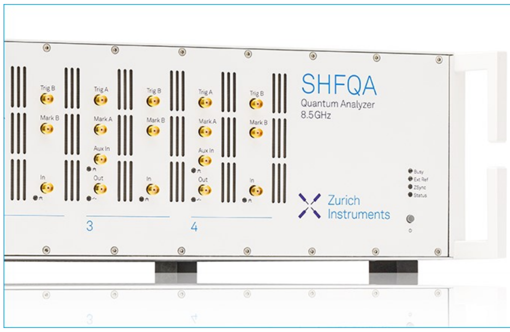
Special labeling for duplicate graph of circular polygon graph with 2 chord

Cite as: AIP Conference Proceedings **2261**, 030064 (2020); https://doi.org/10.1063/5.0017609
Published Online: 05 October 2020

E. Nandagopal, and V. Maheswari



Special Labeling For Duplicate Graph Of Circular Polygon Graph With 2 Chord

E. Nandagopal [1, a)], and V. Maheswari[2,b)],

[1]*Department of Mathematics, Assistant Professor,Sriram Engineering College, Perumalpatu, Tiruvallur 602024, India.*

[2]*Department of Mathematics, Associate Professor, Vels Institute of Science Technology and Advance Studies (VISTAS), Pallavaram, Chennai, India.*

[a)]enandhu77@gmail.com
[b)]Maheswari.sbs@velsuniv.ac.in.

Abstract. In this article, we introduce the special labeling for the Duplicate Graph of behavior thespecial graph of Circular Polygon Graph.$DG(CPG_k)$ $k \geq 2$,On signed product, Bi conditional cordial and Sum divisor cordial labeling are admits.

INTRODUCTION

We need to the currently developing the computer science and engineering. It is required to develop efficient algorithm for solving a challenging problems in science, engineering and technology. Graph theorymeans studiesthevertices and edges. Its role of application to some area in communication, engineering, electrical engineering, civil engineering, operation research, management science, applied physic and mathematics. Rosa introduced the concept of graph labeling in 1967[1].Graph labeling is an assignment of integers to the nodes and lines or both conditions. Graph labeling serve as useful models in a broad range of application such as logical design, circuit design, coding theory, network analysis, computer graphics, complier design, formal languages and topological network etc.BaskarBabujeewas introduced the notion of ‘on signed labeling’ [3] and he proved path graph, cycle graphs, star $K_{1,n}$, Bistar$B_{n,n.}$ etc. Somasundaram and Ponraj have introduced the notion of mean labeling of graphs and they have proved existence thetriangular snakes,wheel and quadrilateral snakes graph etc. Murali, Thirusangu and Madurameenakshihave introduced the Bi-conditional cordial labeling and they have proved existence of flower and coronagraphs [4].Lourdusamy and Patrick have introduced the concept of sum divisor cordial labeling [5]. They have proved existence the path, comb, star, and square graph etc. Nandagopal, Maheswari and Vijayakumarhave proved existence thesum and difference cordial, on signed, sum divisor cordial and sum difference cordial labeling for the duplicate graph of quadrilateral snake graph [6]. Nandagopal and Maheswari have proved the existence of cordial, product cordial and sum divisor of 3-equitable cordial labeling for the duplicate graph of double quadrilateral flow graph[7] and bi conditional cordial labeling for the duplicate graph of quadrilateral snake, double quadrilateral, and triangular ladder graph[8].

Advances in Applicable Mathematics - ICAAM2020
AIP Conf. Proc. 2261, 030064-1–030064-10; https://doi.org/10.1063/5.0017609
Published by AIP Publishing. 978-0-7354-2021-2/$30.00

PRELIMINARIES

In this chapter, we give the basic definitions relevant to this paper. Let $G(N, L)$ be a finite, simple and undirected graph with N Nodes and L Lines.Let $G(N, L)$ be a simple graph, A duplicate graph of G is $DG = (N_1, L_1)$, where the nodes set $N_1 = N \cup N'$ and $N \cap N' = \{\emptyset\}$ and $g: N \to N'$ is bijective (for $n \in N$, we write $g(n) = n'$ Convenience) and the lines set L_1 of DG is defined as the line ab is in L if and only if both ab' and $a'b$ are lines in L_1[2].

Definitions

I. **Circular Polygon Graph:** A Graph G(V, E) is a circle polygon graph is an intersection graph of a set of convex polygons all of whose vertices lie on a common circle.Its denoted CP_k, $k \geq 1$.Clearly it has $5k + 1$ nodes and $8k$ lines. The duplicate graph of circular polygon graph $DG(CPG_k)$ is obtained from the chords graph by joining v_i to v_{i+5} and v_{i+1} and v_{i+4} respectively.Clearly, it has $10k + 2$ nodes and $16k$ lines. Where p is a number of chords(For Convenience $2p = k$ copies)

II. **On Signed product Labeling:** A function $g: N(G) \to \{1, -1\}$ with induced lines labeling of graph $g^*: L(G) \to \{1, -1\}$ defined by $g^*(uv) = f(u).f(v)$ and $|L_g(-1) - L_g(1)| \leq 1$, $|N_g(-1) - N_g(1)| \leq 1$.

III. **Bi-Conditional Cordial Labeling:** A function $g: N(G) \to \{0,1\}$ with induced lines labeling of graph $g^*: L(G) \to \{0,1\}$ defined by $g^*(uv) = \begin{cases} 1: & if\ g(u) = g(v) \\ 0: & if\ g(u) \neq g(v) \end{cases}$ for every $uv \in L(G)$, and $|L_g(0) - L_g(1)| \leq 1$, $|N_g(0) - N_g(1)| \leq 1$.

IV. **Sum Divisor Cordial Labeling:** A function $g: N \to \{1,2,3 \ldots\ldots.. |G(N)|\}$ with induced lines labeling of graph $g^*: L(G) \to \{0, 1\}$ defined by $g^*(uv) = \begin{cases} 1: & if\ \ 2/g(u) + g(v) \\ 0: & otherwise \end{cases}$, further $|L_g(1) - L_g(0)| \leq 1$.

MAIN RESULTS

Algorithm **1**:

Structure the duplicate graph of circular polygon graph $DG(\ CPG_k)$, $k \geq 2$.

Let $V \to \{n_1, n_2, n_3, \ldots\ldots. n_k. n_1', n_2', n_3', \ldots\ldots. n_k'.\}$ $E \to \{l_1, l_2, l_3 \ldots\ldots l_k. l_1', l_2', l_3' \ldots\ldots l_k'.\}$

If $1 \leq m \leq k$

$\emptyset(n_{10m-9}n_{10m-8}') \to l_{16m-15}$, $\emptyset(n_{10m-9}n_{10m-7}') \to l_{16m-14}$, $\emptyset(n_{10m-9,}n_{10m-4}') \to l_{16m-13}$

$(n_{10m-8}n_{10m-6}') \to l_{16m-12}$, $\emptyset(n_{10m-8}n_{10m-5}') \to l_{16m-11}$ $\emptyset(n_{10m-7}n_{10m-5}') \to l_{16m-10}$

$\emptyset(n_{10m-6}n_{10m-4}') \to l_{16m-9}$. $\emptyset(n_{10m-5}n_{10m-4}') \to l_{16m-8}$. $\emptyset(n_{10m-4}n_{10m-3}') \to l_{16m-7}$.

$\emptyset(n_{10m-4,}n_{10m-2}') \to l_{16m-6}$. $\emptyset(n_{10m-4}n_{10m+1}') \to l_{16m-5}$. $\emptyset(n_{10m-3}n_{10m-1}') \to l_{16m-4}$.

$\emptyset(n_{10m-2}n_{10m-1}') \to l_{16m-3}$. $\emptyset(n_{10m-2}n_{10m}') \to l_{16m-2}$. $\emptyset(n_{10m-1}n_{10m+1}') \to l_{16m-1}$.

$\emptyset(n_{10m}n'_{10m+1}) \to l_{16m}$. $\emptyset(n'_{10m-9}n_{10m-6}) \to l'_{16m-15}$, $\emptyset(n'_{10m-9}n_{10m-7}) \to l'_{16m-14}$, $\emptyset\left(n'_{10m-9,}n_{10m-4}\right) \to l'_{16m-13}$,

$(n'_{10m-8}n_{10m-6}) \to l'_{16m-12}$, $\quad \emptyset(n_{10m-8}n_{10m-5}) \to l'_{16m-11}$. $\emptyset(n'_{10m-7}n_{10m-5}) \to l'_{16m-10,}$

$\emptyset(n'_{10m-6}n_{10m-4}) \to l'_{16m-9.}\emptyset(n'_{10m-5}n_{10m-4}) \to l'_{16m-8.}\emptyset(n'_{10m-4}n_{10m-3}) \to l'_{16m-7.}$

$\emptyset\left(n'_{10m-4,}n_{10m-2}\right) \to l'_{16m-6.}\emptyset(n'_{10m-4}n_{10m+1}) \to l'_{16m-5.}\emptyset(n'_{10m-3}n_{10m-1}) \to l'_{16m-4.}$

$\emptyset(n'_{10m-2}n_{10m-1}) \to l'_{16m-3.}\emptyset(n_{10m-2}n_{10m}) \to l'_{16m-2.}\emptyset(n'_{10m-1}n_{10m+1}) \to l'_{16m-1.}$

$\emptyset(n'_{10m}n_{10m+1}) \to l'_{16m.}$

Illustration:

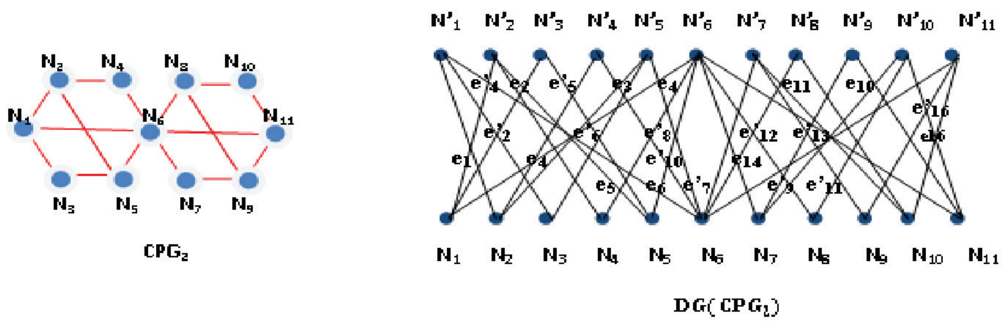


FIGURE 1: $\mathbf{DG(\,CPG_k),\ k \geq 2}$

On signed product labeling for Let

$V \to \{n_1, n_{2,}n_{3,} \dots \dots .\, n_{5k+1}.\, n'_1, n'_2, n'_3, \dots \dots .\, n'_{5k+1}.\}E \to \{l_1, l_2, l_3 \dots \dots l_{8k.}l'_1, l'_2, l'_3 \dots \dots l'_{8k.}\}$

Algorithm 2 : Allocations of Labeled nodes

$For 1 \leq m \leq k$	$For\ 1 \leq m \leq k$
$\emptyset(n_{10m-4}) = \emptyset(n_{10m-6}) = -1.$	$\emptyset(n'_{10m-9}) = \emptyset(n'_{10m-8}) = 1$
$\emptyset(n_{10m-7}) = \emptyset(n_{5m}) = 1.$	$\emptyset(n'_{10m-7}) = \emptyset\left(n'_{10m-5}\right) = -1$
$\emptyset(n_{10m-8}) = 1\emptyset(n_{10m-9}) = 1$	$\emptyset(n'_{10m-6}) = \emptyset(n'_{10m-2}) = 1$
$\emptyset(n_{10m-3}) = \emptyset(n_{10m-1}) = -1$	$\emptyset(n'_{10m-4}) = \emptyset(n'_{10m-3}) = -1$
$\emptyset(n_{10m-2}) = \emptyset(n_{10m}) = 1$	$\emptyset(n'_{10m-1}) = 1, \emptyset(n'_{10m}) = -1$
$if\ m = 5k + 1$	$if\ m = 10k - 4$
$\emptyset(n_m) = -1.$	$\emptyset'_m = -1$

Algorithm: 2.1 Allocation of labeled lines

Let $\mathrm{V} \to \{\mathrm{n}_1, \mathrm{n}_2, \dots \dots, \mathrm{v}_{5\mathrm{k}+1}.\, \mathrm{n}'_1, \mathrm{n}'_2, \dots \dots .\, \mathrm{n}'_{5\mathrm{k}+1}\}$, $\mathrm{E} \to \{\mathrm{l}_1, \mathrm{l}_2, \mathrm{l}_3 \dots \dots, \mathrm{l}_{8\mathrm{m}}.\mathrm{l}'_1, \mathrm{l}_2{}' \dots \dots .\, \mathrm{l}'_{8\mathrm{m}}\}$ be the set of nodes and lines of the duplicate graph of Circular polygon graph $DG(CPG_k)$ using the algorithm 4, each of the $5k + 1$ nodes receive labeled 0 and 1 respectively.

Afunction $\emptyset: N(G) \to \{1, -1\}$ with induced lines labeling of graph $\emptyset^*: L(G) \to \{1, -1\}$ defined by $\emptyset^*(uv) = f(u).f(v)$, and $|L_\emptyset(-1) - L_\emptyset(1)| \leq 1$, $|N_\emptyset(-1) - N_\emptyset(1)| \leq 1$.

Case 1

For k is even

m = 2 to k step 2

do

{

$\emptyset^*(l_1) = \emptyset^*(l_{17}) = \emptyset^*(l_{33}) = \cdots = \emptyset^*(l_{8k-15}) \to -1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l_5 l_6) = \emptyset^*(l_{21} l_{22}) = \emptyset^*(l_{37} l_{38}) = \cdots = \emptyset^*(l_{8k-11} l_{8k-10}) \to -1$ /*up to $k$lines*/

$\emptyset^*(l_8) = \emptyset^*(l_{24}) = \emptyset^*(l_{40}) = \cdots = \emptyset^*(l_{8k-8}) \to -1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l_{10} l_{11} l_{12}) = \emptyset^*(l_{26} l_{27} l_{28}) = \cdots = \emptyset^*(l_{8k-6} l_{8k-5} l_{8k-4}) \to -1$ /*up to $\frac{3k}{2}$lines*/

$\emptyset^*(l_{14} l_{15}) = \emptyset^*(l_{30} l_{31}) = \emptyset^*(l_{46} l_{47}) = \cdots = \emptyset^*(l_{8k-2} l_{8k-1}) \to -1$ /*up to $k$lines*/

$\emptyset^*(l'_3 l'_4) = \emptyset^*(l'_{19} l'_{20}) = \emptyset^*(l'_{35} l'_{36}) = \cdots = \emptyset^*(l'_{8k-13} l'_{8k-12}) \to -1$/*up to $k$lines*/

$\emptyset^*(l'_6 l'_7) = \emptyset^*(l'_{22} l'_{23}) = \emptyset^*(l'_{38} l'_{39}) = \cdots = \emptyset^*(l'_{8k-10} l'_{8k-9}) \to -1$/*up to $k$lines*/

$\emptyset^*(l'_{10}) = \emptyset^*(l'_{26}) = \emptyset^*(l'_{42}) = \cdots = \emptyset^*(l'_{8k-6}) \to -1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l'_{13} l'_{15}) = \emptyset^*(l'_{29} l'_{31}) = \emptyset^*(l'_{45} l'_{47}) = \cdots = \emptyset^*(l'_{8k-3} l'_{8k-1}) \to -1$/*up to $k$lines*/

}

End the processes. **Thus, the $8k$ lines received label -1.**

Similarly, the remaining $8k$lines label 1 as below

{

do

$\emptyset^*(l_2 l_3) = \emptyset^*(l_{18} l_{19}) = \emptyset^*(l_{34} l_{35}) = \cdots = \emptyset^*(l_{8k-14} l_{8k-13}) \to 1$ /*up to $k$lines*/

$\emptyset^*(l_4) = \emptyset^*(l_{20}) = \emptyset^*(l_{36}) = \cdots = \emptyset^*(l_{8k-12}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l_7) = \emptyset^*(l_{23}) = \emptyset^*(l_{39}) = \cdots = \emptyset^*(l_{8k-9}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l_9) = \emptyset^*(l_{25}) = \emptyset^*(l_{41}) = \cdots = \emptyset^*(l_{8k-7}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l_{13}) = \emptyset^*(l_{29}) = \emptyset^*(l_{45}) = \cdots = \emptyset^*(l_{8k-3}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l_{16}) = \emptyset^*(l_{32}) = \emptyset^*(l_{48}) = \cdots = \emptyset^*(l_{8k}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l'_1 l'_2) = \emptyset^*(l'_{17} l'_{18}) = \emptyset^*(l'_{33} l'_{34}) \ldots = \emptyset^*(l'_{8k-15} l'_{8k-14}) \to 1$ /*up to $k$lines*/

$\emptyset^*(l'_5) = \emptyset^*(l'_{21}) = \emptyset^*(l'_{37}) = \cdots = \emptyset^*(l'_{8k-11}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l'_8) = \emptyset^*(l'_{24}) = \emptyset^*(l'_{32}) = \cdots = \emptyset^*(l'_{8k-8}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l'_9) = \emptyset^*(l'_{25}) = \emptyset^*(l'_{41}) = \cdots = \emptyset^*(l'_{8k-7}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l'_{11} l'_{12}) = \emptyset^*(l'_{27} l'_{28}) = \emptyset^*(l'_{43} l'_{44}) \ldots = \emptyset^*(l'_{8k-5} l'_{8k-4}) \to 1$ /*up to $k$lines*/

$\emptyset^*(l'_{14}) = \emptyset^*(l'_{30}) = \emptyset^*(l'_{46}) = \cdots = \emptyset^*(l'_{8k-2}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l'_{16}) = \emptyset^*(l'_{32}) = \emptyset^*(l'_{48}) = \cdots = \emptyset^*(l'_{8k}) \to 1$ /*up to $\frac{k}{2}$lines*/

}

End the processes. **Thus, the $8k$ lines received label 1.**

Hence, the duplicate graph of $\mathrm{DG}(\mathrm{CPG_k})$, $k \geq 2$,admit the on signed product labeling even k copies.

Case 2

For k is odd

$m = 2$ to k step 2

do

{

$\emptyset^*(l_1 l_8) = \emptyset^*(l_{17} l_{24}) = \emptyset^*(l_{33} l_{40}) = \cdots = \emptyset^*(l_{8k-7} l_{8k}) \rightarrow -1$ /*up to $k+1$lines*/

$\emptyset^*(l_5 l_6) = \emptyset^*(l_{21} l_{22}) = \emptyset^*(l_{37} l_{38}) = \cdots = \emptyset^*(l_{8k-3} l_{8k-2}) \rightarrow -1$ /*up to $k+1$lines*/

$\emptyset^*(l_{10} l_{11}) = \emptyset^*(l_{26} l_{27}) = \emptyset^*(l_{42} l_{43}) = \cdots = \emptyset^*(l_{8k-14} l_{8k-13}) \rightarrow -1$ /*up to $k-1$lines*/

$\emptyset^*(l_{14} l_{15}) = \emptyset^*(l_{30} l_{31}) = \emptyset^*(l_{46} l_{47}) = \cdots = \emptyset^*(l_{8k-10} l_{8k-9}) \rightarrow -1$ /*up to $k-1$lines*/

$\emptyset^*(l_{12}) = \emptyset^*(l_{26}) = \emptyset^*(l_{42}) = \cdots = \emptyset^*(l_{8k-12}) \rightarrow -1$ /*up to $\frac{k-1}{2}$lines*/

$\emptyset^*(l'_3 l'_4) = \emptyset^*(l'_{19} l'_{20}) = \emptyset^*(l'_{35} l'_{36}) \ldots = \emptyset^*(l'_{8k-5} l'_{8k-4}) \rightarrow -1$ /*up to $k+1$lines*/

$\emptyset^*(l'_6 l'_7) = \emptyset^*(l'_{22} l'_{23}) = \emptyset^*(l'_{38} l'_{39}) \ldots = \emptyset^*(l'_{8k-2} l'_{8k-1}) \rightarrow -1$ /*up to $k+1$lines*/

$\emptyset^*(l'_{10}) = \emptyset^*(l'_{26}) = \emptyset^*(l'_{42}) = \cdots = \emptyset^*(l'_{8k-14}) \rightarrow -1$ /*up to $\frac{k-1}{2}$lines*/

$\emptyset^*(l'_{13} l'_{15}) = \emptyset^*(l'_{29} l'_{31}) = \emptyset^*(l'_{45} l'_{47}) \ldots = \emptyset^*(l'_{8k-11} l'_{8k-9}) \rightarrow -1$ /*up to $k-1$lines*/

}

End the processes.

Thus, the $8k$ lines received label -1.

Similarly, the remaining $8k$lines label 1 as below

{

do

$\emptyset^*(l_2 l_3) = \emptyset^*(l_{18} l_{19}) = \emptyset^*(l_{34} l_{35}) = \cdots = \emptyset^*(l_{8k-6} l_{8k-5}) \rightarrow 1$ /*up to $k+1$lines*/

$\emptyset^*(l_4 l_7) = \emptyset^*(l_{20} l_{23}) = \emptyset^*(l_{36} l_{39}) = \cdots = \emptyset^*(l_{8k-4} l_{8k-1}) \rightarrow 1$ /*up to $k+1$lines*/

$\emptyset^*(l_9 l_{13}) = \emptyset^*(l_{25} l_{29}) = \emptyset^*(l_{41} l_{45}) = \cdots = \emptyset^*(l_{8k-15} l_{8k-11}) \rightarrow 1$ /*up to $k-1$lines*/

$\emptyset^*(l'_{11} l'_{12}) = \emptyset^*(l'_{27} l'_{28}) = \emptyset^*(l'_{43} l'_{44}) \ldots = \emptyset^*(l'_{8k-13} l'_{8k-12}) \rightarrow 1$ /*up to $k-1$lines*/

$\emptyset^*(l_{16}) = \emptyset^*(l_{32}) = \emptyset^*(l_{48}) = \cdots = \emptyset^*(l_{8k-8}) \rightarrow 1$ /*up to $\frac{k-1}{2}$lines*/

$\emptyset^*(l'_1 l'_2) = \emptyset^*(l'_{17} l'_{18}) = \emptyset^*(l'_{33} l'_{34}) \ldots = \emptyset^*(l'_{8k-7} l'_{8k-6}) \rightarrow 1$ /*up to $k+1$lines*/

$\emptyset^*(l'_5 l'_8) = \emptyset^*(l'_{21} l'_{24}) = \emptyset^*(l'_{37} l'_{40}) \ldots = \emptyset^*(l'_{8k-3} l'_{8k}) \rightarrow 1$ /*up to $k+1$lines*/

$\emptyset^*(l'_9) = \emptyset^*(l'_{25}) = \emptyset^*(l'_{41}) = \cdots = \emptyset^*(l'_{8k-15}) \rightarrow 1$ /*up to $\frac{k-1}{2}$lines*/

$\emptyset^*(l'_{14} l'_{16}) = \emptyset^*(l'_{30} l'_{32}) = \emptyset^*(l'_{46} l'_{48}) \ldots = \emptyset^*(l'_{8k-10} l'_{8k-8}) \rightarrow 1$ /*up to $k-1$lines*/

}

End the processes.

Thus, the $8k$ lines received label 1.

Hence, the duplicate graph of $\mathrm{DG}(\mathrm{CPG}_k)$, $k \geq 2$,admit the on signed product labeling odd k copies.

Illustration:

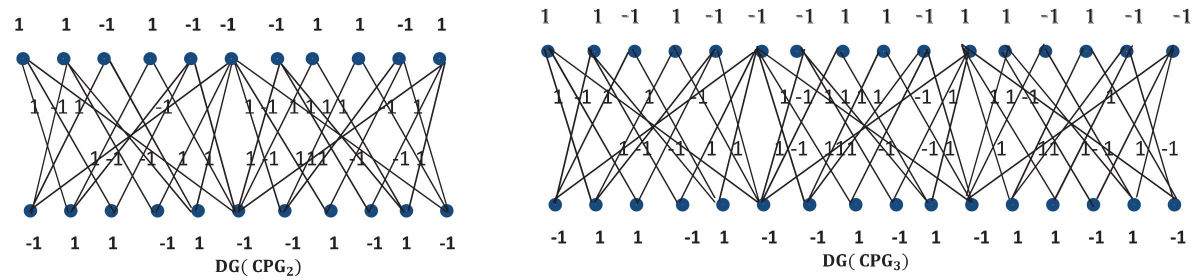


FIGURE 2:Result 2 of Bi Conditional CL for $\mathbf{DG}(\mathbf{CPG_k})$, $\mathbf{k \geq 2}$

Algorithm 3: Allocation of Labeled Nodes

For$1 \leq m \leq k$	For $1 \leq m \leq k$
$fix\ n_1 \rightarrow 1$ $\emptyset(n_{5m-3}) = \emptyset(n_{5m-2}) = \emptyset(n_{5m}) = 0.$ $\emptyset(n_{5m-1}) = 1.$ $if\ m = 5k+1$ $\emptyset(n_m) = 1.$	$fix\ n_1^{'} \rightarrow 0$ $\emptyset(n_{5m-3}^{'}) = \emptyset(n_{5m-1}^{'}) = \emptyset(n_{5m}^{'}) = 1.$ $\emptyset(n_{5m-2}^{'}) = 0.$ $if\ m = 5k+1$ $\emptyset(n_m^{'}) = 0.$

Algorithm: 3.1 Allocation of labeled lines

Let$\mathrm{V} \rightarrow \{\mathrm{n}_1, \mathrm{n}_2, \ldots\ldots, \mathrm{v}_{5k+1}. \mathrm{n}_1^{'}, \mathrm{n}_2^{'}, \ldots\ldots. \mathrm{n}_{5k+1}^{'}\}$, $\mathrm{E} \rightarrow \{\mathrm{l}_1, \mathrm{l}_2, \mathrm{l}_3 \ldots\ldots, \mathrm{l}_{8m}. \mathrm{l}_1^{'}, \mathrm{l}_2^{'} \ldots\ldots. \mathrm{l}_{8m}^{'}\}$ be the set of nodes and lines of the duplicate graph of Circular polygon graph$DG(CPG_k)$using the algorithm 5, each of the $5k+1$ nodes receive labeled 0 and 1 respectively.

A function $\emptyset: N(G) \rightarrow \{0,1\}$ with induced lines labeling of graph $\emptyset^*: \mathrm{L(G)} \rightarrow \{0,1\}$ defined by $\emptyset^*(uv) = \begin{cases} 1: & if\ \emptyset(u) = \emptyset(v) \\ 0: & if\ \emptyset(u) \neq \emptyset(v) \end{cases}$ for every $uv \in L(G)$, and$|L_\emptyset(0) - L_\emptyset(1)| \leq 1$,$|N_\emptyset(0) - N_\emptyset(1)| \leq 1$.

$For\ i = 2\ to\ m$

do

{

$\emptyset^*(\mathrm{l}_1) = \emptyset^*(\mathrm{l}_9) = \emptyset^*(\mathrm{l}_{17}) = \cdots = \emptyset^*(\mathrm{l}_{8k-7}) \rightarrow 1$ /*up to $k$lines*/

$\emptyset^*(\mathrm{l}_8) = \emptyset^*(\mathrm{l}_{16}) = \emptyset^*(\mathrm{l}_{24}) = \cdots = \emptyset^*(\mathrm{l}_{8k}) \rightarrow 1$ /*up to $k$lines*/

$\emptyset^*(\mathrm{l}_1^{'}\mathrm{l}_2^{'}) = \emptyset^*(\mathrm{l}_9^{'}\mathrm{l}_{10}^{'}) = \emptyset^*(\mathrm{l}_{17}^{'}\mathrm{l}_{18}^{'}) = \cdots = \emptyset^*(\mathrm{l}_{8k-7}^{'}\mathrm{l}_{8k-6}^{'}) \rightarrow 1$ /*up to $2k$lines*/

$\emptyset^*(\mathrm{l}_6^{'}\mathrm{l}_7^{'}\mathrm{l}_8^{'}) = \emptyset^*(\mathrm{l}_{14}^{'}\mathrm{l}_{15}^{'}\mathrm{l}_{16}^{'}) = \emptyset^*(\mathrm{l}_{22}^{'}\mathrm{l}_{23}^{'}\mathrm{l}_{24}^{'}) = \cdots = \emptyset^*(\mathrm{l}_{8k-2}^{'}\mathrm{l}_{8k-1}^{'}\mathrm{l}_{8k}^{'}) \rightarrow 1$ /*up to $3k$lines*/

$\emptyset^*(\mathrm{l}_4^{'}) = \emptyset^*(\mathrm{l}_{12}^{'}) = \emptyset^*(\mathrm{l}_{20}^{'}) = \cdots = \emptyset^*(\mathrm{l}_{8k-4}^{'}) \rightarrow 1$ /*up to k lines*/

}

End the processes.**Thus, the $8k$ lines received label 1.**

Similarly, the remaining $8k$lines label 0 as below

{

do

$\emptyset^*(l_7) = \emptyset^*(l_{15}) = \emptyset^*(l_{23}) = \cdots = \emptyset^*(l_{8k-1}) \rightarrow 0$ /*up to $k$lines*/

$\emptyset^*(l_5 l_6) = \emptyset^*(l_{13} l_{14}) = \emptyset^*(l_{21} l_{22}) = \cdots = \emptyset^*(l_{8k-3} l_{8k-2}) \rightarrow 0$ /*up to $2k$lines*/

$\emptyset^*(l_2 l_3 l_4) = \emptyset^*(l_{10} l_{11} l_{15}) = \emptyset^*(l_{18} l_{19} l_{20}) = \cdots = \emptyset^*(l_{8k-6} l_{8k-5} l_{8k-4}) \rightarrow 0$ /*up to $3k$lines*/

$\emptyset^*(l_3') = \emptyset^*(l_{11}') = \emptyset^*(l_{19}') = \cdots = \emptyset^*(l_{8k-5}') \rightarrow 0$ /*up to $k$lines*/

$\emptyset^*(l_5') = \emptyset^*(l_{13}') = \emptyset^*(l_{21}') = \cdots = \emptyset^*(l_{8k-3}') \rightarrow 0$ /*up to $k$lines*/

}

End the processes.**Thus, the $8k$ lines received label 0.**

Hence, the duplicate graph of $DG(CPG_k)$, $k \geq 2$, admit the bi-conditional cordial labeling.

Illustration:

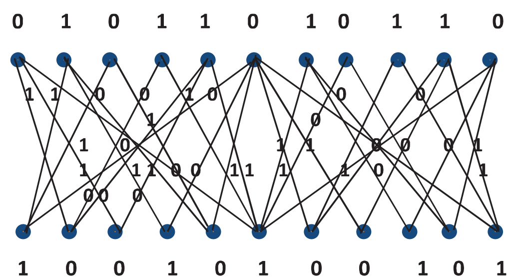


FIGURE 3: $DG(CPG_2)$

Result 3: Sum divisor cordial labeling for $DG(CPG_k)$, $k \geq 2$

Let $V \rightarrow \{n_1, n_2, n_3, \ldots \ldots . n_{5k+1}. n_1', n_2', n_3', \ldots \ldots . n_{5k+1}'.\}$ $E \rightarrow \{l_1, l_2, l_3 \ldots \ldots l_{8k}. l_1', l_2', l_3' \ldots \ldots l_{8k}'.\}$

Algorithm 4: Allocation of labeled nodes

$For\ 1 \leq m \leq k$ $\emptyset(N_{10m-9}) = 20m - 18, \emptyset(N_{10m-8}) = 20m - 17,$ $\emptyset(N_{10m-7}) = 20m - 14, \emptyset(N_{10m-6}) = 20m - 12,$ $\emptyset(N_{10m-5}) = 20m - 9, \emptyset(N_{10m-4}) = 20m - 8,$ $\emptyset(N_{10m-3}) = 20m - 6, \emptyset(N_{10m-2}) = 20m - 4,$ $\emptyset(N_{10m-1}) = 20m - 1, \emptyset(N_{10m}) = 20m.$	$For\ 1 \leq m \leq k$ $\emptyset(N_{10m-9}') = 20m - 19\emptyset(N'_{10m-8}) = 20m - 16,$ $\emptyset(N'_{10m-7}) = 20m - 5, \emptyset(N'_{10m-6}) = 20m - 13,$ $\emptyset(N'_{10m-5}) = 20m - 11, \emptyset(N'_{10m-4}) = 20m - 10,$ $\emptyset(N'_{10m-3}) = 20m - 7, (N'_{10m-2}) = 20m - 5,$ $\emptyset(N'_{10m-1}) = 20m - 7, \emptyset(N'_{10m}) = 20m - 5,$

Algorithm: 5.1 Allocation of labeled lines

Let$V \rightarrow \{n_1, n_2, \ldots \ldots, v_{5k+1}. n_1', n_2', \ldots \ldots . n_{5k+1}'\}$, $E \rightarrow \{l_1, l_2, l_3 \ldots \ldots, l_{8m}. l_1', l_2' \ldots \ldots . l_{8m}'\}$ be the set of nodes and lines of the duplicate graph of Circular polygon graph$DG(CPG_k)$using the algorithm 5, each of the $5k+1$ nodes receive labeled 0 and 1 respectively.

A function $g: N \rightarrow \{1,2,3 \ldots \ldots . |G(N)|\}$ with induced lines labeling of graph $g^*: L(G) \rightarrow \{0,1\}$ defined by$g^*(uv) = \begin{cases} 1: & if\ \ 2/g(u) + g(v) \\ 0: & otherwise \end{cases}$, and$|L_g(1) - L_g(0)| \leq 1$.

Case 1

For k is even

$\mathrm{m} = 2$ to k step 2

do

{

$\emptyset^*(l_7 l_{15}) = \emptyset^*(l_{23} l_{31}) = \emptyset^*(l_{39} l_{47}) = \cdots = \emptyset^*(l_{8k-9} l_{8k-1}) \to 1$ /*up to $k$lines*/

$\emptyset^*(l_1 l_3) = \emptyset^*(l_{17} l_{19}) = \emptyset^*(l_{33} l_{35}) = \cdots = \emptyset^*(l_{8k-17} l_{8k-15}) \to 1$ /*up to $k$lines*/

$\emptyset^*(l_4 l_5) = \emptyset^*(l_{20} l_{21}) = \emptyset^*(l_{36} l_{37}) = \cdots = \emptyset^*(l_{8k-12} l_{8k-11}) \to 1$ /*up to $k$lines*/

$\emptyset^*(l_{14}) = \emptyset^*(l_{30}) = \emptyset^*(l_{46}) = \cdots = \emptyset^*(l_{8k-2}) \to 1$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l'_1 l'_4) = \emptyset^*(l'_{17} l'_{20}) = \emptyset^*(l'_{33} l'_{36}) = \cdots = \emptyset^*(l'_{8k-15} l'_{8k-12}) \to 1$ /*up to $k$lines*/

$\emptyset^*(l'_6 l'_9) = \emptyset^*(l'_{22} l'_{25}) = \emptyset^*(l'_{38} l'_{41}) = \cdots = \emptyset^*(l'_{8k-10} l'_{8k-7}) \to 1$ /*up to $k$lines*/

$\emptyset^*(l'_{10} l'_{11} l'_{12} l'_{13}) = \emptyset^*(l'_{26} l'_{27} l'_{28} l'_{29}) = \cdots = \emptyset^*(l'_{8k-6} l'_{8k-5} l'_{8k-4} l'_{8k-3}) \to 1$ /*up to $2k$lines*/

$\emptyset^*(l'_{16}) = \emptyset^*(l'_{32}) = \emptyset^*(l'_{48}) = \cdots = \emptyset^*(l'_{8k}) \to 1$ /*up to $\frac{k}{2}$lines*/

}

End the processes.

Thus, the $8k$ lines received label 1.

Similarly, the remaining $8k$lines label 0 as below

{

$\emptyset^*(l_{16}) = \emptyset^*(l_{32}) = \emptyset^*(l_{48}) = \cdots = \emptyset^*(l_{8k}) \to 0$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l_{10} l_{11} l_{12} l_{13}) = \emptyset^*(l_{26} l_{27} l_{28} l_{29}) = \emptyset^*(l_{8k-6} l_{8k-5} l_{8k-4} l_{8k-3}) \to 0$ /*up to $2k$lines*/

$\emptyset^*(l_6 l_9) = \emptyset^*(l_{22} l_{25}) = \emptyset^*(l_{38} l_{41}) = \cdots = \emptyset^*(l_{8k-10} l_{8k-7}) \to 0$ /*up to $k$lines*/

$\emptyset^*(l_2) = \emptyset^*(l_{18}) = \emptyset^*(l_{34}) = \cdots = \emptyset^*(l_{8k-14}) \to 0$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l_8) = \emptyset^*(l_{24}) = \emptyset^*(l_{40}) = \cdots = \emptyset^*(l_{8k-8}) \to 0$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l'_2 l'_3) = \emptyset^*(l'_{18} l'_{19}) = \emptyset^*(l'_{34} l'_{35}) = \cdots = \emptyset^*(l'_{8k-14} l'_{8k-3}) \to 0$ /*up to $k$lines*/

$\emptyset^*(l'_5) = \emptyset^*(l'_{21}) = \emptyset^*(l'_{37}) = \cdots = \emptyset^*(l'_{8k-11}) \to 0$ /*up to $\frac{k}{2}$lines*/

$\emptyset^*(l'_7 l'_8) = \emptyset^*(l'_{23} l'_{24}) = \emptyset^*(l'_{39} l'_{40}) = \cdots = \emptyset^*(l'_{8k-9} l'_{8k-8}) \to 0$ /*up to $k$lines*/

$\emptyset^*(l'_{14} l'_{15}) = \emptyset^*(l'_{30} l'_{31}) = \emptyset^*(l'_{46} l'_{47}) = \cdots = \emptyset^*(l'_{8k-2} l'_{8k-1}) \to 0$ /*up to $k$lines*/

}

End the processes.

Thus, the $8k$ lines received label 0.

Hence, the duplicate graph of$DG(CPG_k)$, $k \geq 2$,admit the sum divisor cordial labeling even k copies.

Case 2

For k is odd

m = 2 to k step 2

do

{

$\emptyset^*(l_1) = \emptyset^*(l_{17}) = \emptyset^*(l_{33}) = \cdots = \emptyset^*(l_{8k-7}) \to 1$ /*up to $\frac{k+1}{2}$lines*/

$\emptyset^*(l_4 l_5) = \emptyset^*(l_{20} l_{21}) = \emptyset^*(l_{36} l_{37}) = \cdots = \emptyset^*(l_{8k-12} l_{8k-11}) \to 1$ /*up to $k + 1$lines*/

$\emptyset^*(l_3) = \emptyset^*(l_{19}) = \emptyset^*(l_{35}) = \cdots = \square^*(l_{8k-5}) \to 1$ /*up to $\frac{k+1}{2}$lines*/

$\square^*(l_7) = \square^*(l_{23}) = \square^*(l_{39}) = \cdots = \square^*(l_{8k-1}) \to 1$ /*up to $\frac{k+1}{2}$lines*/

$\square^*(l_{14} l_{15}) = \square^*(l_{30} l_{31}) = \square^*(l_{46} l_{47}) = \cdots = \square^*(l_{8k-10} l_{8k-9}) \to 1$ /*up to $k - 1$lines*/

$\square^*(l'_1) = \square^*(l'_{17}) = \square^*(l'_{33}) = \cdots = \square^*(l'_{8k-7}) \to 1$ /*up to $\frac{k+1}{2}$lines*/

$\square^*(l'_4 l'_6) = \square^*(l'_{20} l'_{22}) = \square^*(l'_{36} l'_{38}) = \cdots = \square^*(l'_{8k-4} l'_{8k-2}) \to 1$ /*up to$k + 1$lines*/

$\square^*(l'_9 l'_{10}) = \square^*\left(l'_{25} l'_{26}\right) = \square^*(l'_{41} l'_{42}) = \cdots = \square^*(l'_{8k-15} l'_{8k-14}) \to 1$ /*up to$k - 1$lines*/

$\square^*(l'_{11} l'_{12}) = \square^*(l'_{27} l'_{28}) = \square^*(l'_{43} l'_{44}) = \cdots = \square^*(l'_{8k-13} l'_{8k-12}) \to 1$ /*up to$k - 1$lines*/

$\square^*(l'_{13} l'_{16}) = \square^*(l'_{29} l'_{32}) = \square^*\left(l'_{45} l'_{48}\right) = \cdots = \square^*(l'_{8k-11} l'_{8k-8}) \to 1$ /*up to$k - 1$lines*/

End the processes.

Thus, the $8k$ lines received label 1.

Similarly, the remaining $8k$lines label 0 as below

{

do

$\square^*(l_2) = \square^*(l_{18}) = \square^*(l_{34}) = \cdots = \square^*(l_{8k-6}) \to 0$ /*up to $\frac{k+1}{2}$lines*/

$\square^*(l_6) = \square^*(l_{22}) = \square^*(l_{38}) = \cdots = \square^*(l_{8k-2}) \to 0$ /*up to $\frac{k+1}{2}$lines*/

$\square^*(l_8) = \square^*(l_{24}) = \square^*(l_{40}) = \cdots = \square^*(l_{8k}) \to 0$ /*up to $\frac{k+1}{2}$lines*/

$\square^*(l_9 l_{10}) = \square^*(l_{25} l_{26}) = \square^*(l_{41} l_{42}) = \cdots = \square^*(l_{8k-15} l_{8k-14}) \to 0$ /*up to $k - 1$lines*/

$\square^*(l_{11} l_{12}) = \square^*(l_{27} l_{28}) = \square^*(l_{43} l_{44}) = \cdots = \square^*(l_{8k-13} l_{8k-12}) \to 0$ /*up to $k - 1$lines*/

$\square^*(l_{13} l_{16}) = \square^*(l_{29} l_{33}) = \square^*(l_{45} l_{52}) = \cdots = \square^*(l_{8k-13} l_{8k-12}) \to 0$ /*up to $k - 1$lines*/

$\square^*(l'_2 l'_3) = \square^*(l'_{18} l'_{19}) = \square^*\left(l'_{34} l'_{35}\right) = \cdots = \square^*(l'_{8k-6} l'_{8k-5}) \to 0$ /*up to$k + 1$lines*/

$\square^*(l'_5) = \square^*(l'_{21}) = \square^*(l'_{34}) = \cdots = \square^*(l'_{8k-3}) \to 0$ /*up to$\frac{k+1}{2}$lines*/

$\square^*(l'_7 l'_8) = \square^*(l'_{23} l'_{24}) = \square^*(l'_{39} l'_{40}) = \cdots = \square^*(l'_{8k-1} l'_{8k}) \to 0$ /*up to$k + 1$lines*/

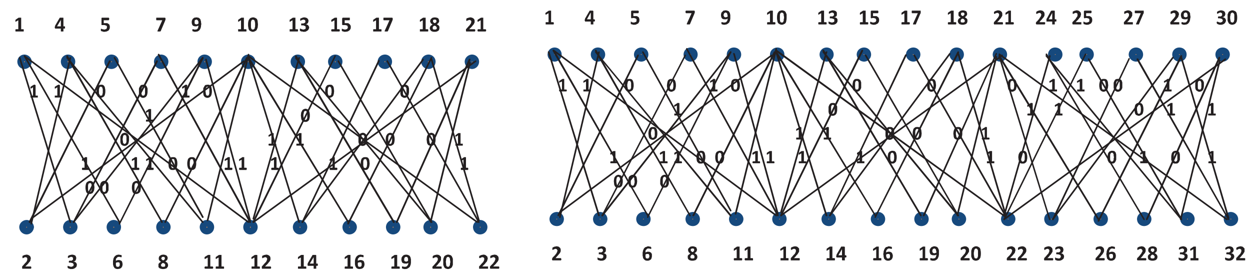
$\square^*(l'_{14} l'_{15}) = \square^*(l'_{30} l'_{31}) = \square^*(l'_{46} l'_{47}) = \cdots = \square^*(l'_{8k-10} l'_{8k-9}) \to 0$ /*up to$k - 1$lines*/

End the processes.

Thus, the $8k$ lines received label 0.

Hence, the duplicate graph of$DG(CPG_k)$, $k \geq 2$,admit the sum divisor cordial labeling odd k copies.

Illustration:



$DG(\ CPG_2)$ $DG(\ CPG_3)$

FIGURE 4: DG(CPG2) and DG(CPG3)

CONCLUSION

We proved that the duplicate graph of two chord of circular polygon graph $DG(CPG_k), k \geq 2$ on signed product, bi conditional and sum divisor cordial labeling.

REFERENCES

1. J. A .Gallian . *A Dynamic survey of Graph Labeling*, TheElectronics Journal of Combinatorics, #DS6(2014).
2. E.Sampath Kumar "*On duplicate graph*", Journal of the Indian Math Society 37, 285 – 293, (1973).
3. Jayapal Baskar Babujee, Shobana Loganathan ,*On Signed Product Cordial labeling*, Applied Mathematics, Scientific Research 1525-1530, (2011).
4. B.J.Murali, K.Thirusangu , R.MaduraMeenakshi, *Bi-conditional cordial labeling of Cycles*, International Journal Applied Engineering Research 188-191, (2015).
5. A.Lourdusamy , F.Patrick , *Sum divisor cordial labeling for star and ladder related graph,* Proyecciones Journal of Mathematics Vol.No.35 N^{0}4, pp437-455, (2016).
6. E.Nandagopal , V.Maheswari , P.Vijaya kumar , *Special labeling for the Extended Duplicate Graph of Quadrilateral Snake Graph*, IJICS Vol.No.2, 59-68, (2019).
7. E.Nandagopal , MaheswariV, Some *Labeling for Duplicate Graph of Double Quadrilateral Flow Graph*, Journal of Physics IOP publishing (ICPPPNS-19) conference series 1362 (2019) 012054.
8. E.Nandagopal ,V.Maheswari Bi-Conditional Cordial Labeling for Extended Duplicate Graph of Certain Classes of Graphs Journal of Physics IOP publishing (PMTIA-19) conference series 1377 (2019) 012001.
9. S.K.Vaidya and N.H.Shah, *Further results on divisor cordial labeling*, Annals of Pure and Applied Mathematics, 4(2) 150-159, (2013).
10. P.Vijayakumar , P.P.Ulaganathan and K.Thirusangu , *"Some Cordial Labeling in Extended Duplicate Graph of Star Graph*", International Journal of Applied Engineering Research, ISSN 0973-4562 Vol. 10 No.80, 171 – 174, (2015).