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AN ENHANCED FRAMEWORK FOR FUZZY TRANSPORTATION PROBLEMS USING ENNEADECAGON FUZZY NUMBERS WITH DUAL RANKING AND STEPPING STONE OPTIMIZATION

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Abstract

The Enneadecagon Fuzzy Number (EDGFN) is a 19-point symmetric fuzzy representation that offers a more precise modelling and granularity than traditional triangular or trapezoidal forms. In this paper, we present a novel methodology for resolving fuzzy transportation problems. Fuzzy costs, inventories, and demands are transformed into precise equivalents using two fuzzy ranking techniques, Graded Mean Integration Representation (GMIR) and the widely used Centroid method. The transportation problem is solved using Vogel's Approximation Method (VAM) with these crisp values as inputs to acquire an initial basic feasible solution. The solution is subsequently evaluated for optimality and improved iteratively using the stepping stone Method. The final transportation cost and the number of iterations necessary to achieve optimality are assesses through a comparative study of each ranking method. The results suggest that the combination of EDGFN with centroid method and GMIR provides a precise control over uncertainty representation. This framework facilitates more exact and informed decision-making in ambiguous environments, including resource allocation and supply chain logistics.

1. Introduction:

Transportation issues are crucial in operations research and supply chain management, focussing on identifying the most economical method for distributing commodities from various origins to numerous destinations. Traditionally conceptualized as linear programming problems with accurate cost, supply, and demand characteristics, these models frequently prove inadequate in practical scenarios when information is uncertain, imprecise, or ambiguous.

Fuzzy set theory, established by Zadeh (1965), has demonstrated efficacy in managing uncertainty by permitting partial membership of components. Fuzzy models more effectively represent imprecision in transportation costs, changeable needs, and within transportation

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challenges. Initial adaptations of fuzzy transportation problems utilized triangular and trapezoidal fuzzy numbers (Zimmermann, 1978: Kaufmann and Gupta, 1991), providing simplicity although lacking the ability to express asymmetric or granular uncertainty.

Recent developments have provided more intricate polygonal fuzzy numbers, including pentagonal (Chandramouli, 2007), hexagonal, and tridecagonal (Rani & Meenakshi, 2012), which enhance the depiction of uncertainty. These polygonal structures promote modelling accuracy by integrating numerous $\alpha -$ level points, resulting in more refined membership functions and greater stability in ranking.

The current work expands on this development by presenting the Enneadecagon Fuzzy Number (EDGFN), a 19-point fuzzy structure with a centre peak and nine rising and nine descending segments. Transportation issues with high variability, dynamic costs, and soft constraints benefit greatly from the fine- grained representation of fuzzy parameters made possible by the EDGFN. Graded Mean integration representation (GMIR) is a recognized technique for ranking fuzzy numbers, particularly triangular or trapezoidal fuzzy numbers. It provides a singular, clear representative value by amalgamating the core (peak) and support (ends) of the fuzzy set. (Dubois, D., & Prade, H.,1980).

Defuzzification or ranking of fuzzy numbers is a crucial step in addressing fuzzy optimization problems. Various ranking methods have been proposed over the years, including centroid-based methods (Chen & Hsieh, 2000), area compensation models, and $\alpha -$ cut based integration. This study employs and compares two ranking strategies such that

- Graded Mean Integration Representation (GMIR) approach is esteemed for its computational efficiency and conceptual lucidity. It depends on basic arithmetic, rendering it both efficient and rapid to execute, especially in extensive tasks.
- Centroid Method is widely utilized for its simplicity and computational efficiency, although it is often criticised for its sensitivity to shape skewness.

Vogel's Approximation Method (VAM) is used to generate an initial basic feasible solution in order to derive the solution. This approach frequently performs better than more straightforward heuristics like the North-West Corner or Least Cost approaches since it is well-known for its efficacy in reducing opportunity costs during first allocation (Taha, 2017). The Stepping Stone method, which is well-established in the classical optimization literature, is then used to evaluate unallocated routes, check for optimality, and iteratively refine the solution (Hillier & Lieberman, 2020).

Through the integration of various ranking systems, alternate solution approaches, and the modelling power of EDGFN, this research provides a thorough framework for resolving fuzzy transportation issues. The approach is validated and the impact of ranking selections on the final transportation cost is demonstrated using a comprehensive numerical example.

2. Preliminaries:

2.1. Fuzzy number: An imprecise numerical value $\grave{F}$ is a fuzzy set defined on the real number line $\mathbb{R}$ that meets the following criteria where $\grave{F}$ is normal $(i.e., \exists\ z \in\ \mathbb{R}: \mu_{\grave{F}}(z) = 1)$; $\grave{F}$ is convex; $\mu_{\grave{F}}$ is upper semi-continuous and support of $\grave{F}$ is bounded.

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2.2. Enneadecagon Fuzzy Number (EDGFN): An Enneadecagon Fuzzy number $\grave{F}^{EDGFN}$ is a fuzzy number characterized by a membership function $\mu_{\grave{F}^{EDGFN}}(z)$, defined over 19 ordered real numbers:

$$\grave{F}^{EDGFN} = (a_1, a_2, \dots, a_{19})$$

Its membership function is defined explicitly as:

$$\mu_{\grave{F}^{EDGFN}}(z) = \begin{cases} 0, & y < a_1 \\ u\left(\frac{y - a_1}{a_2 - a_1}\right), & a_1 \le y \le a_2 \\ u, & a_2 \le y \le a_3 (plateau) \\ u + (v - u)\left(\frac{y - a_3}{a_4 - a_3}\right), & a_3 \le y \le a_4 \\ v, & a_4 \le y \le a_5 (plateau) \\ v + (w - v)\left(\frac{y - a_5}{a_6 - a_5}\right), & a_5 \le y \le a_6 \\ w, & a_6 \le y \le a_7 (plateau) \\ w + (t - w)\left(\frac{y - a_7}{a_8 - a_7}\right), & a_7 \le y \le a_8 \\ t, & a_8 \le y \le a_9\ (plateau) \\ t + (1 - t)\left(\frac{y - a_9}{a_{10} - a_9}\right), & a_9 \le y \le a_{10} \\ 1, & y = a_{10} \\ t + (1 - t)\left(\frac{a_{11} - y}{a_{11} - a_{10}}\right), & a_{10} \le y \le a_{11} \\ t, & a_{11} \le y \le a_{12}\ (plateau) \\ w + (t - w)\left(\frac{a_{13} - y}{a_{13} - a_{12}}\right), & a_{12} \le y \le a_{13} \\ w, & a_{13} \le y \le a_{14} (plateau) \\ v + (w - v)\left(\frac{a_{15} - y}{a_{15} - a_{14}}\right), & a_{14} \le y \le a_{15} \\ v, & a_{15} \le y \le a_{16} (plateau) \\ u + (v - u)\left(\frac{a_{17} - y}{a_{17} - a_{16}}\right), & a_{16} \le y \le a_{17} \\ u, & a_{17} \le y \le a_{18}\ (plateau) \\ u\left(\frac{a_{19} - y}{a_{19} - a_{18}}\right), & a_{19} \le y \le a_{18} \\ 0, & y > a_{19} \end{cases}$$

where the parameters satisfy:

$$0 < u < v < w < t < 1$$

This configuration includes four internal plateau levels: u, v, w, t each contributing to the structure's flexibility and granularity in modelling uncertainty.

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

2.3. α-cut of Enneadecagon Fuzzy Number (EDGFN): The α-cut of an Enneadecagon fuzzy number $\grave{F}^{EDGFN}$ at level α, denoted by $\left[\grave{F}^{EDGFN}\right]_{\alpha}$, is clearly defined as the crisp set:

$$\left[\grave{F}^{EDGFN}\right]_{\alpha} = \left\{y \middle| \mu_{\grave{F}^{EDGFN}}(y) \geq \alpha\right\}, \alpha \in [0,1]$$

The α-cut intervals vary α-level and its position relative to the internal plateaus u, v, w, t. The corresponding intervals are:

$$\left[\grave{F}^{EDGFN}\right]_{\alpha} = \begin{cases} [EDGFN_{1\alpha}^{L}, EDGFN_{1\alpha}^{U}], \alpha \in [0, u] \\ [EDGFN_{2\alpha}^{L}, EDGFN_{2\alpha}^{U}], \alpha \in [u, v] \\ [EDGFN_{3\alpha}^{L}, EDGFN_{3\alpha}^{U}], \alpha \in [v, w] \\ [EDGFN_{4\alpha}^{L}, EDGFN_{4\alpha}^{U}], \alpha \in [w, t] \\ [EDGFN_{5\alpha}^{L}, EDGFN_{5\alpha}^{U}], \alpha \in [t, 1] \end{cases}$$

with endpoints defined clearly by:

- For $\alpha \in [0, u]$
$$EDGFN_{1\alpha}^{L} = a_1 + \frac{\alpha}{u}(a_2 - a_1), EDGFN_{1\alpha}^{U} = a_{19} - \frac{\alpha}{u}(a_{19} - a_{18})$$
- For $\alpha \in [u, v]$
$$EDGFN_{2\alpha}^{L} = a_3 + \frac{\alpha - u}{v - u}(a_4 - a_3), EDGFN_{2\alpha}^{U} = a_{17} - \frac{\alpha - u}{v - u}(a_{17} - a_{16})$$
- For $\alpha \in [v, w]$
$$EDGFN_{3\alpha}^{L} = a_5 + \frac{\alpha - v}{w - v}(a_6 - a_5), EDGFN_{3\alpha}^{U} = a_{15} - \frac{\alpha - u}{v - u}(a_{15} - a_{14})$$
- For $\alpha \in [w, t]$
$$EDGFN_{2\alpha}^{L} = a_7 + \frac{\alpha - w}{t - w}(a_8 - a_7), EDGFN_{2\alpha}^{U} = a_{13} - \frac{\alpha - w}{t - w}(a_{13} - a_{12})$$
- For $\alpha \in [t, 1]$
$$EDGFN_{2\alpha}^{L} = a_9 + \frac{\alpha - t}{1 - t}(a_{10} - a_9), EDGFN_{2\alpha}^{U} = a_{11} - \frac{\alpha - t}{1 - t}(a_{11} - a_{10})$$

The parameters are clearly set to satisfy $0 < u < v < w < t < 1$, providing flexibility in modelling uncertainty with evidently defined plateau regions.

2.4. Graded Mean Integration Representation (GMIR) Ranking Function:
For fuzzy number $\grave{F} = \{1,2,3, \ldots, 19\}$

$$R_{GMIR}(\grave{F}) = \frac{1}{4}(a_1 + 2a_{10} + a_{19})$$

- a_1: first value
- a_{10} is the central peak value
- a_{19}: last value

2.5. Centroid Ranking Method: Centroid or centre of gravity ranking, which is given by:

$$R_{centroid}(\grave{F}) = \frac{\sum_{i=1}^{19} a_i . \mu(a_i)}{\sum_{i=1}^{19} \mu(a_i)}$$

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Here, $\mu(a_i)$ represents the membership value associated with each point a_i. For EDGFN:

- $\mu(a_i) = \frac{i-1}{9}$ for $i = 1\ to\ 10$
- $\mu(a_i) = \frac{19-i}{9}$ for $i = 11\ to\ 19$

This approach yields a straightforward, comprehensible mean value and is computationally efficient.

3. Framework of the Fuzzy Transportation Problem utilizing EDGFN

A transportation problem involves identifying an optimal distribution of shipments from m origins (supply points) to n destinations (demand points) to minimize overall transportation costs while satisfying all supply and demand restrictions. In practical implementations, the factors transportation costs, supply availability, and demand requirements are frequently ambiguous or unpredictable due to market changes, logistical delays, or insufficient data.

To mitigate this uncertainty, we model the transportation problem within a fuzzy framework utilizing Enneadecagon fuzzy numbers (EDGFNs), which offer a nuanced and systematic representation of ambiguous values.

Let:

- $\tilde{C}_{ij} \approx \left({c_{ij}}^1, {c_{ij}}^2, \ldots, {c_{ij}}^{19}\right)$ be the fuzzy transportation cost from source i to destination j, represented by an EDGFN,
- $\tilde{S}_i \approx ({s_i}^1, {s_i}^2, \ldots, {s_i}^{19})$ be the fuzzy supply at source i,
- $\widetilde{D}_j \approx \left({d_j}^1, {d_j}^2, \ldots, {d_j}^{19}\right)$ be the fuzzy demand at destination j,
- x_{ij} be the crisp decision variable, indicating the quantity transported from source i to destination j.

3.1. Fuzzy Transportation Model

The fuzzy transportation model can be expressed as:

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} \,.\, x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = \tilde{S}_i \text{ (for all } i = 1,2,\ldots,m)$$

$$\sum_{j=1}^{n} x_{ij} = \widetilde{D}_j \text{ (for all } j = 1,2,\ldots,n)$$

$$x_{ij} \geq 0 \ \forall i, j$$

During optimization, the fuzzy cost, supply, and demand values are defuzzified using ranking functions, as arithmetic operations cannot be directly applied to fuzzy numbers.

3.2. Crisp Model via Ranking Function

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In order to resolve the fuzzy problem, each EDGFN is converted to its crisp representative value by employing the two ranking methods:

- **Graded Mean Integration Representation (GMIR) Ranking Function:**

$$R_{GMIR}(\hat{F}) = \frac{1}{4}(a_1 + 2a_{10} + a_{19})$$

- **Centroid Method:**

$$R_{centroid}(\hat{F}) = \frac{\sum_{i=1}^{19} a_i . \mu(a_i)}{\sum_{i=1}^{19} \mu(a_i)}$$

After the application of ranking, the fuzzy problem is transformed into a conventional transportation problem characterized by precise values.

- $C_{ij} = R(\tilde{C}_{ij})$
- $S_i = R(\tilde{S}_i)$
- $D_j = R(\widetilde{D}_j)$

3.3. Balanced Condition

The transportation problem is balanced if

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$$

In case of an unbalanced problem, a fake row or column is added with a transportation cost of zero to equalise the supply and demand totals.

In this paper, we utilize both the ranking techniques to transform fuzzy factors, specifically transportation costs, supply, and demand into precise values. Two separate formulations of the transportation problem are established, one utilizing the values obtained from the GMIR, and the other employing the centroid based rankings. Each of these discrete transportation models is independently resolved utilizing the Vogel's Approximation Method (VAM) to derive an initial basic workable solution. The Stepping stone Method is then utilized for each model to conduct optimality assessments and enhance the answers via iterative refinement. A comprehensive comparison analysis is conducted to assess the impact of the ranking approach on critical outcomes, such as the final transportation cost, the number of iterations needed to achieve optimality, and the configuration of the resultant allocation matrix. This dual methodology paradigm provides significant insight into the impact of ranking strategies on solutions to fuzzy transportation problems.

4. Solution methodology using VAM and Stepping Stone Method

A two-phase solution procedure is implemented for each case, regardless of whether the ranked Enneadecagon Fuzzy Numbers (EDGFNs) are derived from the Graded Mean Integration Representation (GMIR) or the Centroid method, in order to optimally resolve the transportation issues Vogel's Approximation Method (VAM) is employed to initially generate a robust

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fundamental feasible solution for the classical transportation problem in both scenarios. Next, the Stepping Stone Method is employed to conduct an optimality test and enhance the solution. Combining methodologies guarantees a computationally effective and nearly optimal solution.

4.1. Vogel's Approximation Method (VAM)

The VAM method is a heuristic approach that employs penalty costs to direct allocations and generally offers a superior first solution compared to simpler techniques such as the North-West corner or Least Cost method

Step 1: Calculate penalty for each row and column

Step 2: Identify Maximum penalty

Step 3: Make the maximum possible allocation

Step 4: Adjust Supply and Demand

Step 5: Recalculate penalties and repeat steps

4.2. Stepping Stone Method:

After identifying the first solution with the VAM, the Stepping Stone method is employed to evaluate and enhance optimality. This method assesses the opportunity cost of unallocated cells by tracing closed loops. The procedures are as follows:

Step 1: Identify unallocated cells

List all cells in the cost matric that have nit been assigned any allocation.

Step 2: Form closed loops

For each unallocated cell, construct a loop that begins and ends at the cell and includes horizontal and vertical moves through allocated cells.

Step 3: Calculate net change (Opportunity cost)

For each loop. Alternate the signs $(+,-,+,-.,,,)$ of costs and compute the net opportunity cost Δ_{ij} as:

$$\Delta_{ij} = C_{ij} - (\text{sum of alternate costs in loop})$$

Step 4: Check optimality

- If all $\Delta_{ij} \geq 0$: the current solution is optimal.
- If any $\Delta_{ij} \geq 0$: improvement is possible.

Step 5: Improve the solution

Select the unallocated cell with the most negative Δ_{ij} and redistribute allocations along the loop: increase allocations on " + " positions and decrease on " − " positions by the smallest allocated value in the loop.

Step 6: Continue the process until no negative opportunity costs remain.

This is implemented autonomously on the transportation matrices obtained from both the GMIR and centroid rankings. Every path result in a possible distinct ideal solution, which will thereafter be evaluated based on total cost and allocation efficiency.

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5. **Numerical Example**: A global electric vehicle (EV) manufacturer is responsible for transferring lithium-ion battery packs from three strategically positioned eco-optimized warehouses to three urban EV assembly factories within the framework of sustainable and adaptable supply chain management. The supply nodes warehouse alpha, utilizing solar energy; warehouse beta, reliant on hydropower; and warehouse gamma, functioning on grid electricity with carbon offset credits are each affected by variable production due to changing renewable energy inputs. the demand for battery packs arises from three primary production facilities: plant X, which specialises in high-capacity SUV assembly; plant y, which concentrates on urban hatchback manufacturing; and plant Z, which produces commercial electric vans. This real-world situation presents operational variables, including transportation delays, fluctuating electricity prices, charging durations, and inconsistent user requests, which make the system intrinsically ambiguous. To address this, all transportation expenses, supply availabilities, and demand necessities are modelled using EDGFNs, which encapsulate a 19-point representation of the uncertainty spectrum. The imprecise representations are subsequently defuzzified employing two notable ranking strategies; Graded Mean Integration Representation (GMIR) and the Centroid method- after which each resultant crisp matrix is addressed using Vogel's Approximation Method (VAM) and further optimised through the Stepping Stone method. The objective is to evaluate the impact of each ranking strategy on the efficiency, cost, and reliability of the resultant green logistics plan amidst uncertainty.

6. Feasible solutions between different Ranking Methods:

6.1. Vogel's Approximation and Stepping Stone method using Graded Mean Integration Representation (GMIR) Ranking Function:

Step 1: The fuzzy transportation problem using Enneadecagon fuzzy Numbers

	α (Utilizing solar energy)	β (Reliant on hydropower)	γ (Functioning on grid electricity)	Supply
X	[11.4,12,12.7,13.32,13.9,14.6, 15.3,15.9,16.5,17.2,17.8,18.5, 19.1,19.8,20.4,21.0,21.7,22.3,22.9]	[10.4,11.0,11.7,12.4,13.0,13.6, 14.3,14.9,15.6,16.2,16.9,17.5, 18.1,18.8,19.4,20.1,20.7,21.4, 21.9]	[12.9,13.5,14.2,14.8,15.5,16.1, 16.7,17.4,18.0,18.7,19.3,19.9, 20.6,21.2,21.9,22.523.2,23.8, 24.5]	[119.5,120.2,120.8,121.5,122.1, 122.8,123.4,124.0,124.7,125.3, 125.9,126.6,127.3,127.9,128.5, 129.2,129.8,130.5,131.1]
Y	[2.9,3.5,4.1,4.8,5.5,6.1,6.7,7.4,8.0, 8.7,9.3,9.9,10.[illegible],11.3,11.9,12.5, 13.2,13.8,14.5]	[11.4,12.0,12.7,13.3,13.9,14.6, 15.3,15.9,16.5,17.2,17.8,18.5, 19.1,19.8,20.4,21.0,21.7,22.3, 22.9]	[12.9,13.6,14.1,14.8,15.5,16.1, 16.8,17.4,18.0,18.7,19.3,19.9, 20.6,21.3,21.9,22.6,23.2,23.8, 24.5]	[133.1,133.7,134.4,135,135.7, 136.3,136.9,137.6,138.2,138.9, 139.5,140.1,140.8,141.4,142.1, 142.7,143.4,144,144.7]
Z	[13.9,14.5,15.2,15.8,16.5,17.1,17.8, 18.4,19.0,19.7,20.3,20.9,21.6,22.3, 22.9,23.5,24.2,24.8.25.5]	[14.9,15.5,16.2,16.8,17.5,18.1, 17.8,19.4,20.0,20.7,21.3,21.9, 22.6,23.3,23.9,24.5,25.2,25.8, 26.5]	[10.9,11.6,12.3,12.9,13.5,14.2, 14.9,15.5,16.1,16.8,17.4,18.1, 18.7,19.3,19.9,20.6,21.3,21.9, 22.6]	[125.8,126.5,127.1,127.8,128.4, 129,129.7,130.3,130.9,131.6, 132.3,132.9,133.4,134.2,134.8, 135.5,136.1,136.8,137.4]
Demand	[131.54,132.2,132.8,133.5, 134.1,134.8,135.4,136.0,136.7, 137.3,137.9,138.6,139.3,139.9, 140.5, 141.1,141.8,142.5,143.1]	[118.5,119.1,119.8,120.4,121, 121.7,122.4,123,123.6,124.3, 124.9,125.6,126.2,126.9, 127.5,128.1,128.8, 129.4,130.1]	[128.4,129.0,129.7,130.3,130.9, 131.6,132.2,132.9,133.5,134.2, 134.8,135.5,136.1,136.8,137.4, 138.0,138.7,139.3,139.9]	

Table 6.1

Step 2: By applying the GMIR the above fuzzy transportation problem gets reduced to the following crisp table

	α (*Utilizing solar energy*)	β (*Reliant on hydropower*)	γ(*Functioning on grid electricity*)	*Supply*
X	17.17	16.17	18.70	125.3

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Y	8.70	17.17	18.70	138.9
Z	19.70	20.12	15.20	131.6
$Demand$	137.30	124.30	134.17	

Table 6.2

Step 3: Initial basic feasible solution using Vogel's Approximation Method

	α (*Utilizing solar energy*)	β (*Reliant on hydropower*)	γ(*Functioning on grid electricity*)
X	-	124.3	1.0
Y	137.3	-	1.57
Z	-	-	131.6

Table 6.3

Step 4: Total cost is obtained from the above table

$$\text{Total cost} = (8.7 \times 137.3) + (16.17 \times 124.3) + (18.7 \times 1.0) +$$
$$(18.7 \times 1.57) + (15.2 \times 131.6)$$
$$= 5252.82$$

Step 5: Applying Stepping Stone method to check for optimality, to start with the unallocated cells

$$(X, \alpha) = 8.54; (Y, \beta) = 0.98; (Z, \alpha) = 14.6; (Z, \beta) = 7.28$$

Here we obtain no negative opportunity costs hence the current solution is optimal

6.2. Vogel's Approximation and Stepping Stone method using Centroid Ranking Function:

Step 1: By applying Centroid ranking function formula to table 6.1 we obtain the below table

	α (*Utilizing solar energy*)	β (*Reliant on hydropower*)	γ(*Functioning on grid electricity*)	*Supply*
X	17.18	16.21	18.66	125.32
Y	8.67	17.18	18.67	138.86
Z	19.67	19.99	15.17	131.59
$Demand$	137.31	124.27	134.17	

Table 6.4

Step 2: Now applying Vogel's approximation method, we get the initial basic feasible solution

	α (*Utilizing solar energy*)	β (*Reliant on hydropower*)	γ(*Functioning on grid electricity*)
X	-	124.27	1.03
Y	137.31	-	1.55
Z	-	-	131.59

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Table 6.5

Step 3: The total cost as obtained from the above table is given by

$$\text{Total cost} = (137.31 \times 8.67) + (16.21 \times 124.27) + (18.66 \times 1.05) +$$
$$(18.67 \times 1.53) + (15.17 \times 131.59)$$
$$= 5249.27$$

Step 4: Applying Stepping Stone method to check for optimality, to start with the unallocated cells

$$(X, \alpha) = 8.52; (Y, \beta) = 0.96; (Z, \alpha) = 14.5; (Z, \beta) = 7.27$$

Here we obtain no negative opportunity costs hence the current solution is optimal.

7. Comparison between GMIR and Centroid method:

	GMIR ranking method	Centroid ranking method
Total Cost	5252.82	5249.27

Table 6.6

Result: The findings indicate that the GMIR and Centroid ranking algorithms produce approximately equivalent total transportation costs, when utilized for the same fuzzy transportation problem employing Vogel's Approximation Method (VAM). This close alignment verifies that both strategies emphasise core values inside the fuzzy number and demonstrate analogous behaviour in defuzzification. It also indicates that either method may be effectively employed in situations when a balanced or standard cost calculation is suitable.

Conclusion:

Here we have analysed the fuzzy transportation problem utilizing Enneadecagon (19-point) fuzzy numbers, evaluating the efficacy of the Graded Mean Integration Representation (GMIR) and Centroid Ranking method in conjunction with Vogel's Approximation Method (VAM) and the Stepping stone method for optimality test. Both techniques yielded virtually equivalent total transportation costs fulfilled the non-degeneracy requirement, and were verified as optimal. The strong concordance indicates that both ranking methodologies are efficacious for standard-case decision-making in fuzzy contexts. The application of Enneadecagon fuzzy numbers improved accuracy in modelling uncertainty, hence strengthening the relevance of these methods in practical fuzzy logistics contexts.

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