

## A NEW APPROACH ON $\kappa$ -REGULAR FUZZY GRAPHS WITH CHROMATIC NUMBER AND ITS APPLICATIONS

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ABSTRACT. The author investigate about this paper to define a Chromatic number of  $\kappa$  regular fuzzy Graph, and also introduce a new concept of fuzzy chromatic number of  $\kappa$  regular fuzzy Graph. Now we consider fuzzy Graph with fuzzy set of vertices and fuzzy set of edges. We extend some application, like as exam timetable problem. Using chromatic number concept we have to solve different type of subject to conduct an exam within a certain time of period.

### 1. INTRODUCTION

The fuzzy graph was invented by Rosenfield in 1935. Graph coloring finds its origin during late 1850 "the total coloring" was independently introduced by Behzad and vizing between 1964 and 1968. The regular fuzzy graph, total degree and totally regular fuzzy graph introduced by Nagoor Gani and Radha. In 1736, the concept of graph theory was first introduced by Euler. The perception of fuzzy set was discussed by L.A. Zadeh, in 1965. In 1973, Kaufmann gave the first definition of a fuzzy graph which was based on Zadeh's fuzzy relations. A. Rosenfeld considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. Zadeh introduced the concept of fuzzy relations, in 1987. The theory of fuzzy graphs was developed J.N. Mordeson studied fuzzy line graphs and developed its basic properties, in 1993. ([1–10])

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In this paper, we discussed the concept of totally  $\kappa$ - regular fuzzy graph (*TRFG*). A comparative study between vertex regular fuzzy graphs (*RFG*) and *TRFG* is made. Also some results on vertex regular fuzzy graphs are studied and examined whether they hold for *TRFG*. Some basic definitions are presented in this paper, kindly see [1–5].

**Definition 1.1.** A fuzzy graph  $G = (\alpha, \beta)$  is a pair of functions  $\gamma : V \rightarrow [0, 1]$  and  $\mu : V \times V \rightarrow [0, 1]$  with,  $\mu(\alpha, \beta) \leq \gamma(\alpha \wedge \beta)$ , for all  $\alpha, \beta \in V$  where  $V$  is a finite non empty set and  $\wedge$  denote the minimum. Where  $\gamma$  a fuzzy sub is set of a non – empty set  $V$  and  $\mu$  is a symmetric fuzzy relation on  $\gamma$ .

**Definition 1.2.** Let  $G = (\alpha, \beta)$  be a fuzzy graph on  $G^* = (\alpha, \beta)$ . The degree of an edge  $\nu$  is  $d_G(\alpha, \beta) = d_G(\alpha) + d_G(\beta) - 2 \wedge \{\phi(\alpha, \beta)\}$ .

**Definition 1.3.** Let  $G = (\alpha, \beta)$  be a fuzzy graph on  $G^* = (\alpha, \beta)$ . The total of an edge  $\nu \in \beta$  is defined by  $Td_G(\alpha, \beta) = \wedge \{\nu_1, \nu_2, \nu_3, \dots, \nu_n\}$ .

**Definition 1.4.** Let  $G = (\alpha, \beta)$  be a fuzzy graph on  $G^* = (\alpha, \beta)$ . The degree of vertex  $\alpha \in \alpha$  is defined by  $d_G(\alpha) = \sum_{\alpha \neq \beta} \wedge(\phi(\alpha, \beta))$ .

**Definition 1.5.** The Minimum degree of  $G$  is defined by  $\delta(G) = \wedge \{d_G(\beta), \forall \beta \in \beta\}$ . The Maximum degree of  $G$  is defined by  $\Delta(G) = \wedge \{d_G(\beta), \forall \beta \in \beta\}$ .

## 2. VERTEX REGULAR FUZZY GRAPH AND TOTALLY VERTEX REGULAR FUZZY GRAPH

**Definition 2.1.** Let  $G = (\alpha, \beta)$  be a fuzzy graph on  $G^* = (\alpha, \beta)$ . If each vertex in  $G$  has same degree  $\kappa$ , then  $G$  is said to be a vertex regular fuzzy graph or  $\kappa$  - regular fuzzy graph.

**Definition 2.2.** Let  $G = (\alpha, \beta)$  be a fuzzy graph on  $G^* = (\alpha, \beta)$ . If each vertex in  $G$  has same total degree  $\kappa$ , then  $G$  is said to be a totally vertex regular fuzzy graph or  $\kappa$  - totally vertex regular fuzzy graph.

**Theorem 2.1.** Let  $G = (\sigma, \mu)$  be a fuzzy graph on  $G^* = (V, E)$ . Then  $\sigma$  is a constant function if and only if the following are equivalent:

1.  $G$  is a vertex regular fuzzy graph.
2.  $G$  is a totally vertex regular fuzzy graph

3. REMARKS

**Remark 3.1.**

- We know that by crisp graph theory, any complete graph is vertex regular. But this result does not follow for fuzzy case.
- A complete fuzzy graph need not be vertex regular. For example  $G$  is not vertex regular, but it is a complete fuzzy graph.

**Remark 3.2.** Very well known that by crisp, any complete graph is vertex regular. In this this result we will show for complete fuzzy graph is a vertex regular fuzzy graph or  $\kappa$  - regular fuzzy graph.

**Example 1.** Consider the following fuzzy graph  $G = (\alpha, \beta)$  (Figure 1)

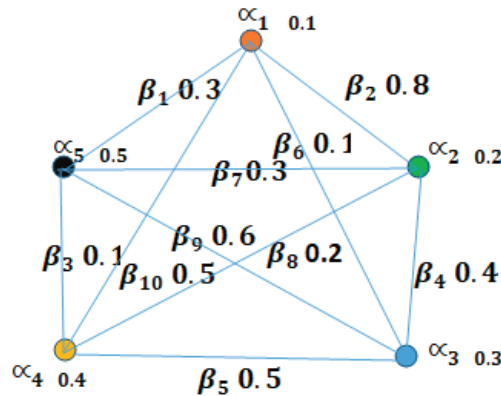


FIGURE 1

$$\begin{aligned} \delta(G) &= \wedge\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\} \\ &= \{0.1, 0.2, 0.3, 0.4, 0.5\} \\ &= 0.1 \end{aligned}$$

$$\begin{aligned} \Delta(G) &= \wedge\{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10}\} \\ &= \{0.3, 0.8, 0.1, 0.4, 0.5, 0.1, 0.3, 0.2, 0.6, 0.5\} \\ &= 0.1 \end{aligned}$$

$$\delta(G) = \Delta(G) = 0.1$$

Therefore the figure 1 is totally vertex regular fuzzy graph and also vertex regular fuzzy graph.

**Theorem 3.1.** *If a fuzzy graph  $G$  is both vertex regular and totally vertex regular, then  $\sigma$  is a constant function. The converse of theorem need not be true*

**Remark 3.3.** *The converse of theorem 3.1 is true, we will prove it can be through the example 1.  $G = (\alpha, \beta)$  is vertex regular and totally vertex regular.*

**Theorem 3.2.** *Let  $\beta = c$  be a constant function in  $G = (\alpha, \beta)$  on  $G^* = (\alpha, \beta)$ . If  $G$  is edge regular, then  $G$  is vertex regular.*

**Theorem 3.3.** *Let  $\beta = c$  be a constant function in  $G = (\alpha, \beta)$  on  $G^* = (\alpha, \beta)$ . If  $G$  is edge regular, then  $G$  is vertex regular.*

#### 4. FUZZY TOTAL CHROMATIC NUMBER OF A TRFG

Consider the following example of fuzzy total coloring of a totally regular fuzzy graph and its associated total chromatic number a scheduling problem is presented in this example.

**Example 2.** *Draw up an examination study schedule involving the minimum number of days for following problem. For 11<sup>th</sup>, 12<sup>th</sup>, UG Students. Sets of students Examination subjects are given below:*

$\theta_1 \rightarrow$  Tamil -I, Tamil- II and Mathematics

$\theta_2 \rightarrow$  English-I, English-II and statistics

$\theta_3 \rightarrow$  Physics, Chemistry, Accounts

$\theta_4 \rightarrow$  Botany, zoology and computer

$\theta_5 \rightarrow$  Combinatory, Topology, and Functional Analysis

$\theta_6 \rightarrow$  Operations Research, Graph Theory and Coding Theory

$\theta_7 \rightarrow$  Operations Research, Graph Theory and Number Theory

$\theta_8 \rightarrow$  Algebra, Number Theory and Coding Theory

$\theta_9 \rightarrow$  Algebra, Operations Research and Real Analysis.

#### **Solution.**

We draw a graph Figure 2 in which the vertices are the minimum number of days and two vertices are adjacent if they have a subject in common. Here we Figure 2 the minimum number of days slots needed to Examination Schedule the entire subjects. The graph is intersection its vertices corresponds to sets and it has edge between two vertices then its are intersect. From the above

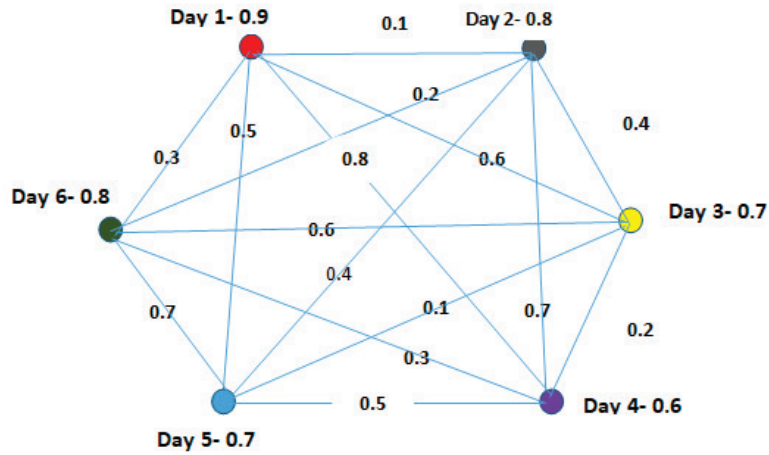


FIGURE 2

graph is totally regular and 6 vertices. Each vertices has degree 5 because the pair of vertices are adjacent. The adjacent vertices has two different color. The  $k$  regular graph where  $k$  is 6 the exam schedule of each subject depends on the intersection of sets.

Solving study time table problem by using chromatic number concept.

**Definition 4.1.** *The chromatic number of a graph is the smallest number of colors needed to color the vertices of so that no two adjacent vertices share the same color.*

*By using this concept we have to take minimum value of each vertex. The fuzzy graph has six vertices and each vertices have same degree. All the vertices are convert in to subject. Each subject we have to give the name of fuzzy numbers.*

- 0.1) - Tamil,
- 0.2) - English,
- 0.3) - Maths,
- 0.4) - Physics,
- 0.5) - Chemistry,
- 0.6) - Botany,
- 0.7) - Zoology,
- 0.8) - Statistics,
- 0.9) - Accounts.

Obviously the examination subjects depend on the intersection of sets. This concept could be fuzzy, and it could be associated with some numerical values and the problem in example 2.

Minimum number of days	Subjects
Day 1-0.9	Tamil and English, Algebra
Day 2-0.8	Physics and Chemistry, OR
Day 3-0.7	Mathematics and Botany
Day 4-0.6	Statistics and Accounts
Day 5-0.7	Combinatory and Number Theory, Real
Day 6-0.8	Computer and Algebra

Consider the Figure 2 a totally regular fuzzy graph and the regular crisp graph is also a regular fuzzy graph with vertex set

$$V = \{V_1, V_2, V_3, V_4, V_5, V_6\} (\alpha = 0.9, 0.8, 0.7, 0.6, 0.7, 0.8)$$

and edge set

$$E = \{V_i V_j / ij = 12, 13, 14, 15, 16, 23, 24, 25, 34, 35, 60, 64, 65, 68, 74\}$$

$$\beta = (\theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}, \theta_{23}, \theta_{24}, \theta_{25}, \theta_{34}, \theta_{35}, \theta_{60}, \theta_{64}, \theta_{65}, \theta_{68}, \theta_{74})$$

the membership functions are defined as follows:

$$\alpha(v_i) = \begin{cases} 0.9 & i = 1, \\ 0.8 & i = 2, 6 \\ 0.7 & i = 3, 5 \\ 0.6 & i = 4 \\ 0 & otherwise; \end{cases} \quad \beta(v_i, v_j) = \begin{cases} 0.1 & ij = \theta_{12}, \theta_{15} \\ 0.2 & ij = \theta_{13}, \theta_{16} \\ 0.3 & ij = \theta_{14}, \theta_{60}, \\ 0.4 & ij = \theta_{23}, \theta_{68} \\ 0.5 & ij = \theta_{24}, \theta_{65} \\ 0.6 & ij = \theta_{25}, \theta_{35} \\ 0.7 & ij = \theta_{34}, \theta_{64} \\ 0.8 & ij = \theta_{74} \\ 0 & otherwise; \end{cases}$$

Now the membership function satisfy the definition totally regular fuzzy graph.

Let  $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$  be a family of fuzzy set defined on  $V \cup E$  as follows

$$\alpha_1(v_i) = \begin{cases} 0.9 & i = 1, \\ 0 & otherwise; \end{cases} \quad \alpha_1(v_i, v_j) = \begin{cases} 0.1 & ij = \theta_{12} \\ 0.3 & ij = \theta_{14} \\ 0.7 & ij = \theta_{64} \\ 0 & otherwise; \end{cases}$$

$$\alpha_2(v_i) = \begin{cases} 0.8 & i = 2, \\ 0 & \textit{otherwise}; \end{cases} \quad \alpha_2(v_i, v_j) = \begin{cases} 0.2 & ij = \theta_{13} \\ 0.4 & ij = \theta_{23} \\ 0.6 & ij = \theta_{25} \\ 0 & \textit{otherwise}; \end{cases}$$

$$\alpha_3(v_i) = \begin{cases} 0.7 & i = 3, \\ 0 & \textit{otherwise}; \end{cases} \quad \alpha_3(v_i, v_j) = \begin{cases} 0.3 & ij = \theta_{60} \\ 0.6 & ij = \theta_{35} \\ 0 & \textit{otherwise}; \end{cases}$$

$$\alpha_4(v_i) = \begin{cases} 0.6 & i = 4, \\ 0 & \textit{otherwise}; \end{cases} \quad \alpha_4(v_i, v_j) = \begin{cases} 0.4 & ij = \theta_{68} \\ 0.8 & ij = \theta_{74} \\ 0 & \textit{otherwise}; \end{cases}$$

$$\alpha_5(v_i) = \begin{cases} 0.7 & i = 5, \\ 0 & \textit{otherwise}; \end{cases} \quad \alpha_5(v_i, v_j) = \begin{cases} 0.5 & ij = \theta_{24} \\ 0.7 & ij = \theta_{34} \\ 0.2 & ij = \theta_{16} \\ 0 & \textit{otherwise}; \end{cases}$$

$$\alpha_6(v_i) = \begin{cases} 0.8 & i = 6, \\ 0 & \textit{otherwise}; \end{cases} \quad \alpha_6(v_i, v_j) = \begin{cases} 0.5 & ij = \theta_{65} \\ 0.1 & ij = \theta_{60} \\ 0 & \textit{otherwise}; \end{cases}$$

Hence the family  $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$  satisfies our definition of total coloring of a totally regular fuzzy graph. We defined that any family of fuzzy sets having less than 5 times periods could not satisfy the definition. From the tables given below, we can see the values of  $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$  clearly.

Hence in this case totally regular fuzzy graph of total chromatic number  $\chi(G)$  is 6.

Set of vertices  $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$  be a family of fuzzy sets defined on as follows.

**Sets of Vertices** ( $v_i$ )

<b>Vertices</b>	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	<b>Maximum</b>
<b>Day 1</b>	0.9	0	0	0	0	0	0.9
<b>Day 2</b>	0	0.8	0	0	0	0	0.8
<b>Day 3</b>	0	0	0.7	0	0	0	0.7
<b>Day 4</b>	0	0	0	0.6	0	0	0.6
<b>Day 5</b>	0	0	0	0	0.7	0	0.7
<b>Day 6</b>	0	0	0	0	0	0.8	0.8

**Sets of Edges** ( $v_i, v_j$ )

<b>Vertices</b>	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	<b>Maximum</b>
12	0.1	0	0	0	0	0	0.1
13	0	0.2	0	0	0	0	0.2
14	0.3	0	0	0	0	0	0.3
15	0	0	0	0	0	0.1	0.1
16	0	0	0	0	0.2	0	0.2
23	0	0.4	0	0	0	0	0.4
24	0	0	0	0	0.5	0	0.5
25	0	0.6	0	0	0	0	0.6
34	0	0	0	0	0.7	0	0.7
35	0	0	0.6	0	0	0	0.6
60	0	0	0.3	0	0	0	0.3
64	0.7	0	0	0	0	0	0.7
65	0	0	0	0	0	0.5	0.5
68	0	0	0	0.4	0	0	0.4
74	0	0	0	0.8	0	0	0.8

**5. CONCLUSION**

We have to discuss this paper, to prove the complete graph need not be a vertex regular but its complete fuzzy graph. Hence it proved that the fuzzy graph  $G$  is both vertex regular and totally vertex regular. This converse part is also true. The  $TRFG$  is very rich both in theoretical developments and applications.



The *TRFG* have been investigated by different authors, and in this paper the concept of *TRFG* adapted for fuzzy graphs.

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