Family of Ladder Graphs are Properly Lucky

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Abstract

The proper labeling is different natural number for adjacent nodes. Lucky labeling is that the total of labels over adjacent nodes within the graph is not same. Lucky labeling is linked with proper labeling. The aim of the paper is to show proper lucky labeling and proper lucky number of a ladder graph, open ladder graph, slanting ladder graph, triangular ladder graph, open triangular graph, diagonal ladder graph and open diagonal ladder graph.

Keywords: Ladder Graph, Open Ladder Graph, Slanting Ladder Graph, Triangular Ladder Graph, Open Triangular Graph, Diagonal Ladder Graph, Open Diagonal Ladder Graph.

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Introduction

Graph labeling is a broad in graph theory and more innovative results in past few decades. Most researchers have initiated several graph labelling in recent decades. The proper labeling is different natural number for adjacent nodes. Lucky labeling is that the total of labels over adjacent nodes within the graph is not same [1]. Lucky and proper labeling was linked. It is represented by $\eta_p(G)$ [2]. And labeling and sum over neighbor nodes is represented by f and s respectively.

Distance irregular, quotient, logarithmic mean, prime, geometric mean, tri magic and sum divisor labeling of ladder graph was derived by researchers. Ladder graphs applications are digital to analog conversion, electrical areas and wireless communication area such as Wi-Fi, cellular phones etc., Our aim was to compute the $\eta_p(G)$ of a ladder graph, open ladder graph, slanting ladder graph, triangular ladder graph, open triangular graph, diagonal ladder graph and open diagonal ladder graph.

Preliminaries

A. Ladder Graph

The ladder graph has vertices a_i and b_j are the two paths in the graph $V(G) = \{a_i b_j : i = j, 1 < i \le n, 1 < j \le n\}$ and the edge of are $E(G) = \{a_i a_{i+1}, b_j b_{j+1} : i = j, 1 \le i \le n, 1 \le j \le n\} \cup \{a_i b_j : i = j, 1 \le i \le n, 1 \le j \le n\}$ refer figure 1. It is denoted by L_n [3].

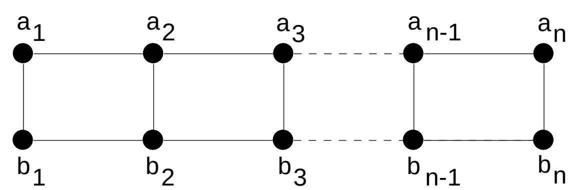

Figure 1: Ladder graph L_n

B. Open Ladder Graph

An Open ladder is generated from a ladder graph with $n > 2$ by excluding the edges $a_i b_j$, for $i = 1 \text{ and } n, j = 1 \text{ and } n$ refer figure 2. It is denoted by OL_n [3].

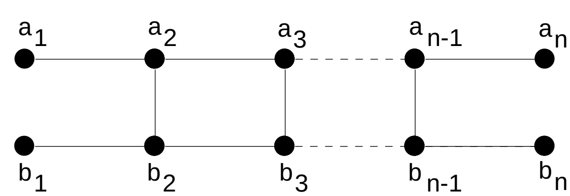

Figure 2: Open ladder graph OL_n

C. Slanting Ladder Graph

A slanting ladder is the graph obtained from two paths a_i and b_j by joining each a_i with b_{j+1}, $1 \le i \le n-1, 1 \le j \le n-1$ refer figure 3. It is denoted by SL_n [3].

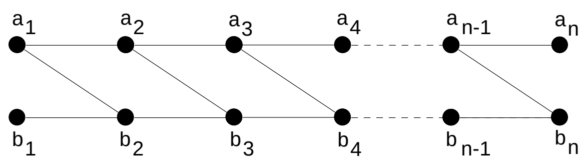

Figure 3: Slanting ladder graph SL_n

D. Triangular Ladder Graph

A triangular ladder graph is obtained from L_n with $n \geq 2$ by adding the edges $E(G) = \{a_{i+1}b_j : i = j, 1 \leq i \leq n-1, 1 \leq j \leq n-1\}$ refer figure 4. It is denoted by TL_n [3].

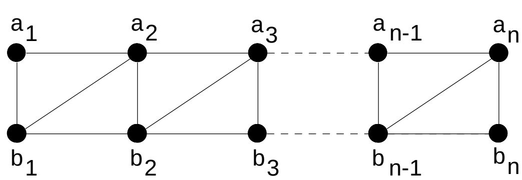

Figure 4: Triangular ladder Graph TL_n

E. Open Triangular Graph

An open Triangular ladder graph is generated from a triangular ladder graph with $n > 2$ with by removing the edges $a_i b_j$, for $i = 1 \; and \; n, j = 1 \; and \; n$ refer figure 5. It is denoted by OTL_n [3].

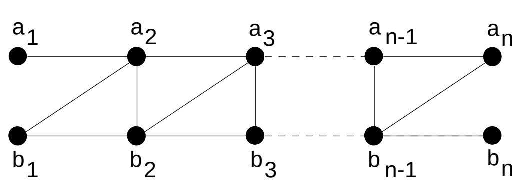

Figure 5: Open triangular ladder graph OTL_n

F. Diagonal Ladder Graph

A diagonal ladder is a graph obtained from L_n by adding the edges

$E\{G\} = \{a_i b_{j+1} : i = j, 1 \leq i \leq n-1, 1 \leq j \leq n-1,\} \cup \{a_{i+1} b_j : i = j \;\; 1 \leq i \leq n-1, 1 \leq j \leq n-1\}$

for all $n \geq 2$ refer figure 6. It is denoted by DL_n [3].

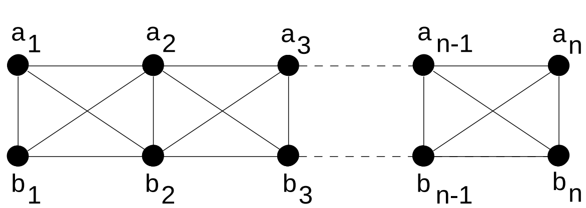

Figure 6: Diagonal ladder graph DL_n

G. Open Diagonal Ladder Graph

The An open diagonal ladder graph is generated from a diagonal ladder graph by excluding the edges $a_i b_j$, for $i = 1 \; and \; n, j = 1 \; and \; n$ refer figure 7. It is denoted by ODL_n [3].

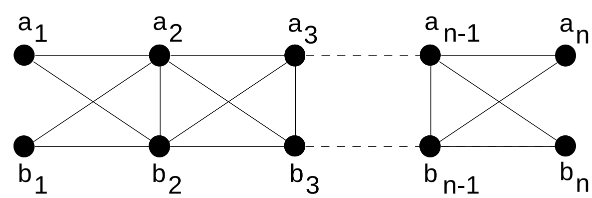

Figure 7: Open diagonal ladder graph ODL_n

Main Results

Theorem 3.1

The Ladder graph L_n for $n > 1$ is proper lucky with $\eta_p(L_n) = 2$.

Proof

Let $f: V(G) \rightarrow \{1,2\}$ for ladder graph L_n for $n > 1$ be defined by,

$$f(a_i) = \begin{cases} 1 & odd \; i \\ 2 & even \; i \end{cases}$$

$$f(b) = \begin{cases} 1 & even \; j \\ 2 & odd \; j \end{cases}$$

$$s(a_1) = 4$$

$$s(b_1) = 2$$

$$s(a_n) = \begin{cases} 2 & even \; n \\ 4 & odd \; n \end{cases}$$

$$s(b_n) = \begin{cases} 2 & odd \; n \\ 4 & even \; n \end{cases}$$

$$s(a_i) = \begin{cases} 3 & even \; i < n \\ 6 & odd \; i \; and \; 3 \leq i < n \end{cases}$$

$$s(b_j) = \begin{cases} 3 & odd \; j \; and \; 3 \leq j < n \\ 6 & even \; j < n \end{cases}$$

The minimum value of $V(G)$ is 2. Therefore it is proper lucky with $\eta_p(L_n) = 2$.

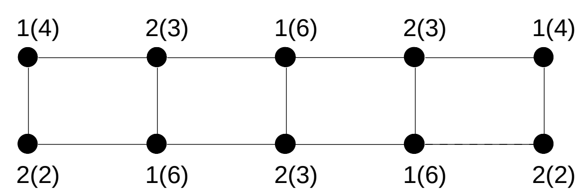

Figure 8: proper lucky ladder graph L_5

Illustration 3.2: The proper lucky labeling of ladder graph with $n = 5$ is shown in figure 8.

Theorem 3.3

The Open Ladder graph OL_n for $n > 2$ is proper lucky with $\eta_p(OL_n) = 2$.

Proof

Let $f: V(G) \to \{1,2\}$ for open ladder graph OL_n for $n > 2$ be defined by,

$$f(a_i) = \begin{cases} 1 & odd\ i \\ 2 & even\ i \end{cases}$$

$$f(b_j) = \begin{cases} 1 & even\ j \\ 2 & odd\ j \end{cases}$$

$$s(a_1) = 2$$

$$s(b_1) = 1$$

$$s(a_n) = \begin{cases} 1 & even\ n \\ 2 & odd\ n \end{cases}$$

$$s(b_n) = \begin{cases} 1 & odd\ n \\ 2 & even\ n \end{cases}$$

$$s(a_i) = \begin{cases} 3 & even\ i < n \\ 6 & odd\ i\ and\ 3 \le i < n \end{cases}$$

$$s(b_j) = \begin{cases} 3 & odd\ j\ and\ 3 \le j < n \\ 6 & even\ j < n \end{cases}$$

The minimum value of $V(G)$ is 2. Therefore it is proper lucky with $\eta_p(OL_n) = 2$.

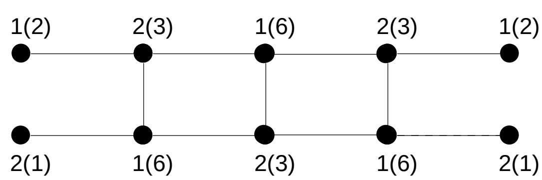

Figure 9: proper lucky open ladder graph OL_5

Illustration 3.4: The proper lucky labeling of open ladder graph with $n = 5$ is shown in figure 9.

Theorem 3.5

The Slanting Ladder graph SL_n for $n > 1$ is proper lucky with $\eta_p(SL_n) = 2$.

Proof

Let $f: V(G) \to \{1,2\}$ for slanting ladder graph SL_n for $n > 1$ be defined by

$$f(a) = \begin{cases} 1 & odd\ i \\ 2 & even\ i \end{cases}$$

$$f(b_j) = \begin{cases} 1 & odd\ j \\ 2 & even\ j \end{cases}$$

$$s(a_1) = 4$$

$$s(b_1) = 2$$

$$s(a_n) = \begin{cases} 1 & even\ n \\ 2 & odd\ n \end{cases}$$

$$s(b_n) = \begin{cases} 2 & even\ n \\ 4 & odd\ n \end{cases}$$

$$s(a_i) = \begin{cases} 3 & even\ i < n \\ 6 & odd\ i\ and\ 3 \le i < n \end{cases}$$

$$s(b_j) = \begin{cases} 3 & even\ j < n \\ 6 & odd\ j\ and\ 3 \le j < n \end{cases}$$

The minimum value of $V(G)$ is 2. Therefore it is proper lucky with $\eta_p(SL_n) = 2$.

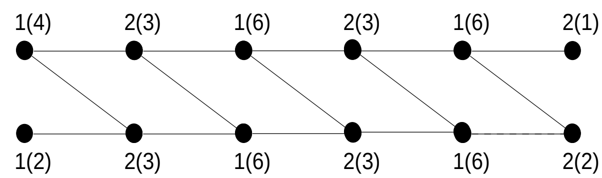

Figure 10: Proper lucky slanting ladder graph SL_6

Illustration 3.6: The proper lucky labeling of slanting ladder graph with $n = 6$ is shown in figure 10.

Theorem 3.7

The Triangular Ladder graph TL_n for $n > 1$ is proper lucky with $\eta_p(TL_n) = 3$.

Proof

Let $f: V(G) \to \{1,2,3\}$ for triangular ladder graph TL_n for $n > 1$ be defined by

$$f(a_i) = \begin{cases} 1 & i = 3k - 1 \\ 2 & i = 3k - 2 \\ 3 & i = 3k \end{cases}$$

$$f(b_j) = \begin{cases} 1 & j = 3k \\ 2 & j = 3k - 1 \\ 3 & j = 3k - 2 \end{cases}$$

$$s(a_1) = 4$$

$$s(b_1) = 5$$

$$s(a_n) = \begin{cases} 4 & n = 3k \\ 7 & n \ne 3k \end{cases}$$

$$s(b_n) = \begin{cases} 3 & n = 3k-2 \\ 4 & n = 3k-1 \\ 5 & n = 3k \end{cases}$$

$$s(a_i) = \begin{cases} 6 & i = 3k \text{ and } i < n \\ 8 & i = 3k-2 \text{ and } 4 \le i < n \\ 10 & i = 3k-1 \text{ and } i < n \end{cases}$$

$$s(b_j) = \begin{cases} 6 & j = 3k-2 \text{ and } 4 \le j < n \\ 8 & j = 3k-1 \text{ and } j < n \\ 10 & j = 3k \text{ and } j < n \end{cases}$$

The minimum value of $V(G)$ is 3. Therefore it is proper lucky with $\eta_p(TL_n) = 3$.

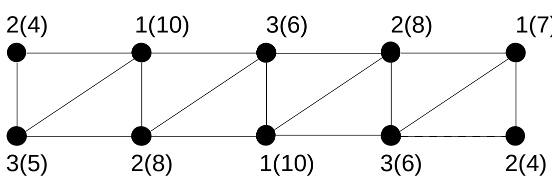

Figure 11: Proper lucky triangular ladder graph TL_5

Illustration 3.8: The Proper lucky labeling of triangular ladder graph with $n = 5$ is shown in figure 11.

Theorem 3.9

The Open Triangular Ladder graph OTL_n for $n > 2$ is proper lucky with $\eta_p(OTL_n) = 3$.

Proof

Let $f: V(G) \to \{1,2,3\}$ for open ladder triangular graph OTL_n for $n > 2$ be defined by

$$f(a_i) = \begin{cases} 1 & i = 3k-2 \\ 2 & i = 3k-1 \\ 3 & i = 3k \end{cases}$$

$$f(b_j) = \begin{cases} 1 & j = 3k-1 \\ 2 & j = 3k \\ 3 & j = 3k-2 \end{cases}$$

$$s(a_1) = 2$$

$$s(b_1) = 3$$

$$s(a_n) = \begin{cases} 3 & n = 3k \\ 4 & n = 3k+2 \\ 5 & n = 3k+1 \end{cases}$$

$$s(b_n) = \begin{cases} 1 & n = 3k \\ 2 & n = 3k+1 \\ 3 & n = 3k+2 \end{cases}$$

$$s(a_i) = \begin{cases} 6 & i = 3k \text{ and } i < n \\ 8 & i = 3k-1 \text{ and } i < n \\ 10 & i = 3k-2 \text{ and } 4 \le i < n \end{cases}$$

$$s(b_j) = \begin{cases} 6 & j = 3k-2 \text{ and } 4 \le j < n \\ 8 & j = 3k \text{ and } j < n \\ 10 & j = 3k-1 \text{ and } j < n \end{cases}$$

The minimum value of $V(G)$ is 3. Therefore it is proper lucky with $\eta_p(OTL_n) = 3$.

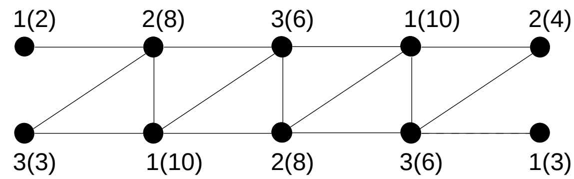

Figure 12: Proper lucky open triangular graph OTL_5

Illustration 3.10: The proper lucky labeling of open triangular ladder graph with $n = 5$ is shown in figure 12.

Theorem 3.11

The Diagonal graph DL_n for $n > 1$ is proper lucky with $\eta_p(DL_n) = 4$.

Proof

Let $f: V(G) \to \{1,2,3,4\}$ for diagonal ladder graph DL_n for $n > 1$ be defined by

$$f(a_i) = \begin{cases} 1 & \text{odd } i \\ 2 & \text{even } i \end{cases}$$

$$f(b_j) = \begin{cases} 3 & \text{odd } j \\ 4 & \text{even } j \end{cases}$$

$$s(a_1) = 9$$

$$s(b_1) = 7$$

$$s(a_n) = \begin{cases} 8 & \text{even } n \\ 9 & \text{odd } n \end{cases}$$

$$s(b_n) = \begin{cases} 6 & \text{even } n \\ 7 & \text{odd } n \end{cases}$$

$$s(a_i) = \begin{cases} 12 & \text{even } i \text{ and } 2 \le i < n \\ 15 & \text{odd } i \text{ and } 3 \le i < n \end{cases}$$

$$s(b_j) = \begin{cases} 10 & \text{even } j \text{ and } 2 \le j < n \\ 13 & \text{odd } j \text{ and } 3 \le j < n \end{cases}$$

The minimum value of $V(G)$ is 3. Therefore it is proper lucky with $\eta_p(DL_n) = 4$.

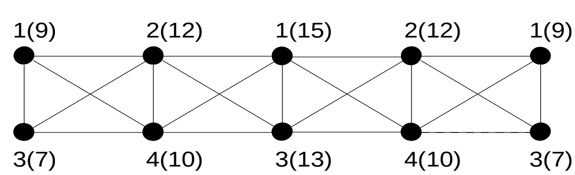

Figure 13: Proper lucky diagonal ladder graph DL_5

Illustration 3.12: The proper lucky labeling of diagonal ladder graph with $n = 5$ is shown in figure 13.

Theorem 3.13

The Open Diagonal graph ODL_n for $n > 2$ is proper lucky with $\eta_p(ODL_n) = 4$.

Proof

Let $f: V(G) \to \{1,2,3,4\}$ for open diagonal ladder graph ODL_n for $n > 2$ be defined by

$$f(a_i) = \begin{cases} 1 & even\ i \\ 2 & odd\ i \end{cases}$$

$$f(b_j) = \begin{cases} 1 & j = 1 \\ 3 & even\ j \\ 4 & odd\ j\ and\ j > 1 \end{cases}$$

$s(a_1) = 5$

$s(b_1) = 5$

$s(a_2) = 10$

$s(b_2) = 9$

$s(a_n) = 5$

$s(b_n) = 5$

$$s(a_i) = \begin{cases} 13 & odd\ i\ and\ 3 \le i < n \\ 14 & even\ i\ and\ 4 \le i < n \end{cases}$$

$$s(b_j) = \begin{cases} 11 & odd\ j\ and\ 3 \le j < n \\ 12 & even\ i\ and\ 4 \le j < n \end{cases}$$

The minimum value of $V(G)$ is 3. Therefore it is proper lucky with $\eta_p(ODL_n) = 4$.

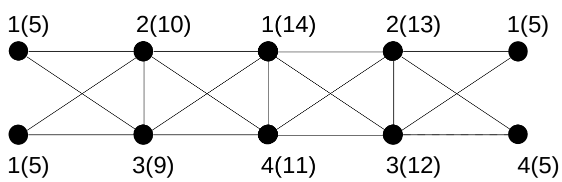

Figure 14: Proper lucky open diagonal ladder graph ODL_5

Illustration 3,14: The proper lucky labeling of open diagonal ladder graph with $n = 5$ is shown in figure 14.

Conclusion

In this article, we found proper lucky labeling for ladder graph, open ladder graph, slanting ladder graph, triangular ladder graph, open triangular ladder graph, diagonal ladder graph and open diagonal ladder. We obtained $\eta_p(L_n) = \eta_p(OL_n) = \eta_p(SL_n) = 2$, $\eta_p(TL_n) = \eta_p(OTL_n) = 3$ and $\eta_p(DL_n) = \eta_p(ODL_n) = 4$ Also we observed that the proper lucky labeling of ladder graphs is one less than the maximum degree of those ladder graphs i.e., $\eta_p(G) = \Delta(G) - 1,$ where $G \in L_n, OL_n, SL_n, TL_n, OTL_n, DL_n, ODL_n$

Further, we are do this to various graphs like triangular family.

References

Kumar TVS and S. Meenakshi. (2021). "Lucky and Proper Lucky Labeling of Quadrilateral Snake Graphs." IOP Conference Series: Materials Science and Engineering. 1085. doi: 10.1088/1757-899X/1085/1/012039.

Kumar, TV Sateesh, and S. Meenakshi. "Proper Lucky Labeling of Graph." Proceedings of First International Conference on Mathematical Modeling and Computational Science: ICMMCS 2020. Springer Nature.

Sumathi, P. & Rathi, A. (2018). Quotient Labelling of Some Ladder Graph. American J. of Engineering Research (AJER), 7(12), 38-42.