Chapter 4

Bivariate Neutrosophic Fuzzy Solid Transportation Problem

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Abstract

Many real-world situations have an element of uncertainty, which is often addressed using fuzzy techniques. This study describes various transportation issues like transportation cost, transportation time, and conveyance capacity. The bivariate neutrosophic fuzzy solid transportation problem is an extension of the neutrosophic solid transportation problem. The bivariate neutrosophic solid fuzzy solid transportation problem consists of three constraints and two objective functions. Two objective functions Cost and time are combined by a single term (order pair), "bivariate neutrosophic fuzzy number," and conveyance capacity, supply and demand are crisp numbers. A bivariate neutrosophic fuzzy number consists of two parts (cost, time) and neutrosophic membership. Optimum allocation is made using the average of neutrosophic confidence, and optimum cost is calculated using the weighted cost-time score function, and it is solved using the row column reduction method, and it is compared with the standard method.

Keywords: Neutrosophic set, Bivariate neutrosophic set, Bivariate neutrosophic solid transportation problem.

ISBN 978-816855385-9

1. Introduction

The foundational work on the transportation problem was pioneered by Hitchcock (1941). In industrial and logistical contexts, transportation remains a critical challenge. This problem is framed as a specialized subset of linear programming, where the core objective is to minimize the cost of shipping goods. The classical transportation model serves as a decision-making tool, guiding the companies on the optimal quantity of a commodity to dispatch from various sources to various destinations by using available logistics services.

Nowadays various logistics services are available with various capacities. Uncertainty exists in selecting the perfect logistics service. This new conveyance constraint included in the classical transportation problem converted it into a solid transportation problem. Neutrosophic set, proposed by Smarandache in 1999, It is an extension of fuzzy set by incorporating three degrees of membership: truth (T), indeterminacy (I), and falsity (F). It is very useful in multi-criteria decision-making problems. The bivariate neutrosophic fuzzy solid transportation problem includes two criteria, cost and time, which are both very important for buyers. Cost and time play important roles in transportation problems. Optimum cost and time lead to customer satisfaction and business efficiency.

1.1 Research gap

Standard solution techniques for the STP may produce results that are infeasible with respect to the third-dimension constraints (conveyance). At this situation, the Row Column Reduction and MODI algorithms are applied to refine these solutions and find the true optimal, feasible answer for the entire three-dimensional problem.

2. Preliminaries

2.1 Fuzzy sets

Let X be a universe of discourse. A fuzzy set $\tilde{A}$ of X is defined by a membership function $f_{\tilde{A}}: X \to [0,1]$ where $f_{\tilde{A}}(x)$ is called the membership function and is represented as

$\tilde{A} = \{(x, f_{\tilde{A}}(x) / \, x \epsilon X\}$

2.2 Normal

A fuzzy set $\tilde{A}$ defined on the universe X is said to be normal iff $Sup f_{\tilde{A}}(x) = 1, x \in X$

2.3 Convex

A fuzzy set $\tilde{A}$ defined on the universe set X is said to be convex iff $f_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq min(f_{\tilde{A}}(x), f_{\tilde{A}}(y), \forall x, y \in X \; and \; \lambda \epsilon [0,1]$

2.4 Fuzzy number

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R must satisfy the following conditions.

(i) $f_{\tilde{A}}(x)$ is piecewise continuous

(ii) There exist at least one $x \epsilon R$ with $f_{\tilde{A}}(x) = 1$

(iii) $\tilde{A}$ must be normal and convex

2.5 Triangular fuzzy number

A fuzzy number $\tilde{A} = (a, b, c)$ is called triangular fuzzy number if its membership function is given by $\mu_{\tilde{A}}(x) = \left\{ \{0 \quad x < a, x > c \; \frac{x-a}{b-a} \; a \leq x \leq b \; \frac{c-x}{c-b} \; b \leq x \leq c \right\}$

2.6 Intuitionistic fuzzy set

Let X be nonempty set an intuitionistic fuzzy set A on X is of the form $A = \{(x, \mu_A(x), v_A(x); \, x \epsilon X\}$ where $\mu_A(x), v_A(x) : U \to [0,1]$ and $\mu_A(x)$ is the degree of membership, $v_A(x)$ is the degree of non-membership for every element

$x \epsilon X, 0 \leq \mu_A(x) + v_A(x) \leq 1$

2.7 Neutrosophic fuzzy set

ISBN 978-816855385-9
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A single valued neutrosophic set A is characterized by truth-membership function $T_A(x)$ an indeterminacy-membership function $I_A(x)$ and a falsity membership $F_A(x)$ for each $x \in X$

For each $x \in X$ $T_A(x)$, $I_A(x)$, $F_A(x) \in [0,1]$ A single valued neutrosophic set A can be written as $A=\{<x, T_A(x), I_A(x), F_A(x)> x \in X\}$

2.8 Bivariate Neutrosophic fuzzy set

A Bivariate neutrosophic fuzzy set A is characterized by truth-membership function $T_A(x,y)$ an indeterminacy-membership function $I_A(x, y)$ and a falsity membership $F_A(x,y)$ for each $(x,y) \in XxY$, For each $(x,y) \in XxY$ $T_A(x, y)$, $I_A(x, y)$, $F_A(x, y) \in [0,1]$ A bivariate neutrosophic fuzzy set A can be written as $\acute{A}=\{((x, y), T_A(x, y), I_A(x, y), F_A(x, y))/ (x, y) \in XxY\}$

2.9 Bivariate Neutrosophic fuzzy set in transportation problem

A bivariate neutrosophic fuzzy set in transportation problem is characterized by truth membership function $T_A(c_{ijk}, t_{ijk})$ and indeterminacy- membership function $I_A(c_{ijk}, t_{ijk})$ and a falsity membership $F_A(c_{ijk}, t_{ijk})$ for each $(c_{ijk}, t_{ijk}) \in CxT$, $T_A(c_{ijk}, t_{ijk})$, $I_A(c_{ijk}, t_{ijk})$, $F_A(c_{ijk}, t_{ijk}) \in [0,1]$ A bivariate neutrosophic fuzzy set can be written as

$$\acute{A} = \{(c_{ijk}, t_{ijk}) \in CxT\}$$

2.10 Symbols and notations

$c_{ijk} -$

$Cost\ of\ transporting\ goods\ to\ j^{th}\ destination\ from\ i^{th}\ source\ by\ k^{th}\ conveyance$

x_{ijk} – –Units of goods transporting to jth destination from ith source by kth Conveyance

t_{ijk} – – Time duration of transporting goods to jth destination from ith source by kth conveyance.

$$O_i \; -- Origin\ i$$

$D_j -- Destination\ j$

$E_k -- Conveyance\ k$

$n(E_k) -- capacity\ of\ conveyance\ k$

S_i −−Number of units available at the source i

D_j −−Number of units required by the destination j

3. Structure of the problem

3.1 Objective function

$$Minimize\ Z = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{t} (\lambda\, c_{ijk} + (1-\lambda) t_{ijk}) x_{ijk}\ , \lambda \epsilon [0,1]$$

3.2 Constraints

$\sum_{J=1}^{m} \sum_{k=1}^{t} x_{ijk} = S_i$ i=1to n

(Supply constraint)

$\sum_{i=1}^{n} \sum_{k=1}^{t} x_{ijk} = D_j$ j=1tom

(Demand constraint)

$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{t} x_{ijk} = \sum_{k=1}^{t} n(E_k)$

(Conveyance constraint)

$$\sum_{i=1}^{n} S_i = \sum_{j=1}^{m} D_j = \sum_{k=1}^{t} E_K$$

4. Proposed Algorithm

This section provides working procedure for Bivariate neutrosophic solid transportation problem. Consider the bivariate solid transportation problem.

The problem must be balanced before starting the allocation

Step 1. Verification

Check that Total supply = Total demand = sum of conveyance capacity If the above condition is not met convert it into balanced problem by adding dummy Source, destination, conveyance depending upon the nature of the problem and then Move to next step.

Step 2. Bivariate Neutrosophic number and membership conversion

Bivariate neutrosophic conversion

Bivariate neutrosophic number convert into single neutrosophic number by using the relation $CT_{ijk} = \lambda\, c_{ijk} + (1-\lambda) t_{ijk}$

Membership conversion

And membership values into crisp value by using

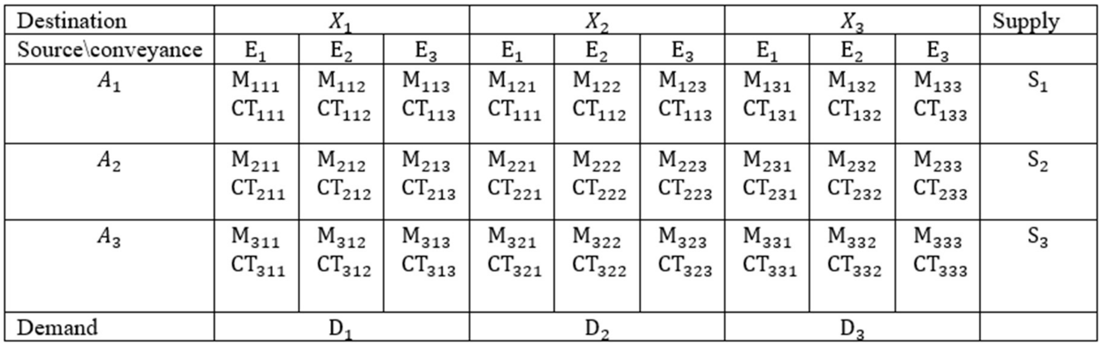
ISBN 978-816855385-9

$$M_{ijk} = \frac{T_{ijk}+(1-F_{ijk})}{2} - I_{ijk}$$

Step 3. Table construction

Construct the table using these new values CT_{ijk}, M_{ijk} express these values in top and bottom in each corresponding cell (M_{ijk} in the top of the and CT_{ijk} in the bottom of the cell).

Table 1: Structure of the table

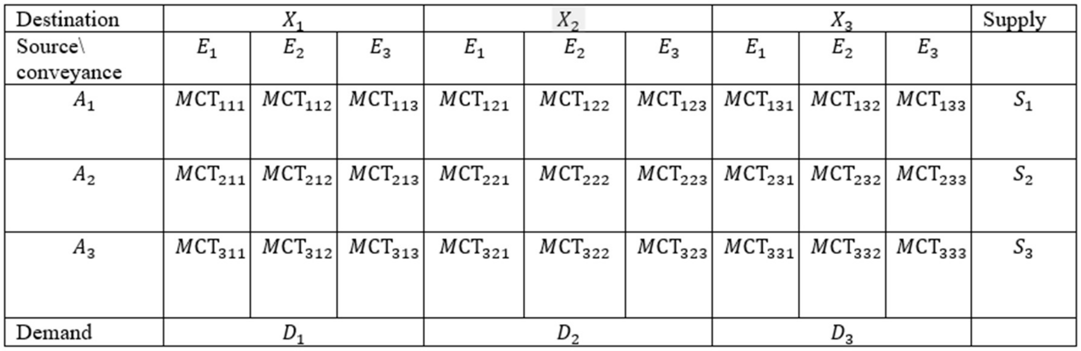


Destination	X_1			X_2			X_3			Supply
Source\conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	M_{111} CT_{111}	M_{112} CT_{112}	M_{113} CT_{113}	M_{121} CT_{111}	M_{122} CT_{112}	M_{123} CT_{113}	M_{131} CT_{131}	M_{132} CT_{132}	M_{133} CT_{133}	S_1
A_2	M_{211} CT_{211}	M_{212} CT_{212}	M_{213} CT_{213}	M_{221} CT_{221}	M_{222} CT_{222}	M_{223} CT_{223}	M_{231} CT_{231}	M_{232} CT_{232}	M_{233} CT_{233}	S_2
A_3	M_{311} CT_{311}	M_{312} CT_{312}	M_{313} CT_{313}	M_{321} CT_{321}	M_{322} CT_{322}	M_{323} CT_{323}	M_{331} CT_{331}	M_{332} CT_{332}	M_{333} CT_{333}	S_3
Demand	D_1			D_2			D_3			

The above table shows the crisp values of membership and the resource factors (time and cost). The final crisp conversion is again obtained by taking their average, using

$$MCT_{ijk} = \frac{M_{ijk} + CT_{ijk}}{2}$$

The final Membership cost time crisp table is presented below.



Destination	X_1			X_2			X_3			Supply
Source\ conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	MCT_{111}	MCT_{112}	MCT_{113}	MCT_{121}	MCT_{122}	MCT_{123}	MCT_{131}	MCT_{132}	MCT_{133}	S_1
A_2	MCT_{211}	MCT_{212}	MCT_{213}	MCT_{221}	MCT_{222}	MCT_{223}	MCT_{231}	MCT_{232}	MCT_{233}	S_2
A_3	MCT_{311}	MCT_{312}	MCT_{313}	MCT_{321}	MCT_{322}	MCT_{323}	MCT_{331}	MCT_{332}	MCT_{333}	S_3
Demand	D_1			D_2			D_3			

Step 4. Allocation procedure

Step i)

Select the membership cost time crisp table and then Subtract each entry of the row by its least value and write the difference in top of the corresponding cost and subtract each entry of the column by its least value and place them in the bottom of the corresponding cost.

Step ii)

Select the maximum value in supply/Demand, choose the corresponding row / column if there is a tie, choose arbitrarily.

Step iii)

Choose the minimum value (sum of row and column reduction value) in the selected row/column. Allocate the minimum value of supply/demand to the selected cell.

Step iv)

Eliminate the row/column corresponding to the supply or demand which is satisfied.

Step v)

Continue the process from step ii) to step v) until the supply demand are met.

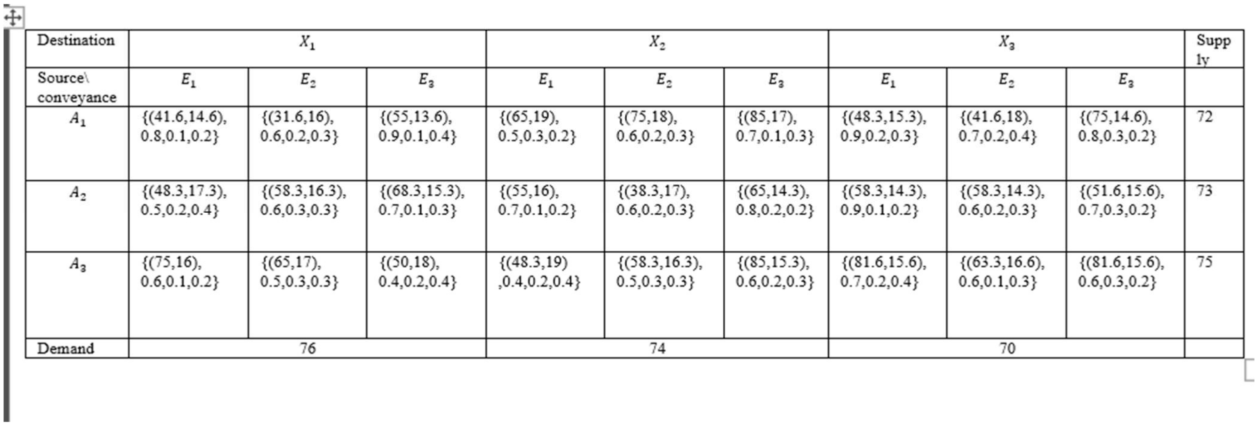
Step 5. Form a table with transportation cost and allocated units. calculate the transportation cost using the formula $\sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij}\, x_{ij}$.

Step 6. Apply Modi method to membership cost time crisp table and then find optimal cost

5. Proposed method

5. Problem

Consider the bivariate neutrosophic solid transportation problem with three origins, A_1, A_2, A_3 three destinations X_1, X_2, X_3 and three conveyance E_1, E_2, E_3 Find the optimal cost by considering cost, time and neutrosophic membership function with conveyance capacity $E_1 = 68$, $E_2 = 74$, $E_3 = 78$ for the following data.



Destination	X_1			X_2			X_3			Supply
Source\ conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	{(41.6,14.6), 0.8,0.1,0.2}	{(31.6,16), 0.6,0.2,0.3}	{(55,13.6), 0.9,0.1,0.4}	{(65,19), 0.5,0.3,0.2}	{(75,18), 0.6,0.2,0.3}	{(85,17), 0.7,0.1,0.3}	{(48.3,15.3), 0.9,0.2,0.3}	{(41.6,18), 0.7,0.2,0.4}	{(75,14.6), 0.8,0.3,0.2}	72
A_2	{(48.3,17.3), 0.5,0.2,0.4}	{(58.3,16.3), 0.6,0.3,0.3}	{(68.3,15.3), 0.7,0.1,0.3}	{(55,16), 0.7,0.1,0.2}	{(38.3,17), 0.6,0.2,0.3}	{(65,14.3), 0.8,0.2,0.2}	{(58.3,14.3), 0.9,0.1,0.2}	{(58.3,14.3), 0.6,0.2,0.3}	{(51.6,15.6), 0.7,0.3,0.2}	73
A_3	{(75,16), 0.6,0.1,0.2}	{(65,17), 0.5,0.3,0.3}	{(50,18), 0.4,0.2,0.4}	{(48.3,19) ,0.4,0.2,0.4}	{(58.3,16.3), 0.5,0.3,0.3}	{(85,15.3), 0.6,0.2,0.3}	{(81.6,15.6), 0.7,0.2,0.4}	{(63.3,16.6), 0.6,0.1,0.3}	{(81.6,15.6), 0.6,0.3,0.2}	75
Demand	76			74			70			

Solution

Proposed method

Step 1

Verify the problem is balanced

Total supply=72+73+75

= 220

Total demand =76+74+70

= 220

Sum of conveyance capacity= 68+74+78

= 220

Therefore, Total supply=Total demand= sum of conveyance capacity

Step 2

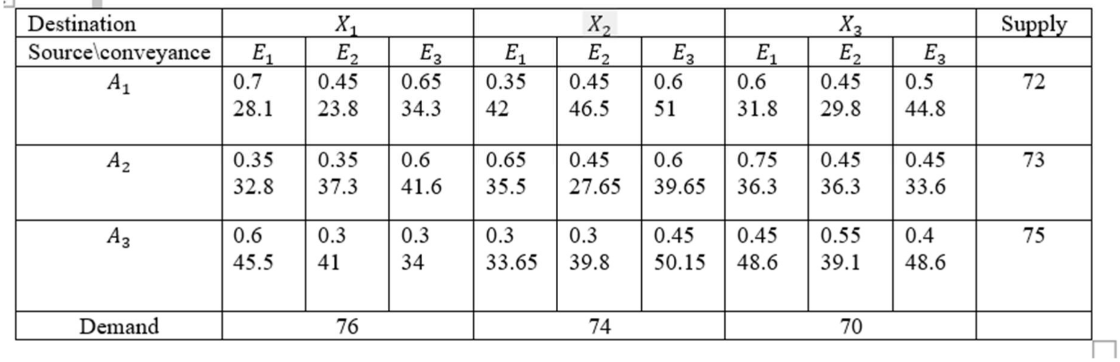
Convert the bivariate neutrosophic solid transportation problem into standard solid transportation problem with membership values in each cell using

$$CT_{ijk} = \lambda\, c_{ijk} + (1 - \lambda)t_{ijk}$$

$$M_{ijk} = \frac{T_{ijk} + (1 - F_{ijk})}{2} - I_{ijk}$$

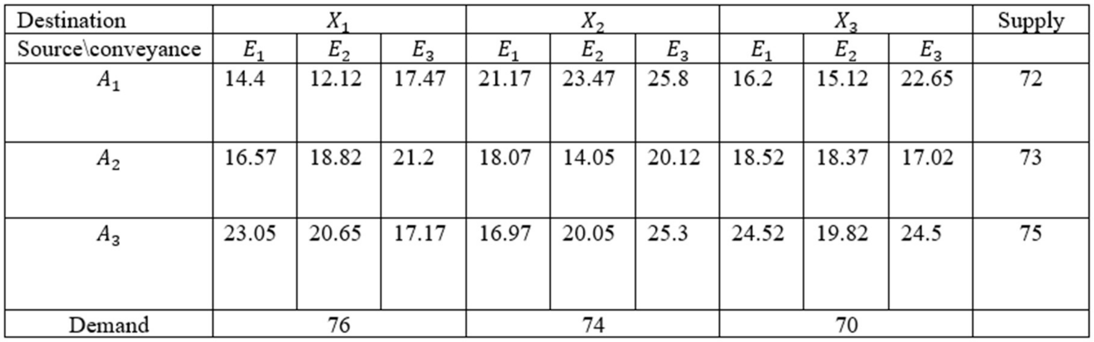
$$MCT_{ijk} = \frac{M_{ijk}+CT_{ijk}}{2}.$$

Step 3 Table construction



Destination	X_1			X_2			X_3			Supply
Source\conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	0.7 28.1	0.45 23.8	0.65 34.3	0.35 42	0.45 46.5	0.6 51	0.6 31.8	0.45 29.8	0.5 44.8	72
A_2	0.35 32.8	0.35 37.3	0.6 41.6	0.65 35.5	0.45 27.65	0.6 39.65	0.75 36.3	0.45 36.3	0.45 33.6	73
A_3	0.6 45.5	0.3 41	0.3 34	0.3 33.65	0.3 39.8	0.45 50.15	0.45 48.6	0.55 39.1	0.4 48.6	75
Demand	76			74			70			

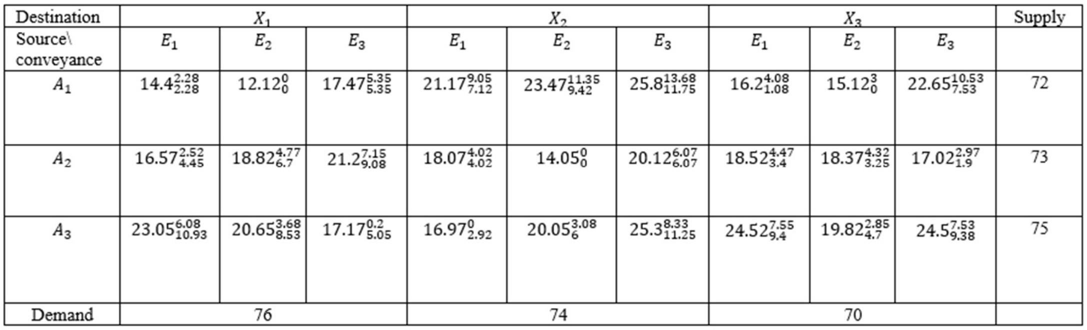
Final crisp table



Destination	X_1			X_2			X_3			Supply
Source\conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	14.4	12.12	17.47	21.17	23.47	25.8	16.2	15.12	22.65	72
A_2	16.57	18.82	21.2	18.07	14.05	20.12	18.52	18.37	17.02	73
A_3	23.05	20.65	17.17	16.97	20.05	25.3	24.52	19.82	24.5	75
Demand	76			74			70			

Step 4 Initial and optimal Allocation

Row column reduced matrix



Destination	X_1			X_2			X_3			Supply
Source\conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	$14.4_{2.28}^{2.28}$	12.12_{0}^{0}	$17.47_{5.35}^{5.35}$	$21.17_{7.12}^{9.05}$	$23.47_{9.42}^{11.35}$	$25.8_{11.75}^{13.68}$	$16.2_{1.08}^{4.08}$	15.12_{0}^{3}	$22.65_{7.53}^{10.53}$	72
A_2	$16.57_{4.45}^{2.52}$	$18.82_{6.7}^{4.77}$	$21.2_{9.08}^{7.15}$	$18.07_{4.02}^{4.02}$	14.05_{0}^{0}	$20.12_{6.07}^{6.07}$	$18.52_{3.4}^{4.47}$	$18.37_{3.25}^{4.32}$	$17.02_{1.9}^{2.97}$	73
A_3	$23.05_{10.93}^{6.08}$	$20.65_{8.53}^{3.68}$	$17.17_{5.05}^{0.2}$	$16.97_{2.92}^{0}$	$20.05_{6}^{3.08}$	$25.3_{11.25}^{8.33}$	$24.52_{9.4}^{7.55}$	$19.82_{4.7}^{2.85}$	$24.5_{9.38}^{7.53}$	75
Demand	76			74			70			

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Destination	X_1			X_2			X_3			Supply
Source\ conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	14.4	12.12 $\boxed{72}$	17.47	21.17	23.47	25.8	16.2	15.12	22.65	72
A_2	16.57	18.82	21.2	18.07	14.05 $\boxed{2}$	20.12 $\boxed{1}$	18.52	18.37	17.02 $\boxed{70}$	73
A_3	23.05	20.65	17.17 $\boxed{4}$	16.97 $\boxed{68}$	20.05	25.3 $\boxed{3}$	24.52	19.82	24.5	75
Demand	76			74			70			

Allocated table

Destination	x_1		x_2			x_3	Supply
Source\ conveyance	E_2	E_3	E_1	E_2	E_3	E_3	
A_1	12.12 $\boxed{72}$	17.47	21.17	23.47	25.8	22.65	72
A_2	18.82	21.2	18.07	14.05 $\boxed{2}$	20.12 $\boxed{1}$	17.02 $\boxed{70}$	73
A_3	20.65	17.17 $\boxed{4}$	16.97 $\boxed{68}$	20.05	25.3 $\boxed{3}$	24.5	75

Transportation cost

Destination	x_1		x_2			x_3	Supply
Source\ conveyance	E_2	E_3	E_1	E_2	E_3	E_3	
A_1	31.6 $\boxed{72}$	55	65	75	85	75	72
A_2	58.3	68.3	55	38.3 $\boxed{2}$	65 $\boxed{1}$	51.6 $\boxed{70}$	73
A_3	65	50 $\boxed{4}$	48.3 $\boxed{68}$	58.3	85 $\boxed{3}$	81.6	75
Demand	76		74			70	

Initial transportation cost = 31.6 x 72 + 50x4+68x48.3+38.3x2+65x1+85x3+51.6x70

= 9768.2

Apply modification method

Optimal transportation cost = 9768.2

6. Comparison table

S.No	Vogels	Modified method	Proposed method
1	10,105.2	Solution violates the capacity of conveyance	9768.2

7. Conclusion

This study focuses on determining the optimal cost and time while satisfying all three constraints in the fewest possible steps. However, in the standard method, the modification applied to handle negative opportunity costs results in a violation of the transportation capacity constraints.

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ISBN 978-816855385-9
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