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ECCENTRIC SEQUENCE OF GRAPHS

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ABSTRACT. The minimum length in a graph G between two vertices is defined to be the distance between the two vertices and is denoted by $d(a, b)$. The farthest vertex distance from a vertex 'a' is known as the eccentricity e(a) of the vertex 'a'. Enumerating the vertex eccentricities in an increasing order is defined as the eccentricity sequence or eccentric sequence of the graph G. The eccentric sequence of some graphs is computed in this paper.

1. INTRODUCTION

By definition, enumerating a set of arithmetical values or numbers is defined as the sequence. In graph theory, it is more preferable to represent a graph as a sequence than an independent number. A single number or an invariant cannot give more information about the graph when compared to a sequence. Hence, using sequence is more convenient than using a single number or an invariant sequence. Some sequences that represent a graph are the degree sequence, eccentric sequence, distance degree sequence, status sequence, path degree sequence.

Hakimi and Havel developed the concept of degree sequence of graphs. They both worked independently on the concept of degree sequence. For undirected graphs, Eccentric sequence was introduced and developed as a distance related sequence. A major contribution in this area was done by Lesniak, Ostrand,

Behzad, Simpson and Nandakumar. Other sequences related to distance are namely path degree sequence and distance degree sequence where Randic made an extensive study in those concepts. These sequence are used in the study and development of stratified graphs.

We determine the eccentric sequence of windmill graph and pan graph in this paper.

For details see [1-9].

2. PREREQUISITES

Assume G as a graph with V as vertex set and E as edge set where G is a finite simple graph. We now define the terminologies required.

Definition 2.1. *Consider two vertices x,y in G. The distance d(x,y) is the shortest x-y path from the 'x' to 'y'.*

Definition 2.2. *Listing of number of edges that are incident with every other vertex of the graph is defined as the degree sequence of the graph.*

Definition 2.3. *Let 'a' be a vertex. The eccentricity of vertex 'a' is the distance of a vertex that is farthest from the vertex 'a'. It is denoted by e(a).*

Definition 2.4. *The diameter of G is the largest values of all the eccentricities and smallest values of the eccentricities is the radius of G.*

Definition 2.5. *If $d(x,y) = x,\ x \neq y$, then x is the eccentric vertex of y.*

Definition 2.6. *The set of all eccentricities of the vertices of graph G is called the eccentric sequence of G. [5]*

Definition 2.7. *A self-centered graph is a graph if all the vertices of G have same eccentricity.*

Definition 2.8. *An undirected graph that is obtained by connecting 'm' copies of the complete graph k_n at a shared common vertex, is called the windmill graph $w(n,m)$. Here $m \geq 2$ and $n \geq 2$.*

Definition 2.9. *A F(l,m) fire cracker graph is constructed by adjoining lm- stars by connecting one leaf from each star. The graph $F(l,m)$ has order lm and size lm-1.*

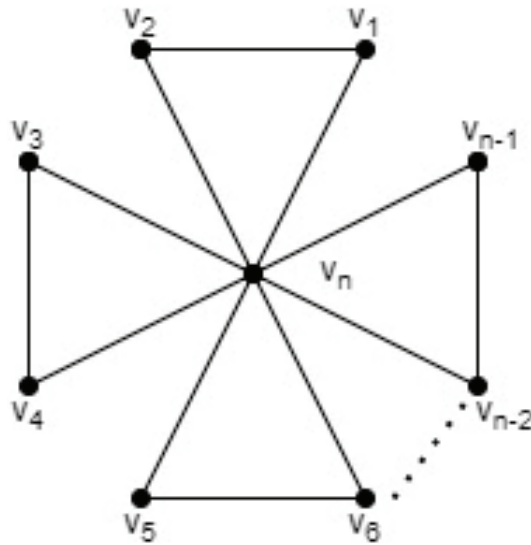FIGURE 1. $G = w_3^n$

3. MAIN RESULTS

Theorem 3.1. *The Eccentric sequence of windmill graph $W(3, n)$ is $\{2, 2, 2, 2, 2, 1\}$.*

Proof. w_3^n or $w(3, n)$ is the windmill graph by joining 'n' copies of a complete graph k_3 at a shared common vertex. We find the eccentricities of all the vertices $e(v) = 2; e(v) = 2; \ e(v) = 2; e(v) = 2; e(v) = 1$. So the eccentric sequence of w_3^2 is $\{2, 2, 2, 2, 1\}$. We find the eccentricities of all the vertices

$$e(v) = 2; e(v) = 2; \ e(v) = 2; e(v) = 2; e(v) = 2; e(v) = 2; e(v) = 1.$$

So the eccentric sequence is 2,2,2,2,2,2,1. The outer vertices ecentricities is 2 and the eccentricity of the central shared vertex is 1. Continuing this way, for w_3^n the eccentric sequence will be $\{2, 2, 2, 2, \ldots, 2, 1\}$. $\qquad\square$

Theorem 3.2. *The Eccentric Sequence of $(n, 2)$ Fire cracker graph $F(n, 2)$ is*
   (i) $((n + 1)^2, \ n^4, \ (n - 1)^4, \ (n - 2)^4, \ldots, (n - (i - 2))^4, \ (n - (i - 1)^3, \ (n - i))$
      for odd 'n'.
   (ii) $((n + 1)^2, \ n^4, \ (n - 1)^4, \ (n - 2)^4, \ldots, (n - (i - 2))^4, \ (n - (i - 1)^2, \ (n - i))$
      for even 'n'.

Proof. Let $G = F(n, 2)$ be the simple, undirected graph (i.e. Firecracker graph)
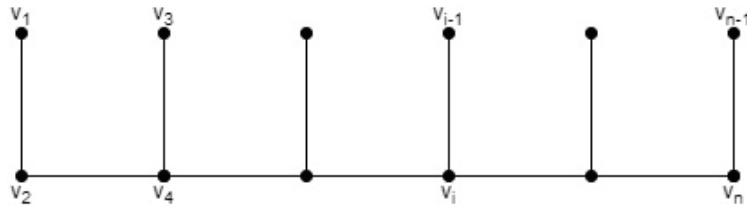
FIGURE 2. $G = F(n, 2)$

We have two cases.

For Odd 'n':

From the figure, we observe that the vertex v_1 and v_{n-1} will have the maximum eccentricity i.e $(n + 1)$. Continuing this way, there are 4 vertices with eccentricity $n, (n - 1), (n - 2) \ldots (n - (i - 2))$. Also there are 3 vertices whose eccentricity is $(n - (i - 1))$. Finally there will be a single vertex say v_k where eccentricity is (n-i).

Continuing all the eccentricities in non- decreasing order as a sequence, we obtain the sequence of odd 'n'.

For Even 'n':

Again from figure, we observe that the vertex v_1 and v_{n-1} will have the maximum eccentricity i.e $(n + 1)$. Continuing this way, there are 4 vertices with eccentricity $n, (n - 1), (n - 2) \ldots (n - (i - 1))$. Finally there will b two veticies say v_k and v_{k+1} with eccentricity is (n-i).

Continuing all the eccentricities in non- decreasing order as a sequence, we obtain the sequence of even 'n'.   □

4. CONCLUSION

Eccentric Sequence was the first distance related sequence to be introduced, Eccentric Sequence plays a vital role as it carries information on the vertex eccentricities and some structural properties of the graph such as diameter, radius and the variations in the vertex eccentricities. Also Eccentric Sequence finds its importance in stratified graphs. Many results have been determined in stratified graphs based on the eccentric sequence. The concept of self centered graphs and almost self centered graphs is showing good development where many researchers have computed various properties and results. Such studies have led

to many open problems in the concept of eccentric sequences. In future, such problems can be discussed and various results and can be determined [1].

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