuzzy Numbers are explained.
Keywords- Fuzzy Numbers, Fractal, Sierpinski triangle, Fuzzy Triangular Numbers, Centroid of the Triangle.

1. Introduction

1.1 Fuzzy numbers

The notion of Fuzzy number's as being convex and normal fuzzy set of some referential set was introduced by Zadeh.[1].

1.2 Fractal

Fractal was coined by Mandelbrot in his fundamental essay from the Latin *fractus* meaning broken, to describe the objects that were too irregular to fit into a traditional geometrical setting [2]. Many fractals have same degree of self-similarity- it made up parts that resemble the whole in some way. Sometimes, the resemblance may be weaker than strict geometrical similarity for example the similarity may be approximate or statistical. Sierpinski Triangle, Cantor Set and Von Koch Curve, Menger sponge, Dragon Curve, Julia Set, Mandelbrot Set are the examples of Fractals.

1.3 Sierpinski triangle [2, 9]

Sierpinski triangle is a fractal described in 1915 by Waclaw Sierpinski. It is a self-similar structure that occurs at different levels of iterations, or magnifications. The Sierpinski Triangle has all the properties of a Fractal.

For triangular area, with each iteration, the side of the inside triangle reduces by a factor of 2. The numbers of these little triangles, on the other had increases not by 4 but by factor of 3. The dimension of self-similar object is then (log3/ log 2) = 1.58 approximately [3]. In this paper in Section 2, Definitions are explained. In Section 3, the operations of Fuzzy Triangular Numbers are discussed using Sierpinski triangle with side 1unit, 2unit, 3unit and 12 unit and Right angle triangle with side 6,8,10 units.[4] Michael Voskoglou explained Fuzzy Triangular Number as a tool for student assessment. And [1] Nagoor Gani discussed a New Operation on Triangular fuzzy Number for solving Fuzzy Linear Programming Problem, Karpagam and Sumathi explained New Approach to solve Fuzzy Linear Programming Problems by the Ranking Function.[8]. Senthilkumar explained about Triangular

Approximation of Fuzzy Numbers- a new Approach[5], they all discussed with randomly chosen Triangular fuzzy numbers. But in this paper Triangular Fuzzy Numbers are chosen from the self-similarity Fractal set Sierpinski triangle and Right angle Triangle explained the basic operations of addition, subtraction ,Approximation of Two fuzzy Triangular Numbers multiplication and division including **α** – Cut operations with graphical representation.

2. Preliminaries

2.1 Fuzzy set [6].

Any set which allows its members to have membership of different grades in the interval [0, 1] is a Fuzzy Set.

2.2 α–cut or α-level set [6].

For any Fuzzy Set $\bar{A}$, $\bar{A}_{\alpha} = \{x/\mu\ \bar{A}(x) \geq \alpha\}$ α is arbitrary, α belongs to [0, 1] is called *α* – Cut or *α*-Level Set.

2.3 Support of fuzzy set

The support of a fuzzy set *F* is a crisp set of all points in the Universe of Discourse *U* (range of all possible values for an input to a Fuzzy system) such that the membership function of F is non-zero [6].

2.4 Fuzzy number [1]

A fuzzy set *A* on *R* must possess at least the following three properties to qualify as a fuzzy number,

- *A* must be a normal fuzzy set;
- A_{α} must be closed interval for every α [0,1]
- The support of *A* , A_{0+} must be bounded

2.5 *Triangular fuzzy numbers (TFN) [4]*

Let *a*, *b*, and *c* be real numbers with $a < b < c$. Then the Triangular Fuzzy Number (TFN) $A= (a, b, c)$ is a Fuzzy Number with membership function:

$$m(x)= \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x<a, x>c \end{cases}$$

2.6 Operation of triangular fuzzy number using function principle [1]

The following are the four operations that can be performed on triangular fuzzy numbers:

Let $C = (a_1, a_2, a_3)$ and $D = (b_1, b_2, b_3)$ then,

(i) Addition: $C + D = (a_1+b_1, a_2+b_2, a_3+b_3)$.

(ii) Subtraction: $C - D = (a_1-b_3, a_2-b_2, a_3-b_1)$.

(iii) Multiplication: $CD = [\min (a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max (a_1b_1, a_1b_3, a_3b_1, a_3b_3)]$.

(iv) Division**:** $C/D = [\min (a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), a_2/b_2, \max (a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3)]$.

2.7 α- cuts of a TFN [4]

The α -cuts A^{α} of a TFN $A = (a, b, c)$, $\alpha \in [0, 1]$ are calculated by the formula

$$A_{\alpha} = [A_l^{\alpha}, A_r^{\alpha}] = [a + \alpha (b - a), c - \alpha (c - b)].$$

2.8 Centroid of the triangle [4]

The coordinates (*X*, *Y*) of the COG of the triangle forming the graph of the TFN (*a*, *b*, *c*) are calculated by the formulas

$$[X = \frac{a+b+c}{3}, Y = \frac{1}{3}]$$

3. Operations on fuzzy triangular numbers

3.1 Equilateral triangle

Figure.1 represents of Fuzzy Triangular Numbers of Equilateral Triangle of side 1 unit

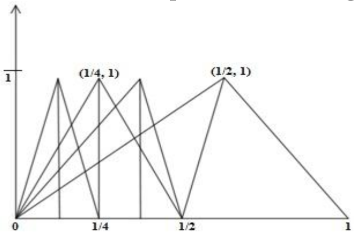

Figure 1. Fuzzy Triangular Numbers

In Figure 2, *G* represents the Centroid of the Fuzzy Triangular numbers triangle *A* (0,0), *B*(1/2,1), and *C*(1,0) and *M* (1/2, 0) is the midpoint of *A* and *C*, and *N* (3/4, 1/2) is the midpoint of *B* and *C.* Based on the definition 2.5 and from the Figure.2

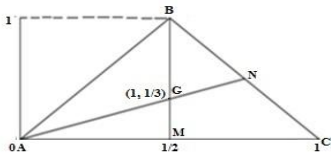

Figure 2. Triangle with Centroid and Median

Equation of the straight line *A* (0, 0) and *N* (3/4, 1/2) on which *AN* lies in

$$6y-4x=0 \tag{1}$$

From the Figure.2 based on the definition 2.5 Equation of the straight line on which *B* (1/2, 1) and *M* (1/2,0)

$$x+1/2=0 \tag{2}$$

The linear system of equations (1), and (2) has a unique solution with the respect to the variables x and y, determining the coordinates of the triangle *COG*, Centroid of the triangle after observing the following

$$D = \begin{vmatrix} 4 & -6 \\ 1 & 0 \end{vmatrix} = 6$$

$$\neq 0 \tag{3}$$

$$D_x = \begin{vmatrix} 0 & -6 \\ 1/2 & 0 \end{vmatrix} = 3 \tag{4}$$

$$D_y = \begin{vmatrix} 4 & 0 \\ 1 & 1/2 \end{vmatrix} = 2 \tag{5}$$

Therefore $x = \frac{1}{2}$, $Y = \frac{1}{3}$

Solving equation (1) and (2), based on the definition 2.8 the coordinates of Centroid of the triangle *G* are (6, 1/3) which is equal to $x = \frac{0+1+1/2}{3}$, $y = \frac{1+0+0}{3}$ that is $(\frac{1}{2}, \frac{1}{3})$.

3.1.1 Approximation of Triangular Fuzzy Numbers [7]

Case 1 For two positive fuzzy numbers

$\tilde{A}$ $<a_1, b_1, c_1> = [(b_1-c_1)\,\alpha + a_1, -(c_1-b_1)\,\alpha + c_1\,] = (S1, \overline{S1})$

$\tilde{B}$ $<a_2, b_2, c_2> = [(b_2-c_2)\,\alpha + a_2, -(c_2-b_2)\,\alpha + c_2\,] = (S2, \overline{S2})$

The product can be calculated,

$$P = \tilde{A} * \tilde{B}$$

$$=[min(s_1s_2, s_1s_2, \overline{s}_1s_2, \overline{s}_1\overline{s}_2), max(s_1s_2, s_1s_2, \overline{s}_1s_2, \overline{s}_1\overline{s}_2)], \alpha \in (0,1) \tag{6}$$

Case II [7]

When any $\tilde{A}$ and $\tilde{B}$ is partial negative and other is positive fuzzy number, the product of $\tilde{A}$ $\tilde{B}$ can not obtain according to the equation (6). The interval of $\alpha \in (0,1]$ will be divided into two parts, according to the intersection point of the two minimum expression in the α cut of *P*. Let us this intersection point is α_s. Then

$$P = \tilde{A} * \tilde{B} = \begin{cases} \min(s_1s_2, s_1s_2, \bar{s}_1s_2, \bar{s}_1\bar{s}_2), \max(s_1s_2, s_1s_2, \bar{s}_1s_2, \bar{s}_1\bar{s}_2], & \alpha \in (0, \alpha s] \\ \min(s_1s_2, s_1s_2, \bar{s}_1s_2, \bar{s}_1\bar{s}_2), \max(s_1s_2, s_1s_2, \bar{s}_1s_2, \bar{s}_1\bar{s}_2], & \alpha \in (\alpha s, 1] \end{cases} \quad (7)$$

Case III

When $\tilde{A}$ is negative and $\tilde{B}$ is positive Fuzzy number. Then the multiplication of $\tilde{A}$ and $\tilde{B}$ can be found as equation (6).

Case IV

When $\tilde{A}$ and $\tilde{B}$ both are negative fuzzy number. Then the multiplication of $\tilde{A}$ and $\tilde{B}$ can also be found as equation (6).The equation (6) and (7) are known as analytical method for fuzzy arithmetic operation.

In actual product the lines connecting the ends points are parabolic and in standard approximation lines connecting the ends points are triangular form.

From the Figure.1, the two positive Fuzzy Triangular Numbers *A (0,1/2,1), B(0,1/4,1/2)* it can derive [5]the approximation of Fuzzy Triangular Numbers shown in the Figure.3

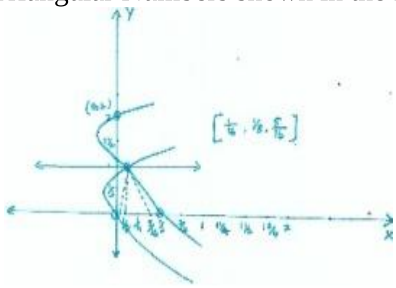

Figure 3. Approximation of Fuzzy Triangular Numbers

3.1.2 Equilateral Triangle with side 12 units

Let us consider the equilateral triangle of side 12 cm. the following are the iterations of equilateral triangle

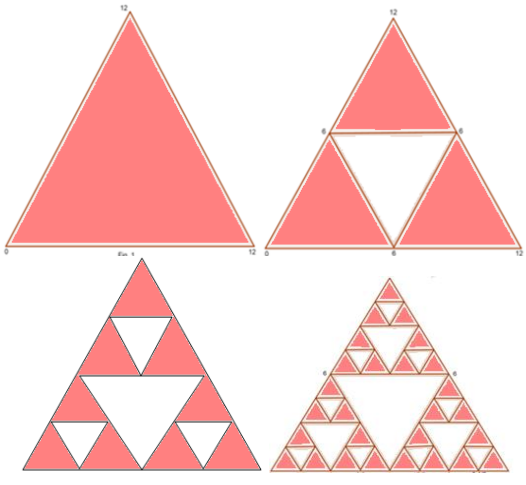

Figure 4. Iteration of Sierpinski Triangle From stage 0 to 3

Table 1. Iteration of Sierpinski Triangle

Iteration number	Scaling	TFNs

I_1	½	(0,6,12)
I_2	1/4	(0,3,6),(6,9,12),
.	.	(0,1.5,3),(3,4.5,6)
.	.	.
.	.	.
.	.	.
I_n	$(1/2)^n$	n-numbers

From the Figure 5 The Triangular Fuzzy numbers are (0,6,12), (0, $\frac{3}{2}$, 3), (0, 3, 6), (0 .6, 9), (3,6,9)……and so on.

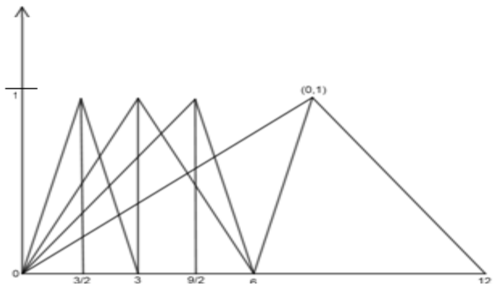

Figure 5. Fuzzy Triangular Numbers

Fuzzy Triangular Numbers (0, 6, 12), (0, 3, 6), (0, $\frac{3}{2}$, 3)……

From the Figure 5 *G* (0, $\frac{3}{2}$, 3) and *F* (0, 3, 6) are the Two TFN

Based on the definition 2.3 (i), (ii)

$G+F=(0+0,\frac{3}{2}+3,3+6)=(0, \frac{9}{2}, 9)$ (8)

$G-F=(0-6,\frac{3}{2}--3,6-3)$ $(-6, -\frac{3}{2}, 3)$ (9)

Based on the definition 2.4 α - Cut,

Operation by G+ F α-Cut

on *G* (0, $\frac{3}{2}$, 3)

$G^{\alpha} = [a_1^{\alpha}, a_3^{\alpha}] = (\frac{3}{2}\alpha, -\frac{3}{2}\alpha + 3)$ (10)

If α = 0

= (0, 3)

If α = 1

= $(\frac{3}{2}, \frac{3}{2})$

Operations by α- cut based on the definition 2.7

F (0, 3, 6),

$F^{\alpha} = [a_1^{\alpha}, a_3^{\alpha}] = (3\alpha, -3\alpha+6)$ (11)

$F^{\alpha} + G^{\alpha} = (\frac{9}{2}\alpha, 9-\frac{9\alpha}{2})$ (12)

α = 0

= (0, 9)

α = 1

= $(\frac{9}{2}, \frac{9}{2})$

$G^{\alpha} + F^{\alpha} = (0, 9/2, 9)$ (13)

From these equations we observe that (8) = (13)

Therefore $G^{\alpha} + F^{\alpha} = G+F$

From the equation (8) and (13) shows that the *sum* and the *difference* of two TFN are also *TFN.*

G (0, $\frac{3}{2}$*, 3), F(0,3,6) are the Two TFN*

Case I Approximation for Multiplication

From the equation (10), (11)

$G^{\alpha} = [\, a_1{}^{\alpha} \;,\; a_3{}^{\alpha}] = (\frac{3}{2}\alpha, -\frac{3}{2}\alpha + 3)$

$F^{\alpha} = [\, a_1{}^{\alpha} \;,\; a_3{}^{\alpha}] = (3\alpha, -3\alpha + 6)$

$G^{\alpha} .\; F^{\alpha} = (\frac{3}{2}\alpha .3\alpha, -\frac{3}{2}\alpha + 3. -3\alpha + 6)$

$\alpha = 0$

$= (0, 18)$

$\alpha = 1$

$= (\frac{9}{2}, \frac{9}{2})$

$$G^{\alpha}(.)\; F^{\alpha} \cong \left(0, \frac{9}{2}, 18\right) \qquad (14)$$

Case II Approximation for Division

F (0, 3, 6), H (3, 6, 9) are the Two TFN

$F^{\alpha} = [a_1{}^{\alpha} \;,\; a_3{}^{\alpha}] = (3\alpha, -\frac{3}{2} + 6)$

$H^{\alpha} = [a_1{}^{\alpha}, \; a_3{}^{\alpha}] = (3\alpha + 3, -3\alpha + 9)$

$F^{\alpha} / \; H^{\alpha} = (\frac{3\alpha}{9-3\alpha} \;,\; \frac{6-3\alpha}{3\alpha+3})$

$\alpha = 0$

$= (0, 2)$

$\alpha = 1$

$= (1/2, 1/2)$

$$F^{\alpha} / \; H^{\alpha} \cong \left(0, \frac{1}{2}, 2\right) \qquad (15)$$

Therefore multiplication and Division of two TFN, although they are FNs, they are not always TFNs. In Figure 6 , G represents the Centroid of the Fuzzy Triangular numbers triangle A (0,0), B(6,1), and C(12,0) and M (6, 0) is the midpoint of A and C, and N (9/2,0) is the midpoint of B and C .

Based on the definition 2.5and from the Figure.6

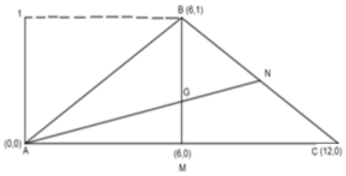

Figure 6. Fuzzy Triangular numbers triangle with Centroid and median

From the Figure.6 Equation of the straight line *A* (0, 0) and *N* (9/2, 1/2) on which *AN* lies in

$$X - 18y = 0 \qquad (16)$$

From the Figure.6 based on the definition 2.5 Equation of the straight line on which B (6, 1) and M (6, 0) lies in

$$X + 0y = 6 \qquad (17)$$

The linear system of equations (16), and (17) has a unique solution with the respect to the variables x and y determining the coordinates of the triangle COG in figure.6, Centroid of the triangle after observing the following

$$D = \begin{vmatrix} 1 & -18 \\ 1 & 0 \end{vmatrix} = 18$$

$$\neq 0 \qquad (18)$$

$$D_x = \begin{vmatrix} 0 & -18 \\ 6 & 0 \end{vmatrix} = 108 \qquad (19)$$

$$D_y = \begin{vmatrix} 1 & 0 \\ 1 & 6 \end{vmatrix} = 6 \qquad (20)$$

Therefore $x=6$, $y=\frac{1}{3}$ Solving equation (16) and (17), based on the definition 2.8 the coordinates of Centroid of the triangle G are $(6, \frac{1}{3})$ which is equal to $x=\frac{0+6+12}{3}$, $y=\frac{1+0+3}{3}$ i.e. $(6, \frac{1}{3})$

Based on Equilateral triangle of side 1 unit and 12 units it is easy to get Fuzzy Triangular Numbers, Approximation of Fuzzy Triangular Numbers, Centroid and Median of the Fuzzy Triangular numbers Triangle for Equilateral Triangle of side 2 units and 3 units.

The following Figures 7, 8, 9 represents side 2 units and Figure.10, 11, 12 represents side 3 units

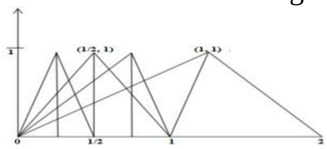

Figure 7. Fuzzy Triangular Numbers (side 2units)

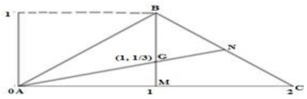

Figure 8. Triangle with Centroid and Median(side 2 units)

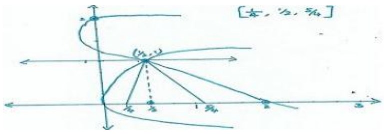

Figure 9. Approximation of Fuzzy Triangular numbers

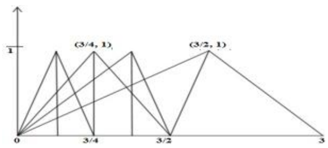

Figure 10. Fuzzy Triangular Numbers (side 3 units)

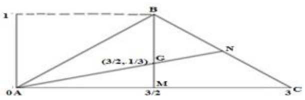

Figure 11. Triangle with Centroid and Median (side 3 units)

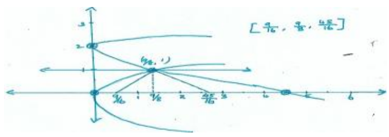

Figure 12. Approximation of Fuzzy Triangular Numbers

3.2. Right angle triangle

Let us consider the right triangle with sides 6cm, 8cm, 10cm respectively. The following are iterations of the right triangle in 5 stages s(0), s(1), S(2), s(3), s(4), s(5) respectively.

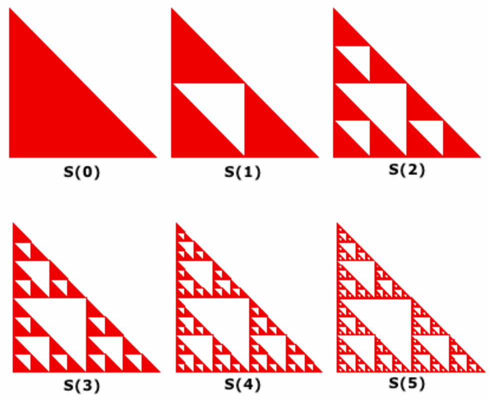

Figure 13. Iteration of Right Angle Triangle from stage 0 to 5

Table 2. Iteration of Right angle triangle

Iteration number	Scaling	TFNs
I_1	1/2	(0,4,8)
I_2	1/4	(0,2,4),(0,1,2),
.	.	(2,3,4,(3,4,5)
.	.	.
.	.	.
I_n	(1/2)n	n-numbers

Multiplication of two fuzzy numbers

.

Based on the Figure.13 Consider two Triangular Fuzzy numbers $A = (0, 4, 8)$ and $B = (3, 4, 5)$
Then their respective α-cuts will be $A^{\alpha} = (4\alpha, -4\alpha+8)$ and $B^{\alpha} = (3 + \alpha, 5- \alpha)$

$A^{\alpha}. B^{\alpha} = (4\ \alpha.\alpha+3,\ \alpha+3.\ - \alpha+5)$

Based on lemma[5]

$$U(\alpha) = 4\alpha^2 + 12\alpha \quad (21)$$

And $$U^1(^{\alpha}) = 4\alpha^2 - 28\alpha + 40 \quad (22)$$

Here $U(\alpha)$ is a parabola meeting α -axis at (0,0) and (0,-3).

$U^1(\alpha)$ is a parabola meeting the same α -axis at(0,2) and (0,5) and meeting u-axis at (40,0).These two curves meet at (16,1). It is obvious that $U(1) = U^1(1) = 16$. Now $m=16, U(0) = 0, U^1(0) = 40$.
Hence the approximate triangular fuzzy from Figure.14is ≅ (8,16, 28) (23)

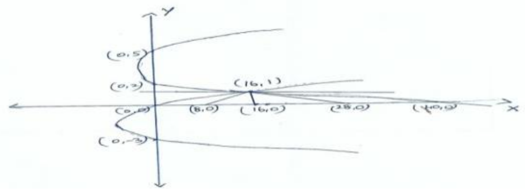

Figure 14. Approximate Fuzzy Triangular numbers

4. Conclusion

Basic operations such as Addition, Subtraction, Multiplication and Division of Fuzzy Triangular Numbers are discussed in Self–Similarity of Sierpinski triangle. This triangle satisfied Fractal property. From this it concludes that Addition and Subtraction of two Fuzzy Triangular Numbers are Fuzzy Triangular Numbers. But Multiplication and Division of two Fuzzy Triangular Numbers are Approximation of Fuzzy Triangular Numbers. Median, Centroid derived from the Triangle formed by

Fuzzy Triangular Numbers triangle are same as normal triangle. Similar discussions are also in Right Angle Triangle.

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