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Fuzzy Modular Lattice Ordered M-Group

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Abstract. In this paper we introduce the notion of fuzzy modular lattice ordered m- groups and some of its properties are investigated. We also study the homomorphic image, pre image of fuzzy modular lattice ordered m- groups using T-norms and some related properties of lattices are discussed.

AMS subject classification:06D72, 06C05,06F15

Keywords: Modular lattice ordered group, fuzzy modular lattice ordered m-group, pre image, direct product.

INTRODUCTION

The notion of fuzzy sets was introduced by L. A. Zadeh[6]. Fuzzy set theory was developed by many researchers in different directions and has evoked great interest among mathematicians working in many fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. N. Ajmal and K. V. Thomas [1] initiated such type of study in the year 1994. It was later independently established by N. Ajmal [1] that the set of all fuzzy normal subgroups of a group constitute a sub lattice of a lattice of all fuzzy subgroups of a given group and is modular. Nanda [9] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in details and in the lattice theoretical aspects of fuzzy subgroups and fuzzy normal subgroups are explored. G. S. V. SatyaSaibaba [3] initiate the study of L-fuzzy lattice ordered groups and sub l-groups. J. A. Goguen [5] replaced the valuation set [0,1] by meansof a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets.

Solairaju and R. Natarajan [12] introduced the concept of lattice valued Q-fuzzy sub modules over near rings with respect to T-norms. Dr. M. Madurai and V. Rajendran [7] modified the definition of fuzzy lattice and introduced the notion of fuzzy lattice of groups and investigated some of its basic properties.

Gu [13] introduced concept of fuzzy groups with operator. Then S. Subramanian, R. Natarajan and Chellappa [11] extended the concept to m-fuzzy groups with operator.

In this paper, we introduce the notion of fuzzy modular lattice ordered m-groups and investigated some of its basic properties. We study the homomorphic image, pre image of fuzzy modular lattice ordered m-groups, arbitrary family of fuzzy modular lattice ordered m-groups and fuzzy modular lattice ordered m-normal groups

PRELIMINARIES

Definition 2.1: Letμ: $X \rightarrow [0,1]$ be a fuzzy set & G $\in \wp$(X) = Set of all fuzzy sets on X. A fuzzy set μ on G is called a fuzzy group if

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i. $\mu\,(xy) \;\geq\; min\,\{\mu\,(x), \mu(y)\}$

ii. $\mu\,(x-1) \;\geq\; \mu\,(x)$, for all x, y∈ G.

Definition 2.2: Letμ: $X \;\rightarrow\; [0,1]$ be a fuzzy set & G ∈ ℘(X). A fuzzy set μ on G is called a normal fuzzy subgroup if, $\mu\,(x-1yx) \;\geq\; \mu(y)$ for all x, y ∈G.

Definition 2.3:(from [4])A lattice ordered group is a system (G, •, ≤) if,

i. (G, •) is a group

ii. (G,≤) is a lattice

iii. x≤ y implies axb≤ayb (compatibility), for a, b, x, y∈ G.

Definition 2.4:A Modular lattice ordered group is a system (G, •, ≤) if,

i. (G,•) is a group

ii. (G,≤) is a modular lattice

iii. x≤y implies x∨(a∧y) = (x∨a)∧y (modular law), for all a, b, x, y ∈ G,

iv. ≤ is the partial relation, ∨, ∧ are the operations of lattices.

Definition 2.5: Letμ: $X \;\rightarrow\; [0,1]$ is a fuzzy set & G is a lattice ordered set, G ∈ ℘(X). A function μ on G is said to be a fuzzy lattice ordered group if,

i. $\mu\,(xy) \;\geq\; min\,\{\mu\,(x), \mu(y)\}$

ii. $\mu\,(x\;-1) \;\geq\; \mu\,(x)$, for all x, y∈ G

Definition 2.6: Let G be a group, M be any set if,

i. $mx \;\epsilon\; G.$

ii. $m(xy) = (mx)y = xmy$, for all x, y∈ G, m ∈M.

Then G is called a m group.

Definition 2.7: Let μ: $X \rightarrow$ [0, 1] be a fuzzy set and G is a M group G. A fuzzy set on G, G ∈ ℘(X) is called a fuzzy m group if,

i. $\mu\;(m(xy)) \;\geq\; min\;\{\mu\;(mx),\;\mu\;(my)\}$

ii. $\mu\;(mx-1) \;\geq\; \mu\;(mx)$, for all x, y∈ G, m∈M.

Definition 2.8: A t-norm T, we mean a function T: [0, 1]×[0, 1]→[0, 1] satisfying the following conditions:

(T1) $T\;(0,\;x) = 0, for\;\;all\;\;x\in$ [0, 1].

(T2) $T(x,\;y) \leq\; T(x,\;z)\;\;if,\;\;y \leq z,\;\;for\;\;all\;\;x,\;\;y\in$ [0, 1].

(T3) $T(x,\;y) = T(y,\;x),\;\;for\;\;all\;\;x,\;\;y\in$ [0, 1].

$(T4)$ $T(x,\;T(y,\;z)) = T(T(x,\;y),\;z)$, for all x, y, z ∈ [0, 1].

Definition 2.9: For any fuzzy m group G and t∈[0, 1], We define the set $U(\mu\colon t) = \{x \in G\,|\,\mu(mx) \geq t\}$ which is called an upper cut off μ and can be used to the characterization of μ .

Definition 2.10: Let θ: $X \to Y$ be a map.A and B are fuzzy lattice ordered m groups in X and Y respectively. Then the inverse image of B under θ is a fuzzy set defined by,
θ^{1} (B) = $\mu\theta^{1}$ $(B)(x) =$ $\mu_B\theta(x)$.

Definition 2.11: Let μ_A be a fuzzy set of G. Letθ: $G \to G'$ be a map.
Define the map $\mu_A\theta: G \to$ [0, 1] by $\mu_A\theta(x)$ = $\mu_A\theta$ (x)

Definition 2.12: Let f: $G \to$ G' be a lattice group homomorphism and A be a fuzzy lattice of G' then $Af(x)$ = $(A\ o\ f)(x)$ = $f^{-1}(A)(x)$.

Definition 2.13: Let μ: $X \to$ [0, 1], G $\in \wp$(X), M $\subset$ X. A function μ on G is said to be a fuzzy lattice ordered m-group if,

i. (G, •) is a M-group.

ii. (G, •, ≤) is a lattice ordered group.

iii. $\mu m(xy))$ $\geq$ $min\{\mu(mx), \mu(my)\}$

iv. $\mu((mx)^{-1})$ $\geq$ $\mu(mx)$

v. $\mu(mxvmy)$ $\geq$ $min\{\mu(mx), \mu(my)\}$

vi. $\mu(mx \wedge my)$ $\geq$ $min\{\mu(mx), \mu(my)\}$, for all x, y $\in$ G

FUZZY MODULAR LATTICE ORDERED M- GROUP

Definition 3.1: Let μ: $X \to$ [0, 1], G $\in \wp$(X), M $\subset$ X. A function μ on G is said to be a fuzzy modular lattice ordered m-group if,

i. (G, •) is a M-group.

ii. (G, •, ≤) is a modular lattice ordered group.

iii. $\mu(m(xy))$ $\geq$ $min\{\mu(mx), \mu(my)\}$

iv. $\mu((mx)^{-1})$ $\geq \mu(mx)$

v. $\mu(mxvmy)$ $\geq$ $min\{\mu(mx), \mu(my)\}$

vi. $\mu(mx \wedge my)$ $\geq$ $min\{\mu(mx), \mu(my)\}$

vii. $\mu(mx\ v\ my\ )\wedge$ $\mu(mx \vee mz)$ $\geq$ $min\{\mu(mx), \mu(my) \wedge \mu\ (mx \vee mz)\}$, for all x, y$\in$G.

PROPERTIES OF FUZZY MODULAR LATTICE ORDERED M- GROUP

Proposition 4.1:
Let G and G' be two fuzzy modular lattice ordered m-groups and θ: $G \to G'$ be a m-homomorphism defined by $\theta(mx)$ = $m\theta(x)$. If B is a fuzzy modular lattice ordered m-group of G' then the pre-image θ^{-1}(B) is a fuzzy modular lattice ordered m-group of G.
Proof:
Assume B is a fuzzy modular lattice ordered m-group of G'.

Let x, y, z $\in$ G

i) $\mu\theta^{1}(B)\ (m(xy)) = \mu B\theta(m(xy))$

$= \mu B\ (m\ \theta(xy)) = \mu B(\ m\ \theta(x)\ \theta(y))$

$\geq min\ \{\mu B(\ m\ \theta(x)),\ \mu B(m\ \theta(y))\}$

$\geq\ min\ \{\mu B(\ \theta(mx),\ \mu B\ (\theta(my)\}$

$\geq\ min\ \{\mu\theta - 1(B)(mx),\ \mu\theta^{1}(B)(my)\}$

ii) $\mu\theta - 1(B)(mx)^{-1}) = \mu B\theta((mx)^{-1}) = \mu B\ (\theta(mx))^{-1} = \mu B\ (m\ \theta(x))^{-1}$
$\geq \mu B\ (m\ \theta(x))$
$\geq \mu B\ (\theta(mx))$
$\geq \mu\theta^{1}(B)(mx)$
iii) $\mu\theta^{1}(B)(mx\ v\ my) = \mu B\theta(mx\ v\ my) = \mu B\theta(mx)\ v\ \theta(my)$
$\geq\ min\ \{\mu B\theta(mx),\ \mu B\theta(my)\}$
$\geq\ min\ \{\mu\theta^{1}(B)(mx),\ \mu\theta^{1}(B)(my)\}$
iv) $\mu\theta^{1}(B)(mx\ \wedge\ my) = \mu B\theta(mx\ \wedge\ my) = \mu B\theta(mx)\ \wedge\ \mu B\theta(my)$
$\geq\ min\ \{\ \mu B\theta(mx),\ \mu B\theta(my)\}$
$\geq\ min\ \{\ \mu\theta^{1}(B)(mx),\ \mu\theta^{1}(B)(my)\}$
v) $\mu\theta^{1}(B)\ (mx\ \vee\ my)\ \wedge\ \mu\theta^{1}(B)(mx\ \vee\ mz)$
$= \mu B\theta\ (mx\ \vee\ my)\ \wedge\ \mu B\theta\ (mx\ \vee\ mz)$
$= (\mu B\theta(mx)\ \vee\ \mu B\theta(my))\ \wedge\ (\mu B\theta(mx)\ \vee\ \mu B\theta(mz))$
$\geq\ min\ \{\ \mu B\theta(mx),\ \mu B\theta(my),\ \mu B\theta(mx)\ \vee\ \mu B\theta(mz)\}$
$\geq\ min\ \{\ \mu B\theta(mx),\ \mu B\theta(my)\ \wedge\ \mu B\theta(mx\ \vee\ mz\ )\}$
$\geq\ min\ \{\ \mu\theta^{1}(B)\ (mx),\ \mu\theta^{1}(B)\ (my)\ \wedge\ \mu\theta^{1}(B)\ (mx\ \vee\ mz)\}$
Therefore θ^{-1}(B) is a fuzzy modular lattice ordered m-group of G.

Proposition 4.2:
Let G and G' be two fuzzy modular lattice ordered m-groups and θ: $G \rightarrow G'$ be a m-epimorphism. B is a fuzzy set in G'. If θ^{-1}(B) is a fuzzy modular lattice ordered m-group of G then B is a fuzzy modular lattice ordered group of G'.
Proof:
Let x, y, z $\in$ G', therefore there exist an element a, b $\in$G such that $\theta(a) = x$, $\theta(b) = y$ and $\theta(c) = z$.
i) $\mu B(m(xy)) = \mu B\ (m(\theta(a)\ \theta(b)) = \mu B(m\theta(ab)) = \mu B\theta(m(ab)) = \mu\theta^{1}(B)\ (m(ab))$
$\geq\ min\ \{\ \mu\theta^{1}(B)(ma),\ \mu\theta^{1}(B)(mb)\}$
$\geq\ min\ \{\ \mu B\theta(ma),\ \mu B\theta(mb)\}$
$\geq\ min\ \{\ \mu B\ m\ \theta(a),\ \mu B\ m\ \theta(b)\}$
$\geq\ min\ \{\ \mu B(mx),\ \mu B(my)\}$
ii) $\mu B((m\ x)^{-1}) = \mu B(m\ \theta(a))^{-1} = \mu B(\theta(ma))^{-1} = \mu B(\theta(ma)^{-1})$
$= \mu\theta^{1}(B)(ma)^{-1}$
$\geq \mu\theta^{1}(B)(ma) \geq \mu B\ \theta^{1}(ma) \geq \mu B\ m\ \theta(a) \geq \mu B\ (mx)$
iii) $\mu B(mx\ v\ my) = \mu B(m\ \theta(a)\ v\ m\ \theta(b)) = \mu B(\theta(ma)\ v\ \theta(mb)) = \mu B\ (\theta(ma\ v\ mb))$
$= \mu\theta^{1}(B)(ma\ v\ mb)$
$\geq\ min\ \{\mu\theta^{1}(B)(ma),\ \mu\theta - 1(B)(mb)\}$
$\geq\ min\ \{\mu B\ \theta(ma),\ \mu B\theta(mb)\}$
$\geq\ min\ \{\mu B\ m\ \theta(a),\ \mu B\ m\ \theta(b)\}$
$\geq\ min\ \{\mu B(mx),\ \mu B(my)\}$
iv) $\mu B(mx\ \wedge\ my) = \mu B\ (m\theta(a)\ \wedge\ m\ \theta(b)) = \mu B(\theta(ma)\ \wedge\ \theta(mb))$

$= \mu B(\theta(ma \wedge mb))$
$= \mu\theta^{1}(B)(ma \wedge mb)$
$\geq min \{\mu\theta^{1}(B) (ma), \mu\theta^{1}(B)(mb)\}$
$\geq min \{\mu B\theta(ma), \mu B\theta(mb)\}$
$\geq min \{\mu B m \theta(a), \mu B m \theta(b)\}$
$\geq min \{ \mu B (mx), \mu B (my)\}$

v) $\mu B(mx\ v\ my) \wedge \mu B(mx\ v\ mz)$
$= \mu B(m\theta(a) \wedge m\theta(b)) \wedge \mu B(m\theta(a) \wedge m\theta(c))$
$= \mu B(\theta(ma\ v\ mb)) \wedge \mu B(\theta(ma\ v\ mc))$
$= \mu\theta^{1}(B)(ma\ v\ mb) \wedge \mu\theta^{1}(B)(ma\ v\ mc)$
$\geq min \{\mu\theta^{1}(B)(ma\ v\ mb), \mu\theta^{1}(B)(ma\ v\ mc)\}$
$\geq min \{ \mu\theta^{1}(B)(ma), \mu\theta^{1}(B)(mb), \mu\theta^{1}(B)(ma\ v\ mc)\}$
$\geq min \{ \mu\theta^{1}(B)(ma), min\{ \mu\theta^{1}(B)(mb), \mu\theta^{1}(B) (ma\ v\ mc)\}\}$
$\geq min \{ \mu\theta^{1}(B)(ma), \mu\theta^{1}(B)(mb) \wedge \mu\theta^{1}(B)(ma\ v\ mc)\}$
$\geq min \{ \mu B m \theta(a), \mu B m \theta(b) \wedge \mu B (\theta(ma\ v\ mc))\}$
$\geq min \{ \mu B(mx), \mu B(my) \wedge \mu B(mx \vee mz)\}$

B is a fuzzy modular lattice ordered group of G'.

Proposition 4.3:
If $\{A_i\}$ is a family of fuzzy modular lattice ordered m-group of G then $\cap A_i$ is a fuzzy modular lattice ordered m-group of G where $\cap Ai = \{ x , \wedge \mu Ai (x) / x \in G \}$.

Proof:

Let x,y,z $\in$ G

i. $(\cap \mu Ai) m(xy) = \wedge\mu Aim(xy) = \wedge\mu Ai(mx\ my)$

$\geq \wedge min \{\mu Ai(mx), \mu Ai(my)\}$
$\geq min \{(\cap \mu Ai)(mx), (\cap \mu Ai)(my)\}$

ii. $(\cap \mu Ai)(mx)^{-1} = \wedge \cap \mu Ai (mx)^{-1}$
$\geq \wedge\mu Ai(mx) \geq (\cap \mu Ai) (mx)$

iii. $(\cap \mu Ai) (mx\ v\ my) = \wedge \mu Ai (mx\ v\ my)$
$\geq \wedge min\{\mu Ai(mx), \mu Ai(my)\}$
$\geq min\{\cap \mu Ai(mx), \cap \mu Ai(my)\}$

iv. $(\cap \mu Ai) (mx \wedge my) = \wedge \mu Ai(mx \wedge my)$
$\geq \wedge min\{\mu Ai(mx), \mu Ai(my)\}$
$\geq min\{\cap \mu Ai(mx), \cap \mu Ai(my)\}$

v. $(\cap \mu Ai) (mx\ v\ my) \wedge (\cap \mu Ai) (mx\ v\ mz)$
$\geq (\wedge \mu Ai)(mx\ v\ my) \wedge (\wedge \mu Ai) (mx\ v\ mz)$
$\geq \wedge min\{\mu Ai(mx), min\{\mu Ai(my), \mu Ai(mx\ v\ mz)\}\}$

Proposition 4.4:
If A is a fuzzy set in G such that all non-empty level subset. U(A;t) is a fuzzy modular lattice ordered m-group of G then A is fuzzy modular lattice ordered m-group of G.

Proof:

Let x,y,z $\in$ U(A;t), $A(mx) \geq t$, $A(my) \geq t$, $A(mz) \geq t$.

So that $A(m(xy)) \geq t$, $A(m(xz)) \geq t$.

i)$A (m(xy)) \geq t$
$\geq min\{t, t\} \geq min\{A(mx), A(my)\}$

ii)$A ((mx)^{-1}) \geq t = A(mx)$

iii) $A(mx\ v\ my) \geq t \geq min \{t, t\}$
$\geq min\{A(mx), A(my)\}$

iv)$A(mx \wedge my) \geq t \geq min \{t, t\}$
$\geq min \{A(mx), A(my)\}$

v) $A(mx\ v\ my) \wedge A(mx\ v\ mz) \geq t$
$\geq min \{t, t, t\}$

$\geq \quad min \quad \{A(mx), \; A(my) \wedge \; A(mx \quad v \quad mz)\}$
Therefore A is a fuzzy modular lattice ordered m-group.

Proposition 4.5:
Let A be a fuzzy modular lattice ordered m-group of G. Let A* be a fuzzy set in G defined by $A^*(x) = \; A(x) + 1 - A \; (e)$ for all x $\in$G. Then A* is a fuzzy modular lattice ordered m-group of G containing A.
Proof:
Let x, y, z$\in$G
i) $A^*(m(xy)) = \; A(m(xy)) + 1 - A(e) \geq \; min\{A(mx), A(my)\} \; + 1 \; - A(e)$
$\geq \; min\{A(mx) + 1 - A(e), A(my) + 1 - A(e)\}$
$\geq \; min \; \{ \; A^*(mx), A^*(my)\}$
ii)$A^*((mx)^{-1}) = A((mx)^{-1}) + 1 - A(e) \geq A(mx) + 1 - A(e)$
$\geq \; A^*(mx)$
iii)$A^*(mx \quad v \quad my) \; = \; A(mx \quad v \quad my)) + 1 - A(e) \geq min\{A(mx), A(my)\} + 1 - A(e)$
$\geq \; min \; \{A(mx) + 1 - A(e), \; A(my) + 1 - A(e)\}$
$\geq \; min \; \{A^*(mx), A^*(my)\}$
iv)$A^*(mx \; \wedge \; my)) = A(mx \; \wedge \; my)) + 1 - A(e)$
$\geq \; min\{A(mx), A(my)\} + 1 - A(e)$
$\geq \; min \; \{A(mx) + 1 - A(e), \; A(my) + 1 - A(e)\}$
$\geq \; min \; \{A^*(mx), A^*(my)\}$
v) $A^*(mx \quad v \quad my) \; \wedge \; A^*(mx \quad v \quad mz) = \; [A(mx \quad v \quad my) \; \wedge \; A(mx \quad v \quad mz)] + \; 1 - A(e)$
$\geq \; [min\{A(mx), \; min\{A(my), \; A(mx \quad v \quad mz)\}\}] + 1 - A(e)$
$\geq \; [min\{A(mx), \; A(my) \; \wedge \; A(mx \quad v \quad mz)\}] + 1 - A(e)$
$\geq \; min\{A(mx) + 1 - A(e), \; (A(my) \; \wedge \; A(mx \quad v \quad mz)) + 1 - A(e)\}$
$\geq \; min\{A^*(mx), \; A^*(my) \; \wedge \; A^*(mx \quad v \quad mz)\}$
Also $A(x) \; \leq \; A^*(x)$, for all x,y $\in$ G.
Therefore A* is a fuzzy modular lattice ordered m-group of G containing A.

Proposition 4.6:
If A is a fuzzy modular lattice ordered m-group of G and θ is a m-homomorphism of G then the fuzzy set $A^\theta = \; \{< mx; \; \mu_A{}^\theta(mx) >, \; x \; \in G \; \}$ is a fuzzy modular lattice ordered m-group.
Proof:
Let x, y, z $\in$ G
i)$\mu_A{}^\theta(m \quad (xy)) = \; \mu_A\theta(m(xy)) = \mu_A \; m \; \theta(xy) = \mu_A{}^\theta m(\theta(x) \quad \theta(y))$
$\geq \; min\{\mu A \; m \; \theta(x), \mu A \; m \; \theta(y)\}$
$\geq \; min \; \{\mu_A\theta(mx), \mu_A\theta(my)\}$
$\geq \; min \; \{\mu_A{}^\theta(mx), \mu_A{}^\theta(my)\}$
ii)$\mu_A\theta(mx)^{-1} = \mu_A\theta(mx)^{-1} \; = \mu_A \; (\theta(mx))^{-1} \; = \mu_A(m\theta(x))^{-1}$
$\geq \mu_A \; (m\theta(x))$
$\geq \mu_A\theta(mx)$
$\geq \mu_A{}^\theta(mx)$
iii)$\mu A\theta(mx \quad v \quad my) = \mu A\theta(mx \quad v \quad my) \; = \mu A\theta \; (mx) \quad v \quad \theta(my)$
$\geq \; min\{\mu A \; \theta(mx), \mu A\theta(my))$
$\geq \; min\{\mu A\theta \; (mx), \mu A\theta(my)\}$
iv)$\mu_A\theta(mx \; \wedge \; my) \; = \mu_A\theta(mx \; \wedge \; my))$
$= \mu_A\theta(mx) \; \wedge \; \theta(mx)$
$\geq min\{\mu_A \; \theta(mx), \mu_A \; \theta(my)\}$
$\geq min\{\mu_A\theta(mx), \mu_A\theta(my)\}$
v) $\mu_A\theta(mx \quad v \quad my) \; \wedge \; \mu_A\theta(mx \quad v \quad mz)$
$= \mu_A\theta(mx \quad v \quad my) \; \wedge \; \mu_A\theta(mx \quad v \quad mz)$
$= (\mu_A\theta(mx) \quad v \quad \theta(my)) \; \wedge \; (\mu_A\theta(mx) \quad v \quad \theta(mz))$
$\geq min\{\mu_A \; \theta(mx), \; \mu_A \; \theta(my), \; \mu_A\theta(mx \quad v \quad mz)\}$

$$\geq min\{\mu_A\ \theta(mx),\ \mu_A\ \theta(my)\ \wedge\ \mu_A\theta(mx\ v\ mz)\}$$
$$\geq min\{\mu_A\theta(mx),\ \mu_A\theta(my)\ \wedge\ \mu_A\theta(mx\ v\ mz)\}$$

Therefore A^{θ} is a fuzzy modular lattice ordered m-group of G.

Proposition 4.7:

Let T be a continuous t- norm and let f be a m-homomorphism on G. If μ is a fuzzy modular lattice ordered m-group on G then μ^f is a fuzzy modular lattice ordered m-group of f (G).

Proof:

Let $A_1 = f^{-1}(my1)$, $A2 = f^{-1}(my2)$, $A3 = f^{-1}(my3)$, $A12 = f^{-1}(m(y1y2)$, $A13 = f^{-1}(m(y1y3)$

Consider $A1A2 = \{mx \in G / mx = mx1\ mx2,\ for\ mx1 \in A1,\ mx2 \in A2\}$

$A1A3 = \{mx \in G / mx = mx1\ mx3,\ for\ mx1 \in A1,\ mx3 \in A3\}$

If mx ∈ A_1A_2 then mx = mx_1 mx_2 and $f(mx) = f(mx1\ mx2) = f(mx1)\ f(mx2) = my1\ my2$

$$= m(y1\ y2)$$

$mx \in f-1(m(y1\ y2)$ therefore $A_1A_2 \in A_{12}$

If mx∈A_1A_2 A_3 then mx= mx_1, mx_2, mx_3 and $f(mx) = f(mx1\ mx2\ mx3) = f(mx1)f(mx2)f(mx3)$

$$= my1\ my2\ my3 = m(y1\ y2\ y3)$$

$mx \in f^{-1}(m(y1\ y2\ y3)$ therefore A_1A_2 $A_3 \in$ A_{123}

i) $\mu f\ (m\ (y1y2)) = sup\ \{\mu(mx)\ /mx \in f^{-1}(m\ (y1y2)\ \} = sup\ \{\mu(mx)\ /mx \in A12\}$

$$\geq sup\ \{\mu(mx)\ /mx \in A1\ A2\}$$
$$\geq sup\ \{\mu(mx1\ mx2)/mx1 \in A1,\ mx2 \in A2\}$$
$$\geq sup\ \{T(\mu(mx1), T(\mu(mx2))/mx1 \in A1,\ mx2 \in A2\}$$
$$\geq T[sup\{\mu(mx1)/mx1 \in A1\},\ sup\{\mu(mx2)\ /mx2 \in A2\}]$$
$$\geq T[sup\{\mu(mx1)/mx1 \in f^{-1}(my1)\},\ sup\{\mu(mx2)/\ mx2 \in f1(my2)\}]$$
$$\geq T\{\mu f(my1),\ \mu f(my2)\}$$

ii) $\mu f\ ((my)^{-1}) = sup\{\mu(mx)^{-1}/(mx)^{-1} \in f^{-1}(my)^{-1}\} = sup\{\mu\ (mx)^{-1}/(mx) \in f^{-1}(my)\}$

$$\geq sup\{\mu(mx)/(mx) \in f^{-1}(my)\} \geq \mu f(my)$$

iii) $\mu f(my1\ v\ my2) = sup\{\mu(mx)/mx \in f^{-1}(my1\ v\ my2)\}$

$$= sup\{\mu(mx)/mx \in A1v2\}$$
$$\geq sup\{\mu(mx)/mx \in A1vA2\}$$
$$\geq sup\{\mu(mx1vmx2)/mx1 \in A1,\ mx2 \in A2\ \}$$
$$\geq sup\{T(\mu(mx1)),\ T(\mu(mx2))/mx1 \in A1,\ mx2 \in A2\}$$
$$\geq T[sup\{\mu(mx1)/mx1 \in A1\},\ sup\{\mu(mx2)/mx2 \in A2\}]$$
$$\geq T[sup\{\mu\ (mx1)/mx1 \in f^{-1}(my1)\},\ sup\{\mu(mx2)/mx2 \in f1(my2)\}] \geq T\ \{\mu f(my1)\ ,\ \mu f(my2)\}$$

iv) $\mu f(my1\ \wedge\ my2)) = sup\ \{\mu(mx)/mx \in f^{-1}(my1\ \wedge\ my2)\}$

$$= sup\{\mu(mx)/mx \in A1\wedge 2\}$$
$$\geq sup\{\mu(mx)/mx \in A1\ \wedge\ A2\}$$
$$\geq sup\{\mu(mx1\ \wedge\ mx2)/mx1 \in A1,\ mx2 \in A2\}$$
$$\geq sup\{T(\mu(mx1)), T(\mu(mx2))/mx1 \in A1,\ mx2 \in A2\}$$
$$\geq T[sup\{\mu(mx1)/mx1 \epsilon A1\},\ sup\{\mu(mx2)/mx2 \in A2\}]$$
$$\geq T[sup\{\mu(mx1)/mx1 \in f^{-1}(my1)\},\ sup\{\mu(mx2)/mx2 \in f^{-1}(my2)\}]$$
$$\geq T\ \{\mu f(my1),\ \mu f(my2)\}$$

v) $\mu f(my1\ v\ my2)\ \wedge\ \mu f(my1\ v\ my3)$

$$= sup\{\mu(mx)/mx \in (A1\ v\ A2\ )\ \wedge\ (A1\ v\ A3\ )\}$$
$$\geq sup\{\mu(mx1\ v\ mx2)\ \wedge\ \mu(mx1\ v\ mx3)/mx1 \in A1,\ mx2 \in A2,\ mx3 \in A3\}$$
$$\geq sup\{T(\mu(mx1)), T(\mu(mx2)), T(\mu(mx1\ v\ mx3))/mx1 \in A1,\ mx2 \in A2,\ mx3 \in A3\}$$
$$\geq T[sup\{(\mu(mx1)),\ mx1 \in A1\},\ sup\{(\mu(mx2))/mx2 \in A2\},\ sup\{(\mu(mx1vmx3)/mx1 \in A1, mx3 \in A3\}]$$
$$\geq T[sup\{\mu(mx1)/mx1 \in f^{-1}(my1)\},\ sup\{\mu(mx2) \wedge\ \mu\ (mx1\ v\ mx3)/mx2 \in\ f1(my2),\ mx3 \in f1(my3)\}]$$
$$\geq T\{\mu f(my1),\ \mu f(my2) \wedge\ \mu f(my1v\ my3)\}$$

Therefore μ^f is a fuzzy modular lattice ordered m-group of f (G).

Proposition 4.8:

An onto m-homomorphic image of fuzzy modular lattice ordered m group with sup property is a fuzzy modular lattice ordered m-group.

Proof:

Let f: G → G' be a onto m-homomorphism of G and let A be a fuzzy modular lattice ordered m-group of G with sup property.

Let mx', my',mz'∈G'

Let $mx_0 \in f^{-1}(mx')$,$my_0 \in f^{-1}(my')$,$mz_0 \in f^{-1}(mz')$

be such that $A(mx0) = \ sup\{A(mx)/mx \in f^{-1}(mx')\}$

$A \ (my0) \ = sup\{A(my)/my \ \in f^{-1}(my')\}$ &

$A \ (mz0) \ = sup\{A(mz)/mz \in f^{-1}(mz')\}$

i) $Af(m(x'y')) = sup\{A(z)/z \ \in f^{-1}(m \ (x'y'))\}$

$= \ sup\{A(z)/z \ \in f^{-1}(mx' \ my'))\}$

$\geq \ sup\{A(mx0my0)/mx0 \in f^{-1}(mx'), my0 \in f^{-1}(my')\}$

$\geq \ sup\{A(mx0y0)/mx0 \in f^{-1}(mx'), \ my0 \in f^{-1}(my')\}$

$\geq sup\{min\{A(mx0), \ A(my0)/mx0 \in f^{-1}(mx'), \ my0 \in f^{-1}(my')\}$

$\geq \ min\{sup\{A(mx0)/mx0 \ \in f^{-1}(mx')\}, \ sup\{A(my0)/my0 \in f^{-1}(my')\}] \ \geq \ min\{Af(mx'), \ Af(my')\}$

ii) $Af((mx')^{-1}) = \ sup\{A(mx0)^{-1}/(mx0)^{-1} \in f^{-1}(mx')^{-1}\}$

$= \ sup\{A(mx0 \ )^{-1}/(mx0) \ \in f^{-1}(mx')\}$

$\geq \ sup\{A(mx0)/(mx0) \ \in f^{-1}(mx')\}$

$\geq Af(mx')$

iii) $Af(mx' \ v \ my') \ = \ sup\{A(z)/z \ \in f^{-1}(mx' \ v \ my')\}$

$\geq \ sup\{A(z)/z \ \in f^{-1}(mx') \ v \ f^{-1}(my')\}$

$\geq \ sup\{A(mx0 \ v \ my0)/mx0 \in f^{-1}(mx'), \ my0 \in f^{-1}(my')\}$

$\geq \ sup\{A(mx0 \ v \ my0)/mx0 \in f^{-1}(mx'), \ my0 \in f^{-1}(my')\}$

$\geq \ sup\{min\{A(mx0), \ A(my0)/mx0 \in f^{-1}(mx'), \ my0 \in f^{-1}(my')\}$

$\geq \ min\{sup\{A(mx0)/mx0 \in f^{-1}(mx')\}, sup\{A(my0)/my0 \in f^{-1}(my')\}$

$\geq \ min\{ \ Af(mx'), \ Af(my')\}$

iv) $Af(mx' \ \wedge \ my') = \ sup\{A(z)/z \ \in f^{-1}(mx' \ \wedge \ my')\}$

$\geq \ sup\{A(z)/z \ \in f^{-1}(mx') \ \wedge \ f^{-1}(my')\}$

$\geq \ sup\{A(mx0 \ \wedge \ my0)/mx0 \in f^{-1}(mx'), \ my0 \in f^{-1}(my')\}$

$\geq \ sup\{A(mx0 \ \wedge \ my0)/mx0 \in f^{-1}(mx'), \ my0 \in f^{-1}(my')\}$

$\geq \ sup\{\min\{A(mx0), \ A(my0)/mx0 \in f^{-1}(mx'), \ my0 \in f^{-1}(my')\}$

$\geq \ min\{sup\{A(mx0)/mx0 \in f^{-1}(mx')\}, sup\{A(my0)/my0 \in f^{-1}(my')\} \geq \ min\{Af(mx'), \ Af(my')\}$

V) $Af(mx'v \ my') \ \wedge \ Af(mx' \ v \ mz')$

$= \ sup\{A(z)/z \in f^{-1}(mx' \ v \ my') \wedge \ f^{-1}(mx' \ v \ mz')\}$

$\geq \ sup\{A(mx0 \ v \ my0) \ \wedge \ A(mx0 \ v \ mz0)/mx0 \in f - 1(mx'), \ my0 \in f - 1(my'), mz0 \in f - 1(mz')\}$

$\geq \ sup\{min\{A(mx0), \ A(my0) \wedge \ A(\ mx0 \ v \ mz0)/mx0 \in f - 1(mx'), \ my0 \in f - 1(my')\}$

$\geq min\{sup\{A(mx0)/mx0 \in f^{-1}(mx')\}, sup\{A(my0) \wedge A(mx0 \ v \ mz0)/my0 \in f^{-1}(my'), mz0 \in f^{-1}(mz')\}$

$\geq min\{Af(mx'), \ Af(my') \wedge \ Af(mx' \ v \ mz')\}$

Proposition 4.9:

Let f: G→G' be a lattice group m-homomorphism and A be a fuzzy modular lattice ordered m-group of G' then $f^{-1}(A)$ is a fuzzy modular lattice ordered m-group of G.

Proof:

Let mx, my, mz∈ G and A be a fuzzy modular lattice ordered m-group of G'.

i) $f^{-1}(A)$ (m(xy)) $= A \ f \ (m(xy))$

$= \ A \ (f(mx)f(my))$

$= A \ (m \ f(x) \ mf(y))$
$= A \ (m \ f(x) \ f(y))$
$\geq min\{A(mf(x)), \ A(mf(y))\}$
$\geq \ min\{A(f(mx)), \ A(f(my))\}$
$\geq \ min\{f^{-1}(A) \ (mx), \ f^{-1}(A) \ (my)\}$

ii) $f^{-1}(A) \ ((m \ x)^{-1}) \ = \ A \ f((mx)^{-1})$
$= \ A \ (f(mx))^{-1}$
$= \ A \ (m \ f(x))^{-1}$
$\geq \ A \ (m \ f(x))$
$\geq \ A \ (f(\ mx))$
$\geq \ f^{-1}(A) \ (mx)$

iii) $f^{-1}(A) \ (mx \ v \ my)) \ = \ A \ f(mx \ v \ my)) \ = \ A \ (f(mx) \ v \ f(my)) \ = \ A \ (m \ f(x) \ v \ m \ f(y))$
$\geq \ min\{A \ (mf(x)), \ A \ (mf(y))\}$
$\geq \ min\{A \ (f(mx)), \ A(f(my))\}$
$\geq \ min\{f^{-1}(A) \ (mx), \ f^{-1}(A) \ (my)\}$

iv) $f^{-1}(A) \ (mx \ \wedge \ my)) \ = \ A \ f(mx \ \wedge \ my)) \ = \ A \ (f(mx) \ \wedge \ f(my)) \ = \ A(m \ f(x) \ \wedge \ m \ f(y))$
$\geq \ min\{A \ (mf(x)), \ A(mf(y))\}$
$\geq \ min\{A \ (f(mx)), \ A(f(my))\}$
$\geq \ min\{f^{-1}(A) \ (mx), \ f^{-1}(A) \ (my)\}$

v) $f^{-1}(A) \ (mx \ v \ my) \ \wedge \ f^{-1}(A) \ (mx \ v \ mz)$
$= \ A \ (mx \ v \ my)) \ \wedge \ A \ (mx \ v \ mz))$
$= \ A \ (m \ f(x \ v \ y) \ \wedge \ m \ f(x \ v \ z))$
$\geq \ min\{A \ (f(mx)), \ A(f(my)\wedge \ f \ (m \ (x \ v \ z))\}$
$\geq \ min\{f^{-1}(A) \ (mx), \ f^{-1}(A) \ (my) \ \wedge \ f^{-1}(A) \ (mx \ v \ mz)\}$

Therefore $f^{-1}(A)$ is a fuzzy modular lattice ordered m-group of G.

DIRECT PRODUCT OF FUZZY MODULAR LATTICE ORDERED M-GROUP

Definition: 5.1:
Let A_i be a fuzzy modular lattice ordered m-group of G_i, for i = 1,2,…,n. Then the product A_i(i = 1,2, ..., n) is the function $A_1 \times A_2 \times ... \times A_n$: $G_1 \times G_2 \times ... \times G_n \rightarrow L$ defined by $(A_1 \times A_2 \times ... \times A_n)$ $m(x_1,x_2,...,x_n)$
$= min\{A_1(mx_1),A_2(mx_2),\dots,A_n(mx_n)\}$

Proposition 5.2:
The direct product of fuzzy modular lattice ordered m groups is a fuzzy modular lattice ordered m-group.

Proof:
Let $x = (x_1, x_2, \ \dots, \ x_n), \ y \ = (y_1, y_2, \ \dots, y_n), \ z = (z_1, \ z_2, \dots, z_n) \in G_1 \times G_2 \times \dots \times G_n$
Let $A_1 \times A_2 \times \dots \times A_n \ = \ A$

i) $A(m(xy)) = A(m(x_1y_1, \ x_2y_2, \dots, x_ny_n))$
$= \ min\{A_1(mx_1y_1), \ A_2(mx_2y_2), \ \dots, \ A_n(mx_ny_n)\}$
$\geq \ min\{min[A_1(mx_1), A_1(my_1)], \ min[A_2(mx_2), A_2(my_2)], \dots, \ min[A_n(mx_n), \ A_n(my_n)]\}$
$\geq \ min\{min[A_1(mx_1), \ A_2(mx_2), \dots, A_n(mx_n)], \ min[A_1(my_1), \ A_2(my_2), \ \dots, \ A_n(my_n)]\}$
$\geq \ min\{(A_1 \times A_2 \times, \dots, \ \times A_n) \ m(x_1, x_2, \ \dots, x_n), (A_1 \times A_2 \times \dots \times A_n) \ m(y_1, y_2, \ \dots, y_n)\}$
$\geq \ min\{A(mx), \ A(my)\}$

ii) $A(m \ x)^{-1} \ = \ Am(x_1^{-1}, x_2^{-1}, \dots, \ x_n^{-1})$
$= \ min\{A_1(mx_1^{-1}), \ A_2(mx_2^{-1}), \ \dots, \ A_n(mx_n^{-1})\}$
$\geq \ min\{A_1(mx_1), \ A_2(mx_2), \dots, \ A_n(mx_n)\}$

$$\geq \; Am(x_1, \; x_2, \ldots, \; x_n)$$
$$\geq \; A(mx)$$

iii) $A(mx \;\; v \;\; my) = \; A(mx_1 \; v \; my_1, \; mx_2 \; v \; my_2, \; \ldots, \; mx_n \; v \; my_n)$
$$= \; min\{A_1(mx_1 \; v \; my_1), A_2(mx_2 \; v \; my_2), \; \ldots, A_n(mx_n \; v \; my_n)\}$$
$$\geq \; min\{min[A_1(mx_1), \; A_1(my_1)], \; min[A_2(mx_2), \; A_2(my_2)], \ldots, \; min[\; A_n(mx_n), A_n(my_n)]\}$$
$$\geq \; min\{min[A_1(mx_1), A_2(mx_2), \; \ldots, \; A_n(mx_n)], \; min[A_1(my_1), \; A_2(my_2), \; \ldots, \; A_n(my_n)]\}$$
$$\geq \; min\{(A_1{\times}A_2{\times}\ldots{\times}A_n) \;\; m(x_1, \; x_2, \ldots, \; x_n), \; (A_1{\times}A_2{\times}, \ldots, {\times}A_n) \;\; m(y_1, y_2, \ldots, y_n)$$
$$\geq \; min\{A(\; mx), \; A(my)\}$$

iv) $A(mx \;\; \wedge \;\; my) \;\; = \;\; A(mx_1 \; \wedge \; my_1, \; mx_2 \; \wedge \; my_2, \; \ldots, \; mx_n \wedge \; my_n \;)$
$$= \; min\{A_1(mx_1 \; \wedge \; my_1), \; A_2(mx_2 \; \wedge \; my_2), \; \ldots, \; A_n(mx_n \; \wedge \; my_n)\}$$
$$\geq \; min\{min[A_1(mx_1), \; A_1(my_1)], \; min[A_2(mx_2), \; A_2(my_2)], \ldots, \; min[A_n(mx_n), \; An \;\; (my_n)]\}$$
$$\geq \; min\{min[A_1(mx_1), \; A_2(mx_2), \ldots, \; A_n(mx_n)], \; min[A_1(my_1), \; A2(my_2), \; \ldots, \; A_n(my_n)]\}$$
$$\geq \; min\{(A1{\times}A2{\times} \;\; \ldots \;\; {\times}An) \;\; m(x1, x2, \ldots, xn), \; (A1{\times}A2{\times} \;\; \ldots \;\; {\times}An) \;\; m(y1, y2, \; \ldots, yn)$$
$$\geq \; min\{A(mx), \; A(my)\}$$

v) $A(mx \;\; v \;\; my) \;\; \wedge \;\; A(mx \;\; v \;\; mz)$
$$= min\{A1(mx1 \;\; v \;\; my1) \;\; \wedge \;\; A1(mx1 \;\; v \;\; mz1), A2(mx2 \;\; v \;\; my2) \wedge \;\; A2(mx2 \;\; v \;\; mz2), \ldots, An(mxnv \;\; myn) \;\; \wedge \;\; An(mxnv \;\; mzn)\}$$
$$\geq \; min\{min[A1(mx1), \; A2(mx2), \ldots, \; An(mxn)],$$
$$min[A1(my1), A2(my2), \ldots, \; An(myn)] \;\; \wedge min[A1(mx1 \;\; v \;\; mz1), A2(mx2 \;\; v \;\; mz2), \ldots, An(mxnv \;\; mzn)\}$$
$$\geq \; min\{(A1{\times}A2{\times}\ldots{\times}An) \;\; m(x1, x2, \;\; \ldots, xn),$$
$$(A1{\times}A2{\times}\ldots{\times}An) \;\; m(y1, y2, \ldots, yn), (A1{\times}A2{\times}\ldots{\times}An) \;\; m(x1 \;\; v \;\; z1), \; (x2 \;\; v \;\; z2), \; \ldots, \; (xn \;\; v \;\; zn)\}$$

CONCLUSION

In this paper we studied the notion of fuzzy modular lattice ordered m-group and investigated some of it's properties. We also studied the homomorphic image, pre-image of fuzzy modular lattice ordered m-group, arbitrary family of fuzzy modular lattice ordered m-groups and fuzzy modular lattice ordered m group using T-norms.

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