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Row Column Reduction Method for Solving Solid Transportation Problem

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Abstract: The solid transportation problem (STP) is an advancement of the traditional transportation model that integrates several kinds of conveyance, rendering it applicable to contemporary logistics systems. To mitigate uncertainty in costs, supply, and capacity, we offer an innovative method utilizing the Row Column Reduction Method (RCRM) within a fuzzy framework. The problem parameters are expressed as triangular fuzzy numbers (TFN), and the approach entails resolving upper and lower bound issues in conjunction with the MODI method for optimization. This method diminishes computing complexity and adeptly addresses imprecision providing a systematic solution framework for fuzzy solid transportation issues.

Keywords: Fuzzy transportation, Triangular fuzzy number, Row-column reduction, Solid transportation, MODI method

1. Introduction

The classical transportation is a foundational model in operations research that focusses on the optimal distribution of commodities from several sources to different destinations to minimize transportation expenses [1]. In practical supply chains, decision-makers frequently encounter added complexity due to the involvement of many transportation modes, including road, rail, air, and maritime. This STP, three-dimensional extension of the traditional transportation problem (TP) that integrates conveyance as an additional constraint alongside supply and demand [2,3].

In numerous actual situations transportation characteristics like cost, capacity, supply, and demand are not accurately known due to variable economic conditions, seasonal fluctuations, or subjective assessments. The uncertainties render deterministic models insufficient. Fuzzy set theory, explored by Zadeh [4] in 1965, offers a comprehensive framework for representing imprecision. Triangular fuzzy numbers (TFNs) are frequently employed because of their simplicity and efficacy in depicting unknown values.

Numerous academicians have adapted classical and robust transportation models to accommodate fuzzy situations. Haley [5] established the groundwork for STP, whereas Sobana and Anuradha [6], together with Pandian and Anuradha [7], introduced heuristic and zero-point methodologies, respectively. To address fuzzy variants of STP, recent studies have shown diverse fuzzy ranking methodologies and metaheuristic algorithms to tackle imprecision [8,9]. Nonetheless, numerous solutions either entail intricate calculations or lack a systematic framework for equilibrating fuzzy parameters [10].

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To mitigate these restrictions, we present an innovative solution approach utilising the Row Column Reduction Method (RCRM) within a fuzzy framework. This method transforms triangular fuzzy parameters into upper and lower bound issues, implements systematic row and column reductions, and employs the Modified Distribution Method (MODI) to optimize allocation. The strategy streamlines calculations, enhances transparency in allocation processes, and yields reliable outcomes despite uncertainty.

2. Preliminaries

2.1 Fuzzy Number

A "fuzzy number $\tilde{A}$ is a fuzzy set on the real line R must satisfy the following conditions.

(i) $\tilde{A}$ must be normal and convex

(ii) There exist at least one $x_0 \in R$ with $f_{\tilde{A}}(x_0)=1$

(iii) $f_{\tilde{A}}(x_0)$ is piecewise continuous

2.2 Triangular fuzzy number (TFN)

Three points are used to represent fuzzy numbers in the following way: A=(a_1, a_2, a_3). The membership function is used to interpret this representation.

$$\mu_A(x) = \begin{cases} 0 & x < a_1, x > a_3 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \le x \le a_3 \end{cases}$$

2.3 Operation of Triangular Fuzzy Number

Let $\widetilde{A_1} = (a_1, a_2, a_3)$ and $\widetilde{A_2} = (b_1, b_2, b_3)$ be two non-negative triangular fuzzy number then

(i) $\widetilde{A_1} - \widetilde{A_2} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

(ii) $\widetilde{A_1} \oplus \widetilde{A_2} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

2.4 Mathematical Formation of Solid Transportation" Problem

Let:

- O_i : origin i for $i = 1, 2, \ldots, m$
- D_j: destination j for $j = 1, 2, \ldots, n$
- E_k : conveyance mode k for $k = 1, 2, \ldots, l$
- C_{ijk}: transportation cost from O_i to D_j
- x_{ijk} : units transported from O_i to D_j
- a_i : supply at origin O_i
- b_j : demand at destination D_j
- e_k : capacity of conveyance E_k

The mathematical model of Fuzzy solid transportation problem is

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} C_{ijk} x_{ijk}$$

Subject to the constraints:

- Supply constraint

$$\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} = a_i i = 1, 2, \ldots..m$$

- Demand constraint

$$\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} = b_j j = 1, 2, \ldots..n$$

- Conveyance capacity constraint:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = e_k k = 1, 2, \ldots..l$$

- Non-negativity:

$$x_{ijk} \ge 0 \text{ for all } i,j,k$$

If fuzzy parameters are considered, then:

- a_i, b_j, e_k, C_{ijk} become triangular fuzzy numbers $\tilde{a}_i, \tilde{b}_j, \tilde{e}_k, \tilde{C}_{ijk}$

3. Proposed Algorithm

Step 1: Construct the Upper bound problem by choosing maximum value from each entry (cost/Supply/Demand/ Capacity of conveyance). (Fig. 70.1).

Step 2: Check the problem is balanced. If not convert it into balanced problem.

Step 3: The cost matrix is reduced by using Row reduction formula,

$$R_{ijk} = C_{ijk} - min_{j=1 to 3\, k=1 to 3} \{C_{ijk}\}, i = 1 \text{ to } 3$$

Step 4: Again, the cost matrix is reduced by using Column reduction formula,

$$CO_{ijk} = C_{ijk} - B_j \quad = 1 \text{ to } 3$$

where, $B_j = \{C_{ijk}\}$, minimum value is extracted from each of the destinations D_j, $j = 1$ to 3.

Step 5: Choose the row / column, according to maximum value is present in supply /demand.

Step 6: Select the cell having the minimum value from the selected row/column

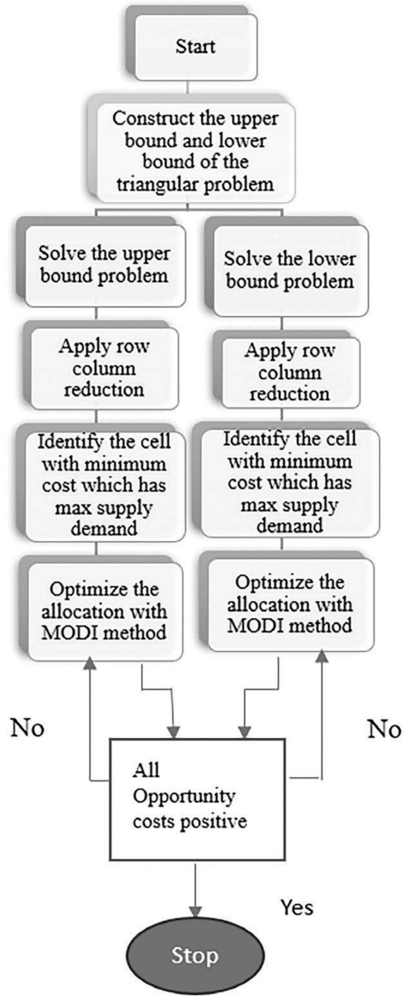

Fig. 70.1 Flowchart of the proposed Algorithm

Preference given to the cell with both $R_{ijk} = 0, CO_{ijk} = 0$. Otherwise, choose the Cost cell using the formula

$$\text{Cost cell} = min\left\{R_{ijk} + CO_{ijk}\right\}$$

Step 7: The minimum value of (supply, demand, capacity of conveyance) is allocated to the selected cell. It is subtracted from supply, demand and capacity then mark the remaining value. After allocation, any one the term (supply, demand, capacity) will be zero. Remove the corresponding row /column /conveyance value.

Step 8: Steps 5 through 7 should be repeated until the conveyance's capacity and supply demand are zero.

Step 9: Find the optimal solution using MODI method.

Step 10: Repeat the steps from 2 to 9 to get the optimal solution for Lower bound problem.

4. Formulation of the Problem

In light of increasing environmental concerns and carbon emission laws, a major distribution company is enhancing its supply chain through the implementation of green logistics strategies. The company manages three eco-certified warehouses O_1, O_2, O_3, that provide items to three urban distribution centres D_1, D_2, D_3. Goods are conveyed through three eco-friendly transportation methods electric trucks(E_1), railway cargo(E_2), hybrid-powered air freight (E_3).Transportation costs, supply constraints, demand fluctuations, and conveyance capacity are represented as triangular fuzzy numbers due to changeable electricity prices, fluctuating fuel surcharges, and inconsistent vehicle availability. The purpose is to create a sustainable and cost-effective logistics strategy that meets all demand while minimising total transportation costs under uncertainty by addressing a fuzzy solid transportation problem (FSTP) (Tables 70.1–70.7).

The capacity of E_1, E_2, E_3 are (9,11,13), (11,13,15), (10,14,17)

Solution:

In the above problem,

Total supply = Total demand = Total capacity of conveyance

Therefore, the TP is balanced.

The Upper bound problem of triangular FSTP is given by

Table 70.1 Numerical example 1

Origin	Destinations									
	D_1			D_2			D_3			
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	SUPPLY
O_1	(2,5,7)	(1,2,4)	(5,8,11)	(8,9,12)	(3,7,11)	(1,2,3)	(3,4,6)	(6,7,9)	(5,6,7)	(10,13,16)
O_2	(8,10,11)	(3,7,9)	(2,3,4)	(2,3,5)	(9,12,15)	(5,6,8)	(4,6,8)	(2,3,5)	(9,10,12)	(12,14,16)
O_3	(3,5,7)	(7,9,10)	(4,6,8)	(4,7,9)	(2,3,5)	(8,10,12)	(7,9,11)	(3,5,6)	(2,3,4)	(8,11,13)
Demand	(9,11,13)			(11,13,14)			(10,14,18)			

Table 70.2 Upper bound TFN

	D_1			D_2			D_3			
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	**Supply**
O_1	7	4	11	12	11	3	6	9	7	16
O_2	11	9	4	5	15	8	8	5	12	16
O_3	7	10	8	9	5	12	11	6	4	13
DEMAND	13			14			18			

where the capacity of E_1, E_2, E_3 are 13,15,17

Applying steps 3 and 4 we get the below table

Table 70.3 Row column reduction method

	D_1			D_2			D_3			
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	**SUPPLY**
O_1	7_3^4	4_0^1	11_7^8	12_9^9	11_8^8	3_0^0	6_2^3	9_5^6	7_3^4	16
O_2	11_7^7	9_5^5	4_0^0	5_2^1	15_{12}^{11}	8_5^4	8_4^4	5_1^1	12_8^8	16
O_3	7_3^3	10_6^6	8_4^4	9_6^5	5_2^1	12_9^8	11_7^7	6_2^2	4_0^0	13
DEMAND	13			14			18			

Applying steps 5 to 9 we get the final allocation as given below in the table

Table 70.4 Allocation table

	D_1			D_2			D_3			
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	**SUPPLY**
O_1	$7_3^{4[2]}$	$4_0^{1[10]}$	11_7^8	12_9^9	11_8^8	$3_0^{0[4]}$	6_2^3	9_5^6	7_3^4	16
O_2	$11_7^{7[1]}$	9_5^5	4_0^0	$5_2^{1[10]}$	15_{12}^{11}	8_5^4	8_4^4	$5_1^{1[5]}$	12_8^8	16
O_3	7_3^3	10_6^6	8_4^4	9_6^5	5_2^1	12_9^8	11_7^7	6_2^2	$4_0^{0[13]}$	13
Demand	13			14			18			

The above-mentioned problem satisfies all the constraints including supply, demand and conveyance

Therefore, the solution is a non-degenerate basic feasible solution.

The optimal transportation cost as obtained from the allocation table is given by Rs.204

The Lower bound problem of triangular FSTP is given by

Table 70.5 Lower bound triangular fuzzy number

	D_1			D_2			D_3			
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	**Supply**
O_1	2	1	5	8	3	1	3	6	5	10
O_2	8	3	2	2	9	5	4	2	9	12
O_3	3	7	4	4	2	8	7	3	2	8
Demand	9			11			10			

where the capacity of E_1, E_2, E_3 are 9,11,10

Table 70.6 Row column reduction method

	D_1			D_2			D_3			
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	**SUPPLY**
O_1	2_1^1	1_0^0	5_4^4	8_7^7	3_2^2	1_0^0	3_1^2	6_4^5	5_3^4	10
O_2	8_7^6	3_2^1	2_1^0	2_1^0	9_8^7	5_4^3	4_2^2	2_0^0	9_7^7	12
O_3	3_2^1	7_6^5	4_3^2	4_3^2	2_1^0	8_7^6	7_5^5	3_1^1	2_0^0	13
DEMAND	9			11			10			

Utilizing step 5 to 9 and we obtain the allocation table

Table 70.7 Allocation table

	D_1			D_2			D_3			
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	**SUPPLY**
O_1	2_1^1	$1_0^{0[\varepsilon]}$	5_4^4	8_7^7	3_2^2	$1_0^{0[10]}$	3_1^2	6_4^5	5_3^4	10
O_2	8_7^6	$3_2^{1[1]}$	2_1^0	$2_1^{0[1]}$	9_8^7	5_4^3	4_2^2	$2_0^{0[10]}$	9_7^7	12
O_3	$3_2^{1[8]}$	7_6^5	4_3^2	4_3^2	2_1^0	8_7^6	7_5^5	3_1^1	$2_0^{0[\varepsilon]}$	13
DEMAND	9			11			10			

So, the final Table 70.7 assures all the constraints comprising supply, demand and conveyance

Then the solution is non degenerate basic feasible solution.

The Optimal Transportation cost = Rs. 59

According to proposed method the problem with three supplies, demand and conveyance transportation cost lies between Rs.59 and Rs.204.

4.1 Proposed Method

Table 70.8 below illustrates the selection of optimal conveyance from various origins to destinations from the allocations table of upper and lower bound.

Table 70.8 Conveyance for upper and lower bound in proposed method

Problem	Origin	Destination	Conveyance
Upper bound problem	O_3	D_3	E_2, E_3
	O_2	D_2	E_1, E_3
	O_1	D_1	E_1, E_2
Lower bound problem	O_3	D_3	E_2, E_3
	O_2	D_2	E_1, E_3
	O_1	D_1	E_1, E_2

4.2 Vogel's Method

Similarly to the proposed strategy, Vogel's method also achieves optimal transportation from several starting points to destinations (Table 70.9).

Table 70.9 Conveyance for upper and lower bound in Vogel's method

Problem	Origin	Destination	Conveyance
Upper bound problem	O_3	D_3	E_1, E_2, E_3
	O_2	D_2	E_1, E_3
	O_1	D_1	E_1
Lower bound problem	O_3	D_3	E_2, E_3
	O_2	D_2	E_1, E_3
	O_1	D_1	$(2n+$

4.3 Result

The comparison demonstrates that the proposed method maintains consistent conveyance options between origins and destinations for both upper and lower bound problems, suggesting a stable and uniform transport strategy. In contrast, Vogel's method demonstrates a more cost-sensitive and adaptive approach, as evidenced by the variation in conveyance choices between the two scenarios. This emphasis that the proposed method guarantees consistency, whereas Vogel's method optimizes efficiency by adjusting conveyance based on constraints.

5. Comparison Table

Below is the comparison table for numerical example 1 (Table 70.10 & Fig. 70.2).

Table 70.10 Comparison of Vogel's and MODI method with proposed method

S. no	Problem	Transportation cost	
		Vogel's and MODI method	Proposed Method
1	Upper bound cost	Rs.205	Rs.204
2	Lower bound cost	Rs.59	Rs.59

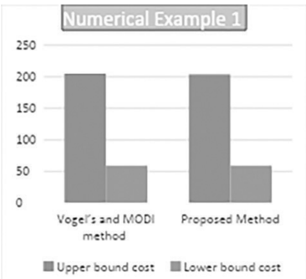

Fig. 70.2 Comparison chart of numerical example

6. Conclusion

The ambiguous solid transportation problem is effectively resolved by the proposed Row Column Reduction Method, which establishes a precise range defined by optimal upper and lower bounds within which the transportation cost is contained. This method consistently produces a more cost-effective and narrower range than Vogel's method, as evidenced by numerical example. Additionally, the propose method guarantees uniform conveyance selections across both bounds, thereby minimising allocation variability and improving stability. This renders the procedure computationally efficient, logically consistent, and practically appropriate for uncertain logistics environments.

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Note: All the tables and the figures in this chapter were made by the authors.