

Profit Maximization of Balanced Fuzzy Transportation Problem Using Ranking Method

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ABSTRACT

The aim of this paper is to find out the maximum profit cost of some commodities through a capacited network, when the supply and demand of nodes and the capacity and cost of nodes are represented as triangular fuzzy numbers. Using Yager's Ranking Method fuzzy quantities are transformed in to crisp quantities. Finally a numerical illustration is given to check the validity of the proposal.

Keywords: fuzzy balanced transportation problem, triangular fuzzy number (TFNs), Yager's ranking method, profit maximization, Vogel's Approximation Method (VAM).

1. INTRODUCTION

Transportation models provide a powerful framework to meet the challenge of how to supply the commodities to the customers in more efficient ways. They ensure the efficient movement and timely availability of raw materials and finished goods. In 1941, Hitchcock⁴ originally developed the basic transportation problem. In 1953, Charnes *et al.*³ developed the stepping stone method which provided an alternative way determining the simplex method information. In 1963, Dantzig² used the simplex method to the transportation problems as the primal simplex transportation method. Till-date, most of the researchers studied extensively to get cost minimizing transportation problem in different ways. In real world application, all the parameters of the transportation problems may not be known precisely due to uncontrollable

factors. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy numbers introduced by Zadeh in 1965. Zimmermann⁷ showed that solutions obtained by fuzzy linear programming are always efficient.

A fuzzy transportation problem is transportation problem in which the transportation cost supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. In this paper the objective is to maximize the total profit, subject to some fuzzy constraints, the objective function is also considered as a fuzzy number.

This paper consist as follows: Section 2 briefly discussed basic definition of fuzzy number. In Section 3, briefly explained a new proposed algorithm. In Section 4, the proposed method was illustrated by numerical examples with triangular fuzzy number. In Section 5, we provide conclusion of this article.

2. PRELIMINARIES

In this section we define some basic definitions which will be used in this paper.

2.1 Definition (Fuzzy Set)

A fuzzy set is characterized by a membership function mapping objects of a domain, space or universe of discourse X to the unit interval $[0, 1]$. (ie) $\hat{A} = \{ x, \mu_A(x); x \in X \}$, here $\mu_A : X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set \hat{A} and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set. These membership grades are often represented by real numbers ranging from $[0, 1]$.

2.2 Definition (Triangular Fuzzy Number)

For a triangular fuzzy number $A(x)$, it can be represented by $\hat{A} = (a,b,c;1)$ with membership function $\mu_A(x)$ given by

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

α -cut

The α -cut of a fuzzy number $A(x)$ is defined as $A(x) = \{x / \mu(x) \geq \alpha, \alpha \in [0,1]\}$

2.3 Definition: (Defuzzification)

Defuzzification is the process of finding singleton value (crisp value) which represents the average value of the TFNs. Here use Yager's ranking to defuzzify the TFNs because of its simplicity and accuracy.

2.4 Definition: (Yager's Ranking Technique)

Yager's ranking technique⁶ which satisfies compensation, linearity, additivity properties and provides results which consists of human intuition. $Y(R)$ represents the set of all TFNs. It R be any ranking function, then,

$$Y(R) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U) d\alpha,$$

Where $(a_\alpha^L, a_\alpha^U) = \{(b-a)\alpha + a, c - (c-b)\alpha\}$.

2.5 Definition: (Fuzzy Balanced Transportation Problem)

The balanced fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as LPP as follows

$$\text{Minimize } \sum_{i=1}^p \sum_{j=1}^q c_{ij} * x_{ij}$$

Subject to

$$\sum_{j=1}^q x_{ij} = a_i, \quad i = 1, 2, \dots, p, \quad \sum_{i=1}^p x_{ij} = b_j, \quad j = 1, 2, \dots, q$$

$$\sum_{i=1}^p a_i = \sum_{j=1}^q b_j$$

Here x_{ij} is a non negative trapezoidal fuzzy number, where

p = total number of sources

q = total number of destinations

a_i = the fuzzy availability of the product at i^{th} source

b_j = the fuzzy demand of the product at j^{th} destination

c_{ij} = the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination

x_{ij} = the fuzzy quantity if the product that should be transported from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost, $\sum_{i=1}^p a_i$ = total fuzzy availability of the product,

$\sum_{j=1}^q b_j$ = total fuzzy demand of the product,

$\sum_{i=1}^p \sum_{j=1}^q c_{ij} * x_{ij}$ = total fuzzy transportation cost.

In L.P.P minimize (Z) = - maximize ($-Z$), i.e., maximize the product is equal to minimize the cost. If $\sum_{i=1}^p a_i = \sum_{j=1}^q b_j$, then the fuzzy transportation problem is said to be balance fuzzy transportation problem. Consider transportation with m fuzzy origins (rows) and n fuzzy destination (columns). Let $c_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}]$ be the cost of transporting one unit of the product from i^{th} fuzzy origin to j^{th} fuzzy destination, $a_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}]$ be the quality of commodity available at fuzzy origin i and $b_j = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}]$ be the quantity of commodity requirement at fuzzy destination j , $X_{ij} = [X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}]$ is the quantity transported from i^{th} fuzzy origin to j^{th} fuzzy destination.

3. ALGORITHM FOR VOGEL'S APPROXIMATION METHOD

Step 1: Determine the crisp value of each cell of the given intuitionistic fuzzy transportation problem by using Yager's Ranking.

Step 2: Balance the given intuitionistic transportation problem if either (total supply > total demand) or (total supply < total demand).

Step 3: Convert the given maximization problem into a minimization problem by multiplying the cost elements by -1.

Step 4: Determine the penalty cost for each row (column) by subtracting the lowest cell cost in the row (column) from the next lowest cell cost in the same row (column).

Step 5: Identify the row or column with highest penalty cost. If tie happen in any place choose arbitrarily (if there are any).

Step 6: Allocate the least transportation cost to the feasible cell in the row or column with the highest penalty cost.

Step 7: Repeat 2, 3 & 4 until all necessities have been made.

Step 8: Calculate total transportation cost for the feasible allocations.

4. NUMERICAL EXAMPLE

Consider the fuzzy transportation problem for maximizing the profit. A firm owns facilities at seven places. It has manufacturing plants at places A, B and C with daily production of (13, 23, 33), (34, 44, 54), (23, 33, 43) units respectively. At point D, E, F and G it has four warehouses with daily demands of (13, 23, 33), (21, 31, 41), (6, 16, 26) and (20, 30, 40) units respectively. Per unit shipping cost are given in the following table. Find the maximum profit of the firm?

Since the given problem is a maximization type, first convert this into a minimization problem by multiplying the cost elements by -1. Since $\sum_{i=1}^p a_i = \sum_{j=1}^q b_j = 100$ there exist a basic feasible solution to this problem and is displayed in the following table by using VAM.

Table 1: Triangular Fuzzy Transportation Table

	D	E	F	G	Fuzzy Available
A	(10,15,20)	(41,51,61)	(32,42,52)	(23,33,43)	(13,23,33)
B	(70,80,90)	(32,42,52)	(16,26,36)	(71,81,91)	(34,44,54)
C	(80,90,100)	(30,40,50)	(56,66,76)	(50,60,70)	(23,33,43)
Fuzzy Requirement	(13,23,33)	(21,31,41)	(6,16,26)	(20,30,40)	

Table 2: Defuzzified transportation Table

	D	E	F	G	Fuzzy Available
A	15	51	42	33	23
B	80	42	26	81	44
C	90	40	66	60	33
Fuzzy Requirement	23	31	16	30	100

Table 3: Minimization Transportation table

	D	E	F	G	Fuzzy Available
A	-15	-51	-42	-33	23
B	-80	-42	-26	-81	44
C	-90	-40	-66	-60	33
Fuzzy Requirement	23	31	16	30	

Table 4: Using Vogel's Approximation Method (VAM)

	D	E	F	G	Fuzzy Available
A	-15	-51(23)	-42	-33	23
B	-80(6)	-42(8)	-26	-81(30)	44
C	-90(17)	-40	-66(16)	-60	33
Fuzzy Requirement	23	31	16	30	

Hence, Maximum profit cost = $(-51 \times 23) + (-80 \times 6) + (-42 \times 8) + (-81 \times 30) + (-90 \times 17) + (-66 \times 16) = 7005$.

5. CONCLUSION

In this paper, the transportation costs are considered as imprecise numbers by fuzzy number which are more realistic and general in nature. More over fuzzy transportation problem of triangular numbers has been transformed into crisp transportation problem using Yager's ranking indices. In future we can extend this idea in Trepezoidal, Pentagonal Fuzzy numbers and etc. Numerical examples are solved to illustrate the fuzzy transportation problem with triangular fuzzy number.

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