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Fuzzy Congruence Relations in Generalized Almost Distributive Fuzzy Lattices

T. Sangeetha[1] and Dr. S. Senthamil Selvi[2]

[1]Research Scholar, Department of Mathematics
Vels Institute of Science, Technology & Advanced Studies.
e-mail: sangeethavenkadesh@gmail.com
[2]Assistant Professor, Department of Mathematics
Vels Institute of Science, Technology & Advanced Studies.
e-mail: msselvi2305@gmail.com
[1]corresponding author

Abstract. This paper defines the fuzzy congruence relation of GADFL (Generalized nearly distributive fuzzy lattices). The ideas of θ - ideal and θ - Prime ideal are introduced in GADFL, and the fuzzy congruence relation is used to explain these ideals.

AMS subject classification: 06D72, 06F15, 08A72.
Index Terms—Fuzzy congruence relation, Fuzzy ideal, θ- ideal and θ- Prime ideal.

1. Introduction

G.C.Rao, Ravi Kumar, Banadaru, and N.Rafi proposed the concept of Generalized Almost Distributive Lattices (GADFL) as a generalisation of Almost Distributive Lattices (ADLs), which were a common abstraction of almost all existing ring theoretic generalisations of a Boolean algebra on the one hand and distributive lattices on the other. However, L.A.Zadeh [7] introduced the concept of a fuzzy set in 1965. A fuzzy ordering, according to L.A. Zadeh [8, is a transitive fuzzy relation that is a generalisation of the concept of ordering. A fuzzy partial ordering is a reflexive and antisymmetric fuzzy ordering in particular. N.Ajmal and K.V.Thomas [1] established a fuzzy lattice as a fuzzy algebra in 1994 and defined fuzzy sub lattices in 1995. In 2009, I.Chon [4] presented a novel notion of fuzzy lattices and examined the level sets of fuzzy lattices, based on Zadeh's fuzzy order concept. He also discussed the basic features of fuzzy lattices and presented the concepts of distributive and modular fuzzy lattices.

In this paper, the fuzzy congruence relation of GADFL (Generalized nearly distributive fuzzy lattices). The ideas of θ - i deal and θ - Prime ideal are introduced in GADFL, and the fuzzy congruence relation is used to explain these ideals.

2. Basic Definitions

The basic definitions of GADFL ideals are offered in this section, which is helpful in developing the subsequent sections.

2.1 Definition

Let $L(R,\vee,\wedge,0)$ be a fuzzy poset and (2,2,0) be an algebra type. If $L(R,A)$ satisfies the following axioms, we call $L(R,A)$ is a Generalized Almost Distributive Fuzzy Lattice.

1. $A\big((a\wedge b)\wedge c,\ a\wedge(b\wedge c)\big) = A(a\wedge(b\wedge c),(a\wedge b)\wedge c) = 1$
2. $A(a\wedge(b\vee c),(a\wedge b)\vee(a\wedge c)) = A\big((a\wedge b)\vee(a\wedge c),\ a\wedge(b\vee c)\big) = 1$
3. $A\big(a\vee(b\wedge c),(a\vee b)\wedge(a\vee c)\big) = A\big((a\vee b)\wedge(a\vee c),\ a\vee(b\wedge c)\big) = 1$
4. $A(a\wedge(a\vee b),\ a) = A\big(a,\ \ a\wedge(a\vee b)\big) = 1$
5. $A\big((a\vee b)\wedge a,\ a\big) = A(a,(a\vee b)\wedge a) = 1$
6. $A\big((a\wedge b)\vee b,\ b\big) = A(b,(a\wedge b)\vee b) = 1\ \forall a,b,c\in R.$

2.2 Definition

Let $L(R,A)$ be a GADFL and I be any R set that is not empty. If I meets the following characteristics, it is said to be an ideal of a GADFL $L(R,A)$.

1. $a,b\in I\Rightarrow a\vee b\in I$
2. $a\in I,\ b\in R\Rightarrow a\wedge b\in I$

2.3 Definition

An equivalence relation θ on an ADL, $L(R,A)$ is called a congruence relation on L if $(a\wedge c, b\wedge d), (a\vee c, b\vee d)\in\theta\ \forall\ (a,b),(c,d)\in\theta.$

2.4 Definition

For any congruence relation θ on an ADL and $a\in L$, we define $[a]_\theta = \{b\in L \mid (a,b)\in\theta\}$ and it called the congruence class containing a.

2.5 Definition

Let (R,A) be a GADFL. An equivalent relation θ on (R,A) is called a congruence on (R,A)if, for $a,b,c,d\in R$, holds $(a,b),(c,d)\in\theta\Rightarrow(a\vee c,b\vee d),(a\wedge c,b\wedge d)\in\theta.$

3. Fuzzy Congruence relations in GADFL

3.1 Definition

A fuzzy congruence relations in GADFL 'A' is defined as a fuzzy relation that meets the following requirements.

1. $\theta(a,\ a) = 1\ for\ all\ a\in A$
2. $\theta(a,b) = \theta(b,y)\ for\ all\ a,b\in A$
3. $\theta(a,\ b)\geq\theta(a,\ b)\wedge\theta(b,c), for\ all\ a,b,c\in A$
4. $\theta(a\vee c,b\vee d)\wedge\theta(a\wedge c,\ b\wedge d)\geq\theta(a,b)\wedge\theta(c,\ d) for\ all\ a,b,c,d\in A$

3.2 Definition

For all $a\in A$ is a fuzzy ideal of A, the fuzzy subset $\mu_\theta(a)$ is defined by $\mu_\theta(a\ n)$=$(a,0)$..

3.3 Definition

In GADFL A, let θ be a fuzzy congruence relation. The fuzzy congruence class that includes $a\in A$ is defined as follows: $[a]_\theta = \{\ b\in A \mid (a,b)\in\theta\ \}$.

3.4 Example

Let $R=\{a,b,c\}$ On GADFL, define the two binary operations ∨ and ∧ follows.

∨	*a*	*b*	*c*
a	*a*	*b*	*a*
b	*b*	*b*	*b*

c	c	c	c

$\wedge$	a	b	c
a	a	a	c
b	a	b	c
c	a	a	c

Cayley's tables 1 & 2

And let $R = \{0', a'\}$ be a discrete GADFL with the Hasse diagram shown in the diagram.

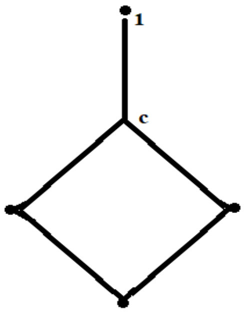

Figure 1 Hasse diagram, discrete GADFL R = {0′, a′}

Define the following fuzzy relation A: $R \times R \to [0,1]$:
$L(R, A) = \{(0', 0), (0', a), (0', b), (0', c), (0', 1), (a', 0), (a', a), (a', b), (a', c), (a', 1)\}$ is GADFL under point-wise operations.
Take $\theta = \{((0',0),(0',0)), ((0',a),(0',a)), ((0',b),(0',b)), ((0',c),(0',c)), ((0',1),(0',1)), ((a',0),(a',0)), ((a',a),(a',a)), ((a',b),(a',b)), ((a',c),(a',c)), ((a',1),(a',1)), ((0',c),(0',1)), ((0',1),(0',c))\}$.
Clearly, θ is a fuzzy congruence relation on $L(R, A)$.

3.5 Theorem
In GADFL 'A', let θ be a fuzzy congruence relation. Then μ_θ is a fuzzy ideal of A.
Proof
Let θ is a fuzzy congruence relation in GADFL 'A'. The μ_θ is the mapping defined by $\theta: A \to B$and $\mu_\theta(a)$ is defined by $\mu_\theta(a) = \theta(a, 0)$ for all $a \epsilon A$. Therefore, we get $\mu_\theta(0) = \theta(0,0) = 1$.
For any $x, y \epsilon \theta$this implies
$\mu_\theta(x \vee y) = \theta((x \vee y), 0)$
$= sup\{\theta(x, a) \wedge \theta(y, b) \mid a, b \epsilon A\}$
$\mu_\theta(x \vee y) \geq \{\theta(x, a) \wedge \theta(y, b) \mid a, b \epsilon A\}$
Assume $b = 0$, we get
$\mu_\theta(x \vee y) \geq \{\theta(x, a) \wedge \theta(y, 0)\}$
$\mu_\theta(x \vee y) \geq 1 \wedge \mu_\theta(y)$
$\mu_\theta(x \vee y) \geq \mu_\theta(y)$
Similarly, we find
$\mu_\theta(x \wedge y) = \theta((x \vee y), 0)$
$= sup\{\theta(x,a) \wedge \theta(y, b) \mid a, b \in A\}$
$\mu_\theta(x \wedge y) \geq \{\theta(x, a) \wedge \theta(y, b) \mid a, b \in A\}$
Assume $a = 0$, we get
$\mu_\theta(x \wedge y) \geq \theta\{(x \vee 0) \wedge \theta(y, b)\}$
$\mu_\theta(x \wedge y) \geq \mu_\theta(x, 0) \wedge 1$
$\mu_\theta(x \wedge y) \geq \mu_\theta(x, 0)$
$\mu_\theta(x \wedge y) \geq \mu_\theta(x)$
Therefore, we get $\mu_\theta(x \wedge y) \geq \mu_\theta(x) \wedge \mu_\theta(y)$. This implies $\mu_\theta(a)$ is a fuzzy ideal of GADFL 'A'.
Hence Proved.

3.6 Theorem

In GADFL 'A', let θ be a fuzzy congruence relation. Then V_θ is a fuzzy ideal of A. $V_\theta(x)$ is defined as follows: $V_\theta(x) = Inf\{\theta(a \wedge x, x) \forall a \in A\}$.

Proof

Let θ is a fuzzy congruence relation in GADFL 'A'. Then V_θ is a fuzzy ideal of A. Here $V_\theta(x)$ is defined by $V_\theta(x) = Inf\{\theta(a \wedge x, x) \forall a \in A\}$.

Consider $V_\theta(x) = Inf\{\theta(a \wedge x, x) \forall a \in A\}. = \theta(0, 0) = 1$

For any $x, y \in A$, this implies

$$V_\theta(x \vee y) = Inf\{\theta(a \wedge (x \vee y), x \vee y) | \forall a \in A\}$$
$$= Inf\{\theta(a \wedge (x \vee y), x \vee y) | \forall a \in A\}$$
$$\geq Inf\{\theta(a \wedge x, x) \wedge \theta(a \wedge y, y) | \forall a \in A\}$$
$$= Inf\{\theta(a \wedge x, x) | \forall a \in A\} \wedge Inf\{\theta(a \wedge y, y) | \forall a \in A\}$$
$$= V_\theta(x) \wedge V_\theta(y)$$

Therefore $V_\theta(x \vee y) \geq V_\theta(x) \wedge V_\theta(y)$

$$V_\theta(x \vee y) = Inf\{\theta(a \wedge (x \wedge y), x \wedge y) | \forall a \in A\}$$
$$= Inf\{\theta(a \wedge x, x) \wedge \theta(y \vee y) | \forall a \in A\}$$
$$\geq Inf\{\theta(a \wedge x), x) | \forall a \in A\}$$
$$= Inf\{\theta(a \wedge x, x) | \forall a \in A\}$$
$$= V_\theta(x)$$
$$V_\theta(x \vee y) \geq V_\theta(x)$$

Similarly, we get $V_\theta(x \vee y) \geq V_\theta(y)$. Therefore, we get $V_\theta(x \vee y) \geq V_\theta(y) \wedge V_\theta(y)$. Hence v_θ is a fuzzy ideal of GADFL 'A'.

3.7 Definition

If there θ is a fuzzy congruence relation in A that implies $[a]_\theta \subseteq I$., then an ideal 'I' of GADFL 'A' is said to be a θ - ideal of A.

3.8 Theorem

The following criteria are identical if θ is a fuzzy congruence relation in GADFL 'A' and I is an ideal of A.

1. I is a θ – Ideal
2. For any $a, b \in A (a, b) \in \theta$ and $a \in I \Longrightarrow b \in I$.
3. $I = \cup_{a \in I} [a]_\theta$.

Proof

Let θ is a GADFL 'A' fuzzy congruence relation, and I is an ideal of A. Assume I is a θ –Ideal, which means that if there is a fuzzy congruence relation in A for each $a \in I$, it entails $[a]_\theta \subseteq I$.

Hence (1) ⇒ (2).

Assume for any $a, b \in A, (a, b) \in \theta$ and $a \in I \Rightarrow b \in I$. Let $a \in I$, this implies $a \in [a]_\theta$. therefore we get $I = \cup_{a \in I} [a]_\theta$. Conversely, we assume $b \in \cup_{a \in I} [a]_\theta$. Then, $(a, b) \in \theta$ for some $b \in I$. Using condition (2) we get $a \in I$ this implies $I = \cup_{a \in I} [a]_\theta$. Hence, we get (2) ⇒ (3).

Assume $I = \cup_{a \in I} [a]_\theta$. Let $b \in I$, this implies $(a, b) \in \theta$, for some $a \in I$. Let $x \in [a]_\theta$ this implies we get $(x, a) \in \theta$. Therefore, we get $(x, a) \in \theta \subseteq I$. This implies I is a θ – Ideal of A. Hence (3) ⇒ (1).

3.8. Definition

In GADFL 'A', let θ be a fuzzy congruence relation. If any $a, b \in I$, such that $a \wedge b \in [a]_\theta$ implies either $a \in P$ or $b \in P$, then a proper θ –Ideal P of a GADFL 'A' is said to be a θ –Prime ideal of A.

3.9. Theorem

In GADFL 'A', let θ be a fuzzy congruence relation. Then, in A, every prime ideal is a θ –Prime ideal of A.

Proof

Let θ is a fuzzy congruence relation in GADFL 'A' and P be a prime ideal in A.
Therefore, for any $a, b \in I$, such that $a \wedge b \in [a]_\theta$ this implies either $a \in P$ or $b \in P$.
This implies $[a]_\theta \subseteq P$.
Therefore, P is a θ–Ideal of A. Let $a, b \in I$, such that $a \wedge b \in [a]_\theta$ this implies either $[a \wedge b]_\theta \in [0]_\theta$ since θ be a fuzzy congruence relation in GADFL 'A'.
Therefore, we get $a \wedge b = 0 \in P$.
Hence, we get P is a θ – Prime ideal of A.

3.10. Theorem

Let A be a θ– ideal of θ and be a fuzzy congruence relation in GADFL 'A'. Then the statements that follow are equivalent.

1. P is a θ – Prime ideal of A.
2. Any ideals I, $J \in A$ with $I \cap J \subseteq [0]_\theta \Rightarrow I \subseteq P$ or $J \subseteq P$.
3. Any $a, b \in A$, $[a]_\theta \cap [b]_\theta = [0]_\theta \Rightarrow a \in P$ or $b \in P$

Proof

Assume θ be a fuzzy congruence relation in GADFL 'A' and P be a θ – Prime ideal of A. Let any ideals $I, J \in A$ with $I \cap J \subseteq [0]_\theta$. For any $a \in I\ or\ b \in J$, such that $a \wedge b \in I \cap J \subseteq [0]_\theta$ this implies either $a \in P$ or $b \in P$. Hence $I \subseteq P\ or\ J \subseteq P$. (1) $\Rightarrow$ (2).
Assume any ideals $I, J \in A$ with $I \cap J \subseteq [0]_\theta \Rightarrow I \subseteq P\ or\ J \subseteq P$. Suppose that $[a]_\theta \cap [b]_\theta$ for any $a, b \in A$, therefore we get $a \wedge b \in [a]_\theta$ this implies $a \in P\ or\ b \in P$.
Hence (2) $\Rightarrow$ (3).
Assume for any $a, b \in A$, $[a]_\theta \cap [b]_\theta = [0]_\theta \Rightarrow a \in P$ or $b \in P$. Therefore, we get $a \wedge b \in [0]_\theta$. This implies $[a]_\theta \cap [b]_\theta = [a \wedge b]_\theta = [0]_\theta$. Hence, we get $a \in P\ or\ b \in P$. This implies P is a θ –Prime ideal of A.
Hence (3) $\Rightarrow$ (1).

3.11. Definition

Let θ is a fuzzy congruence relation in GADFL 'A' and$V_\theta$be a fuzzy ideal of A. The subset $V_\theta(x)$ is defined by $V_\theta(x) = Inf\{\theta(a \wedge x, x) \forall\, a \in A\}$.

3.12. Theorem

In GADFL 'A', let θ be a fuzzy congruence relation. Then $V_\theta$be a fuzzy ideal of A.

Proof

Let θ is a fuzzy congruence relation in GADFL 'A'. By the definition of $0 \in V_\theta$.
Let $x, y \in A$ such that $(x, a) \in \theta$ & $(y, b) \in \theta$ for some $a, b \in \theta$.
Therefore, we get $((x \vee y), (a \vee b)) \in \theta$. Since θ is fuzzy congruence relation in GADFL 'A'. This implies we get $(x \vee y) \in V_\theta$ Now assume $x \in V_\theta$ and $y \in A$. This implies $(x, a) \in \theta$ for some $a \in \theta$. Hence, we get $((x \wedge y), (a \wedge y)) \in \theta$.
Since θ is fuzzy congruence relation in GADFL 'A'. This implies $(x \wedge y) \in V_\theta$. Therefore $V_\theta$is a fuzzy ideal of A.

3.13. Theorem

Let θ is a fuzzy congruence relation in GADFL 'A' and m be a maximal element in A, then every maximal ideal M such that $M \cap [m]_\theta = \emptyset$, is a θ – ideal of A.

Proof

Let θ is a fuzzy congruence relation in GADFL 'A' and m be a maximal element in A. Let M be a maximal ideal in A such that $M \cap [m]_\theta = \emptyset$. Let $x, y \in$ A such that $(x, y) \in \theta$ and $x \in M$. Assume $y \notin$ M, therefore we get $M \cap (y] = A \Rightarrow a \vee y$ is a maximal element of A for some $x \in M$.
Let $(x, y) \in \theta$ therefore we get $((a \vee x), (a \vee y)) \in \theta$ this implies $(a \vee x) \in [a \vee y]_\theta$ since $(a \vee x) \in M$ we get $M \cap [a \vee y] = \emptyset$. This is contradicting to our assumption. Therefore, we get $y \notin M$. Hence M is a θ – ideal of A.

3.14. Theorem

Let θ be fuzzy congruence relation GADFL (R, θ). Then the following are equivalent

1. (R, θ) is a GADFL.
2. For any fuzzy ideal V_θ of (R, θ), φV_θ is a fuzzy congruence relation on (R, θ).
3. φa_θ is a fuzzy congruence relation on $(R, \theta)\ \forall\ a \in R$.

Proof
(1) ⇒ (2)
Assume (1), Let V_θ be an ideal of (R, θ). Clearly, φV_θ is an equivalence relation
(1) Let $(a, a) \in \varphi V_\theta$.Then $\theta(a \vee x,\ a \vee x) = 1$ and $(a \vee y,\ a \vee y) = 1$ for some , $y \in V_\theta$.
Let $(a,\ b) \in \varphi V_\theta$.Then $\theta(a \vee x,\ b \vee x) = 1$ and $\theta(a \vee y,\ b \vee y) = (b \vee y,\ a \vee y) = 1$for some $x, y \in V_\theta$. Therefore $\theta(a,\ b) = \theta(b, a)\ \forall\ a, b \in \varphi V_\theta$.
(2) Let $(a,\ b, c) \in \varphi V_\theta$Then $\theta(a \vee x,\ c \vee x) \geq \theta(a \vee x,\ b \vee x) \wedge \theta(b \vee x,\ c \vee x) = 1$ and $\theta(a \vee y,\ c \vee y) \geq \theta(a \vee y,\ b \vee y) \wedge \theta(b \vee y,\ c \vee y) = 1$ for some $x,\ y, \in V_\theta$
Therefore $\theta(a,\ c) \geq (a,\ b) \wedge (b,\ c)\ \forall\ a,\ b,\ c \in \varphi V_\theta$
(3) Let $(a,\ b), (c,\ d) \in \varphi V_\theta$Then $\theta(a \vee x,\ b \vee x) = \theta(b \vee x,\ a \vee x) = 1$ and
$\theta(c \vee y,\ d \vee y) = \theta(d \vee y,\ c \vee y) = 1$ for some $x,\ y, \in V_\theta$
Since V_θ is a fuzzy ideal of (R, θ), $x,\ y, \in V_\theta$
Now, $\theta(a \vee c \vee x \vee y, b \vee d \vee x \vee y)$
$= \theta(a \vee x \vee c \vee y,\ b \vee d \vee x \vee y)$
$= \theta(a \vee x \vee d \vee y,\ b \vee d \vee x \vee y)$
$= \theta(b \vee x \vee d \vee y,\ b \vee d \vee x \vee y)$
$= \theta(b \vee d \vee x \vee y,\ b \vee d \vee x \vee y) = 1$
Similarly, $\theta(b \vee d \vee x \vee y,\ a \vee c \vee x \vee y) = 1$ and hence $(a \vee c,\ b \vee d) \in \varphi V_\theta$
Also, $\theta((a \wedge c) \vee x \vee y, (b \vee d) \vee x \vee y)$
$= \theta((a \vee x \vee y) \wedge (c \vee x \vee y), (b \wedge d) \vee x \vee y)$
$= \theta((b \vee x \vee y) \wedge (x \vee c \vee y), (b \wedge d) \vee x \vee y)$
$= \theta([b \vee (x \vee y)] \wedge [(x \vee d \vee y], (b \wedge d) \vee x \vee y)$
$= \theta([b \vee (x \vee y)] \wedge [(d \vee (x \vee y)], (b \wedge d) \vee x \vee y)$
$= \theta((b \wedge d) \vee x \vee y, (b \wedge d) \vee x \vee y) = 1$
Similarly, $\theta((b \wedge d) \vee x \vee\ , (a \wedge c) \vee x \vee y) = 1$
Therefore $(a \wedge c,\ b \wedge d) \in \varphi V_\theta$
Thus $\varphi V_\theta$is a fuzzy congruence relation on (R, θ)
(2) ⇒ (3) It is obivious (3) ⇒ (1)
Assume (3) Let $a, b \in R$
Since $\theta(a \vee b, (a \vee b) \vee b) = \theta((a \vee b) \vee b, a \vee b) = 1$
Then $(a, a \vee b) \in \varphi a_\theta$. Also $\theta(b \vee b, b \vee d) = 1$
Hence $(b, b) \in \varphi a_\theta$
Since $\varphi a_\theta$is a congruence relation on $(R, \theta), (a \wedge b, (a \vee b) \wedge b) \in \varphi a_\theta$
Hence $\theta((a \wedge b) \vee b, [(a \vee b) \wedge b] \vee b)$
$= \theta([(a \vee b) \wedge b] \vee b, (a \wedge b) \vee b) = 1$
$\Rightarrow \theta((a \wedge b) \vee b\ ,\ b \vee b) = \theta(b \vee b, (a \wedge b) \vee b) = 1$
$\Rightarrow \theta((a \wedge b) \vee b,\ b) > 0$ and $\theta(b, (a \wedge b) \vee b) > 0$
Therefore (R, θ) is a GADFL.

4. Conclusion

The fuzzy congruence relation of Generalized Almost Distributive Fuzzy Lattices is defined in this study (GADFL). The ideas of θ - ideal and θ - Prime ideal are introduced in GADFL. with the fuzzy congruence relation used to characterize these ideals. We will also address the features and fundamental theorem of fuzzy homomorphism on Generalized Almost Distributive Fuzzy Lattices in the future paper

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