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GMJ Coding Using Even Felicitous Labeling and Bistar Graphs

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Abstract. In this paper,using GMJ(Graph Message Jumbled) code, we developed a technique for coding a confidential message by applying the even felicitous labeling to the bistar graph $B_{\upsilon,\omega}$.We presented two illustrations for transfering plain text into cipher text(picture coding), method for labeling an even felicitous graph and flowchart is outlined.
Keywords: GMJ coding, Star, FNNO, ANFON, Even felicitious labeling.
2010 AMS Subject Classification : $05C78$.

1. Introduction

Let $\mathbb{A} = (N(\mathbb{A}), L(\mathbb{A}))$ be a simple graph, for notations and terminology referred by [1],[2]. The art and science of hiding messages in order to introduce confidetiality in information security is recognised as cryptography. Encryption is the method of disguising a message in such a way as to conceal its content. The cipher text is an encrypted message. In [4], they have introduced the definition Even felicitous labeling and proved that the Bistar graph $B_{\upsilon,\omega}$ is an even felicitious graph. In [5][6] [7], they have introduced a new coding technique using super mean labeling and difference cordial labeling on two star graph, three star graph and Fibonacci web. In[3] they have developed a technique for coding a confidential message by applying the mean labeling to the two star graph. Motivated by all these findings, we found some technique for coding a message using Bistar graph and even felicitous labeling.

Definition 1.1 *The Bistar $B_{\upsilon,\omega}$ is the graph obtained from K_2 by joining υ pendent links to one end of K_2 and ω pendent links to other end of K_2. The link of K_2 is called the central link of $B_{\upsilon,\omega}$ and the nodes of K_2 are called the central nodes of $B_{\upsilon,\omega}$.*

Definition 1.2 *An injective function $\Pi : N \rightarrow \{0, 1, 2, \cdots q\}$ where $L = q$ is called felicitous, if the link labels induced by $\{\Pi(\alpha) + \Pi(\beta)\}\ (mod\ q)$ for each link $\alpha\beta$ are distinct.*

Definition 1.3 *An injective function* $\Pi : N \to \{0,1,2,\cdots 2q-1\}$ *and* $\Pi : L \to \{0,2,4,\cdots 2q-2\}$ *is called even felicitous, if the link labels induced by* $\{\Pi(\alpha)+\Pi(\beta)\}\, mod\ (2q-1)$ *for each link* $\alpha\beta$ *are distinct.*

2. Description for Even Felicitous Labeling on Bistar Graph:

By refering [4], The Bistar graph $B_{\upsilon,\omega}$ is an even felicitous graph. Let $\mathbb{A} = B_{\upsilon,\omega}$. Let $\{\alpha\} \cup \{\alpha_i : 1 \le i \le \zeta\}$; $\{\beta\} \cup \{\beta_j : 1 \le j \le \delta\}$ be the nodes of $\mathbb{A}$. Then $\mathbb{A}$ has $\zeta+\delta+2$ nodes and $\zeta+\delta+1$ links. We have $N(\mathbb{A}) = \{\alpha,\beta\} \cup \{\alpha_i : 1 \le i \le \zeta\} \cup \{\beta_j : 1 \le j \le \delta\}$. We need to show that $\mathbb{A}$ is an even felicitous graph.
The node labeling $\Pi : N(\mathbb{A}) \to \{0,1,2,\cdots 2q-1\}$ is defined as follows
$\Pi(\alpha) = 0$; $\Pi(\beta) = 2q-1$; $\Pi(\alpha_i) = 2i$ for $1 \le i \le \zeta$;
$\Pi(\beta_j) = \Pi(\alpha_\zeta) + 2j$ for $1 \le j \le \delta$. The corresponding link labels are as follows: The link label of $\alpha\alpha_i$ is $\{2i\}\, mod\ (2q-1)$ for $1 \le i \le \zeta$, $\beta\beta_j$ is $\{2q-1+\Pi(\alpha_\zeta)+2j\}\, mod\ (2q-1)$ for $1 \le j \le \delta$ and $\alpha\beta$ is 0. Hence the induced link labels are distinct.
Therefore the Bistar graph $B_{\upsilon,\omega}$ is an even felicitous graph.

3. Description for GMJ (Graph Message Jumbled) coding method:

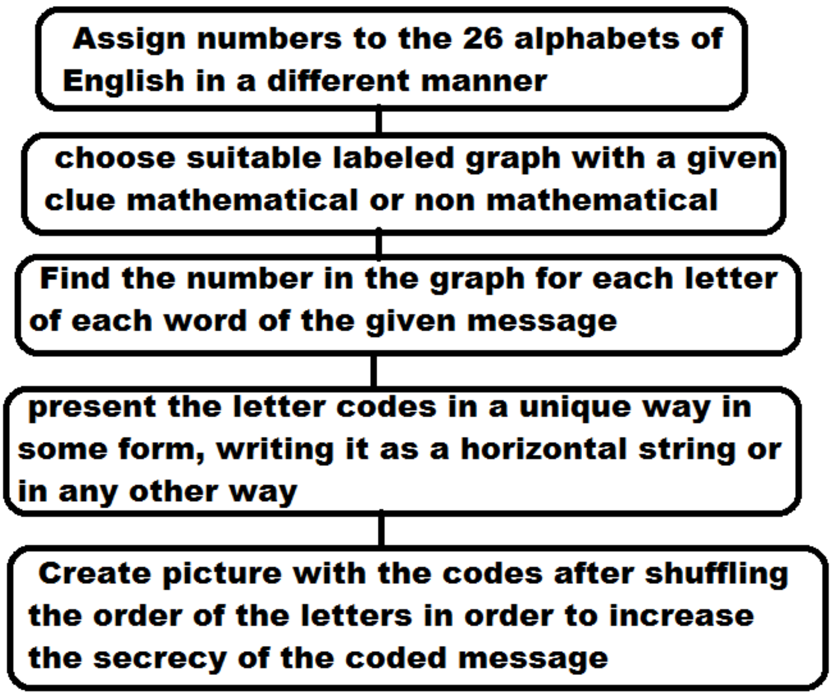

Figure 1. GMJ Coding

4. Results and Discussions

4.1. ILLUSTRATION 1:

(i) **Message:** Strawberry and Raspberry are sitting in the Bamboo Bridge.

(ii) **Clue:** Hiding Peak and Annapurna II are highing to the top. [Hiding peak is the eleventh highest mountain in the world and Annapurna II is the sixteenth highest mountain in the world. From this information, we have to understood that the graph is $B_{11,16}$.]

(iii) **Graph:** The bistar graph $B_{11,16}$

(iv) **Labeling:** The Even Felicitous Labeling done for $B_{11,16}$.

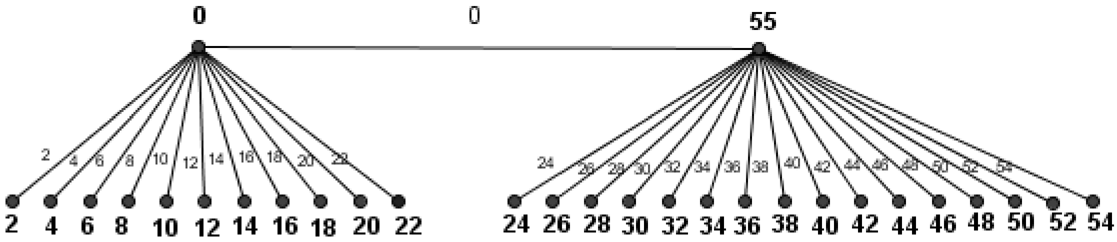

Figure 2. $B_{11,16}$

(v) **Even Numbering of Alphabets:** First Neon Next Others (FNNO)

0	4	6	8	10	12	14	16	2	18	20	22	24
A	B	C	D	E	F	G	H	I	J	K	L	M
26	28	30	32	34	36	38	40	42	44	46	48	50
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

The alphabets in the position of Neon number(A,I) are given the even numbers (0,2) and for remaining alphabets the even numbers from 4 to 50 are alloted.
The encoded function is $Z(b_{r_m}) = 2m - 2,\ \ m = 1, 2.$
Here r_m indicates the m^{th} neon number from 1 to 26 and b_{r_m} indicates r_m^{th} alphabetical letter.
$h(b_{s_n}) = 2 + 2n,\ \ n = 1, 2, \cdots 24.$
Here s_n indicates the n^{th} non neon number from 1 to 26 and b_{s_n} indicates s_n^{th} alphabetical letter.
$b_{r_m} \neq b_{s_n}$, Here b_{r_m} and b_{s_n} indicates twenty six alphabets.

(vi) **Coding a Letter:** Here $\begin{pmatrix} & \\ & \end{pmatrix}$ represents the 1^{st} star and $\begin{bmatrix} & \\ & \end{bmatrix}$ represents the 2^{nd} star respectively. Here, $\begin{pmatrix} 7 & 0 \\ 0 & 0 \end{pmatrix}$ represents the number alloted to seventh node of 1^{st} star and $\begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}$ represents the number alloted to the fourth link of 2^{nd} star and $\begin{pmatrix} 0 & T \\ T & 0 \end{pmatrix}$ represents the number alloted to the top node of 1^{st} star and $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$ represents the central link value.

(vii) **Coding(wordwise):**

STRAWBERRY - $\begin{bmatrix}7&0\\0&0\end{bmatrix}\begin{bmatrix}0&0\\0&8\end{bmatrix}\begin{bmatrix}6&0\\0&0\end{bmatrix}\begin{vmatrix}1&1\\1&1\end{vmatrix}\begin{bmatrix}0&0\\0&11\end{bmatrix}\begin{pmatrix}2&0\\0&0\end{pmatrix}\begin{pmatrix}0&0\\0&5\end{pmatrix}$
$\begin{bmatrix}6&0\\0&0\end{bmatrix}\begin{bmatrix}0&0\\0&6\end{bmatrix}\begin{bmatrix}13&0\\0&0\end{bmatrix}$

AND - $\begin{vmatrix}1&1\\1&1\end{vmatrix}\begin{bmatrix}2&0\\0&0\end{bmatrix}\begin{pmatrix}4&0\\0&0\end{pmatrix}$

RASPBERRY - $\begin{bmatrix}6&0\\0&0\end{bmatrix}\begin{vmatrix}1&1\\1&1\end{vmatrix}\begin{bmatrix}7&0\\0&0\end{bmatrix}\begin{bmatrix}0&0\\0&4\end{bmatrix}\begin{pmatrix}0&0\\0&2\end{pmatrix}\begin{pmatrix}5&0\\0&0\end{pmatrix}$
$\begin{bmatrix}0&0\\0&6\end{bmatrix}\begin{bmatrix}6&0\\0&0\end{bmatrix}\begin{bmatrix}13&0\\0&0\end{bmatrix}$

ARE - $\begin{pmatrix}0&T\\T&0\end{pmatrix}\begin{bmatrix}0&0\\0&6\end{bmatrix}\begin{pmatrix}5&0\\0&0\end{pmatrix}$

SITTING - $\begin{bmatrix}0&0\\0&7\end{bmatrix}\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{bmatrix}0&0\\0&8\end{bmatrix}\begin{bmatrix}8&0\\0&0\end{bmatrix}\begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{bmatrix}0&0\\0&2\end{bmatrix}\begin{pmatrix}0&0\\0&7\end{pmatrix}$

IN - $\begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{bmatrix}2&0\\0&0\end{bmatrix}$

THE - $\begin{bmatrix}0&0\\0&8\end{bmatrix}\begin{pmatrix}8&0\\0&0\end{pmatrix}\begin{pmatrix}5&0\\0&0\end{pmatrix}$

BAMBOO - $\begin{pmatrix}0&0\\0&2\end{pmatrix}\begin{vmatrix}1&1\\1&1\end{vmatrix}\begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{pmatrix}0&0\\0&2\end{pmatrix}\begin{bmatrix}0&0\\0&3\end{bmatrix}\begin{bmatrix}0&0\\0&3\end{bmatrix}$

BRIDGE - $\begin{pmatrix}0&0\\0&2\end{pmatrix}\begin{bmatrix}0&0\\0&6\end{bmatrix}\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}0&0\\0&4\end{pmatrix}\begin{pmatrix}0&0\\0&7\end{pmatrix}\begin{pmatrix}0&0\\0&5\end{pmatrix}$

(viii) **Horizontal string:**

$$\begin{bmatrix}7&0\\0&0\end{bmatrix}\begin{bmatrix}0&0\\0&8\end{bmatrix}\begin{bmatrix}6&0\\0&0\end{bmatrix}\begin{vmatrix}1&1\\1&1\end{vmatrix}\begin{bmatrix}0&0\\0&11\end{bmatrix}\begin{pmatrix}2&0\\0&0\end{pmatrix}\begin{pmatrix}0&0\\0&5\end{pmatrix}\begin{bmatrix}6&0\\0&0\end{bmatrix}\begin{bmatrix}0&0\\0&6\end{bmatrix}\begin{bmatrix}13&0\\0&0\end{bmatrix}\begin{vmatrix}1&1\\1&1\end{vmatrix}$$
$$\begin{bmatrix}2&0\\0&0\end{bmatrix}\begin{pmatrix}4&0\\0&0\end{pmatrix}\begin{bmatrix}6&0\\0&0\end{bmatrix}\begin{vmatrix}1&1\\1&1\end{vmatrix}\begin{bmatrix}7&0\\0&0\end{bmatrix}\begin{bmatrix}0&0\\0&4\end{bmatrix}\begin{pmatrix}0&0\\0&2\end{pmatrix}\begin{pmatrix}5&0\\0&0\end{pmatrix}\begin{bmatrix}0&0\\0&6\end{bmatrix}\begin{bmatrix}6&0\\0&0\end{bmatrix}\begin{bmatrix}13&0\\0&0\end{bmatrix}$$
$$\begin{pmatrix}0&T\\T&0\end{pmatrix}\begin{bmatrix}0&0\\0&6\end{bmatrix}\begin{pmatrix}5&0\\0&0\end{pmatrix}\begin{bmatrix}0&0\\0&7\end{bmatrix}\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{bmatrix}0&0\\0&8\end{bmatrix}\begin{bmatrix}8&0\\0&0\end{bmatrix}\begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{bmatrix}0&0\\0&2\end{bmatrix}\begin{pmatrix}0&0\\0&7\end{pmatrix}\begin{pmatrix}0&0\\0&1\end{pmatrix}$$
$$\begin{bmatrix}2&0\\0&0\end{bmatrix}\begin{bmatrix}0&0\\0&8\end{bmatrix}\begin{pmatrix}8&0\\0&0\end{pmatrix}\begin{pmatrix}5&0\\0&0\end{pmatrix}\begin{pmatrix}0&0\\0&2\end{pmatrix}\begin{vmatrix}1&1\\1&1\end{vmatrix}\begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{pmatrix}0&0\\0&2\end{pmatrix}\begin{bmatrix}0&0\\0&3\end{bmatrix}\begin{bmatrix}0&0\\0&3\end{bmatrix}\begin{pmatrix}0&0\\0&2\end{pmatrix}\begin{bmatrix}0&0\\0&6\end{bmatrix}$$
$$\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}0&0\\0&4\end{pmatrix}\begin{pmatrix}0&0\\0&7\end{pmatrix}\begin{pmatrix}0&0\\0&5\end{pmatrix}.$$

(ix) **Picture Coding:** In this picture coding, the entries in square represents the matrix assigned to first star and the entries in circle represents the matrix assigned to second

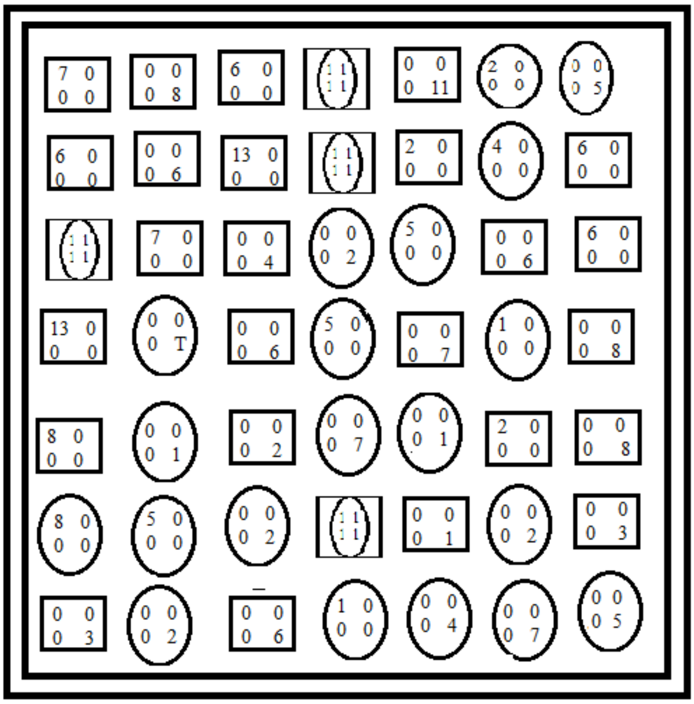

Figure 3. Picture coding FNNO

star and the entries in circle inside a square represents the matrix assigned to central link.

4.2. ILLUSTRATION 2:

(i) **Message:** Strawberry and Raspberry are sitting in the Bamboo Bridge.

(ii) **Clue:** Silicon and Magnesium are joining individually. [In periodic table, the fourteenth element is silicon and twelth element is Magnesium. So from this data, we have to understood that the graph is $B_{14,12}$.]

(iii) **Graph:** The Bistar graph $B_{14,12}$.

(iv) **Labeling:** The Even Felicitous Labeling done for $B_{14,12}$.

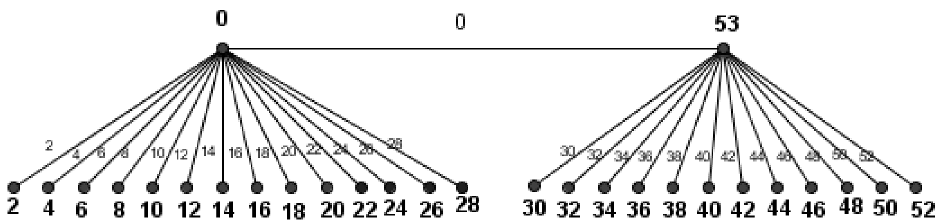

Figure 4. $B_{14,12}$

(v) **Even Numbering of Alphabets:** Automorphic Numbers First Others Next (ANFON)

0	8	10	12	2	4	14	16	18	20	22	24	26
A	B	C	D	E	F	G	H	I	J	K	L	M
28	30	32	34	36	38	40	42	44	46	48	6	50
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

The alphabets in the position of Automorphic number (A,E,F,Y) are given the even numbers (0,2,4,6) and for remaining alphabets the even numbers from 8 to 50 are alloted.
The encoded function is $Z(b_{r_m}) = 2m - 2,\ \ m = 1, 2, 3, 4.$
Here r_m indicates the m^{th} Automorphic Number from 1 to 26 and b_{r_m} indicates r_m^{th} alphabetical letter.
$Z(b_{s_n}) = 6 + 2n,\ \ n = 1, 2, \cdots 22.$
Here s_n indicates the n^{th} non Automorphic Number from 1 to 26 and b_{s_n} indicates s_n^{th} alphabetical letter.
$b_{r_m} \neq b_{s_n}$. Here b_{r_m} and b_{s_n} indicates the twenty six alphabets.

(vi) **Coding a Letter:** Let $\lceil\ \rceil$ represents the 1^{st} star and $\lfloor\ \rfloor$ represents the 2^{nd} star respectively. Let C, Δ_i, and ∇_i denotes the center node, the i^{th} node value and the i^{th} link value in order. For instance, $\lceil\Delta_7\rceil$ represents the number alloted to the seventh node of 1^{st} star and $\lfloor\nabla_6\rfloor$ represents the number alloted to the sixth link of second star respectively.

(vii) **Coding(wordwise):**

STRAWBERRY	-	$\lfloor\Delta_5\rfloor\ \lfloor\nabla_6\rfloor\ \lfloor\Delta_4\rfloor\ \lceil c\rceil\ \lfloor\Delta_9\rfloor\lceil\nabla_4\rceil\ \lceil\Delta_1\rceil\ \lfloor\Delta_4\rfloor\ \lfloor\nabla_4\rfloor\ \lceil\Delta_3\rceil$
AND	-	$\lceil C\rceil\ \lceil\Delta_{14}\rceil\ \lceil\nabla_6\rceil$
RASPBERRY	-	$\lfloor\Delta_4\rfloor\ \lceil W\rfloor\ \lfloor\Delta_5\rfloor\ \lfloor\nabla_2\rfloor\ \lceil\Delta_4\rceil\lceil\nabla_1\rceil\ \lfloor\Delta_4\rfloor\ \lfloor\nabla_4\rfloor\ \lceil\nabla_3\rceil$
ARE	-	$\lceil W\rfloor\ \lfloor\Delta_4\rfloor\ \lceil\nabla_1\rceil$
SITTING	-	$\lfloor\Delta_5\rfloor\ \lceil\nabla_9\rceil\ \lfloor\Delta_6\rfloor\ \lfloor\nabla_6\rfloor\lceil\Delta_9\rceil\ \lceil\Delta_{14}\rceil\lceil\Delta_7\rceil$
IN	-	$\lceil\Delta_9\rceil\ \lceil\nabla_{14}\rceil$
THE	-	$\lfloor\Delta_6\rfloor\ \lceil\nabla_8\rceil\ \lceil\Delta_1\rceil$
BAMBOO	-	$\lceil\Delta_4\rceil\ \lceil W\rfloor\ \lceil\Delta_{13}\rceil\ \lceil\Delta_4\rceil\lfloor\Delta_1\rfloor\ \lfloor\nabla_1\rfloor$
BRIDGE	-	$\lceil\Delta_4\rceil\ \lfloor\nabla_4\rfloor\ \lceil\Delta_9\rceil\ \lceil\Delta_6\rceil\lceil\nabla_7\rceil\ \lceil\Delta_1\rceil$

(viii) **Horizontal string:**
$\lfloor\Delta_5\rfloor\ \lfloor\nabla_6\rfloor\ \lfloor\Delta_4\rfloor\ \lceil c\rceil\ \lfloor\Delta_9\rfloor\ \lceil\nabla_4\rceil\ \lceil\Delta_1\rceil\ \lfloor\Delta_4\rfloor\ \lfloor\nabla_4\rfloor\ \lceil\Delta_3\rceil\lceil C\rceil\ \lceil\Delta_{14}\rceil\ \lceil\nabla_6\rceil\ \lfloor\Delta_4\rfloor$
$\lceil W\rfloor\ \lfloor\Delta_5\rfloor\ \lfloor\nabla_2\rfloor\ \lceil\Delta_4\rceil\ \lceil\nabla_1\rceil\ \lfloor\Delta_4\rfloor\lfloor\nabla_4\rfloor\ \lceil\nabla_3\rceil\ \lceil W\rfloor\ \lfloor\Delta_4\rfloor\ \lceil\nabla_1\rceil\ \lfloor\Delta_5\rfloor\ \lceil\nabla_9\rceil\ \lfloor\Delta_6\rfloor$
$\lfloor\nabla_6\rfloor\ \lceil\Delta_9\rceil\lceil\Delta_{14}\rceil\lceil\Delta_7\rceil\ \lceil\Delta_9\rceil\ \lceil\nabla_{14}\rceil\ \lfloor\Delta_6\rfloor\ \lceil\nabla_8\rceil\ \lceil\Delta_1\rceil\ \lceil\Delta_4\rceil\ \lceil W\rfloor\ \lceil\Delta_{13}\rceil\lceil\Delta_4\rceil\ \lfloor\Delta_1\rfloor$
$\lfloor\nabla_1\rfloor\ \lceil\Delta_4\rceil\ \lfloor\nabla_4\rfloor\ \lceil\Delta_9\rceil\ \lceil\Delta_6\rceil\ \lceil\nabla_7\rceil\ \lceil\Delta_1\rceil$.

(ix) **Picture Coding:** In the below picture, the coded letters are shuffled in the order of clockwise direction starting from the top to bottom of each ray from first star to second star respectively.

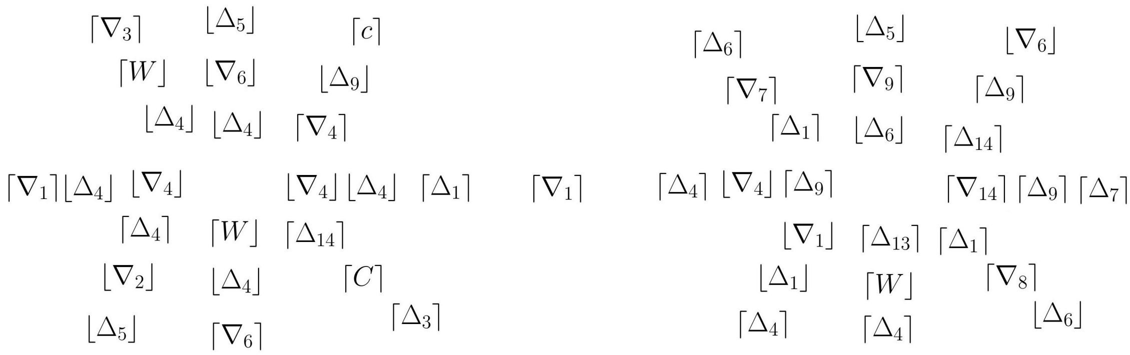

Figure 5. Picture coding ANFON

The sender and receiver has to maintain the following flowchart.

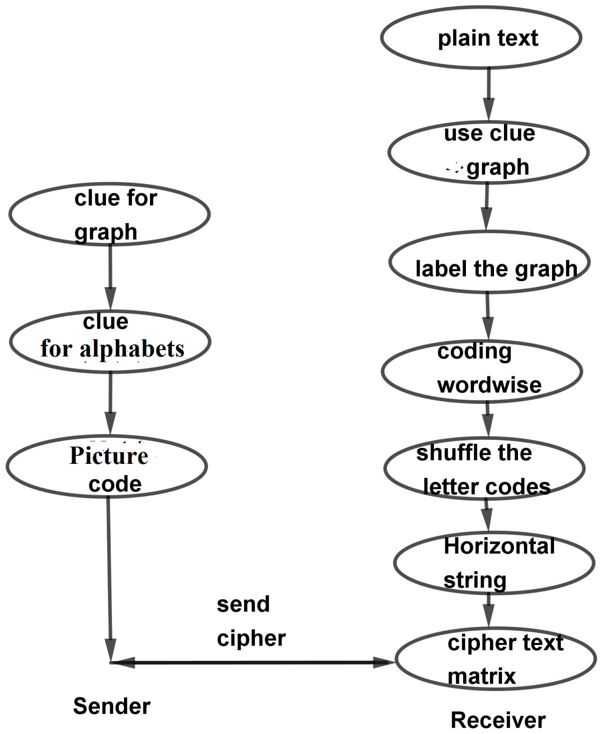

Figure 6. Flow Chart

5. Conclusion

Even felicitous labeling on the Bistar graph for communicating secret messages using different method of alphabets numbering (FNNO) and (ANFON) with mathematical functions and different methods for coding a letter and appropriate coded picture are presented in this paper. The methods outlined in this paper can be used to code an extremely secretive message. In future, we decided to do more coding techniques using different labeling on a star graph.

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