# Integrated Coordination Inventory Model for Buyer – Vendor Using Lagrange Multiplier Technique

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**Abstract---** This study proposes integrated inventory model for buyer – vendor under coordination and non coordination situations. Integrated system cost is developed for system optimization and optimal values are found by using analytical geometry and algebraic method. In addition, the model considered inventory level constraint for both situations. Lagrange's multiplier technique is used to solve this type of problems. Numerical examples are presented to illustrate the developed model.

**Keywords---** Inventory, Lagrange Multiplier Technique, Order Quantity, Coordination.

## I. Introduction

In real life situations, the unique intention of traditional inventory models is to minimize the total inventory cost. These models could not consider the defective items in lot and rework of them. Mostly, these models have a common assumption like, all units purchased or manufactured are perfect in quality. But, in practice due to some reasons like workers skill, imperfect raw materials, capability of machine, etc., they purchase or produce defective items also.

Muniappan et al. [10] studied multi-echelon multi-constraints inventory model using Lagrange multiplier technique. Sugirtha et al. [16] developed a model for buyer – vendor involving back orders, screening process and transportation cost. Ravithammal et al. [14] analyzed inventory model for price discount involving shortage, back ordering and rework. Muniappan et al. [9] developed a production inventory model for fixed life time products involving quantity discount, backordering and rework. Chang et al. [2] studied economic order quantity model for imperfect quality. Ganesh et al. [3] developed vendor - buyer coordination model with shortages and screening process.

Cardenas-Barron [1] and Mari Selvi et al. [6] considered the derivation of EOQ/EPQ inventory models using analytic geometry and algebra. J. T. Hsu and L. F. Hsu [4] developed EPQ models with imperfect production processes, inspection errors, planned backorders, and sales returns. Jawla and Singh [5] studied multi-item production quantity model for imperfect items with multiple production setups and rework. Mukhopadhyay and Goswami [7] analyzed production quantity model for three type imperfect items with rework and learning in setup.

Muniappan et al. [8] developed mathematical technique for computing optimal replenishment polices. Ravithammal et al. [15] developed an optimal pricing model with positive exponential function of price discount rate of demand. Muniappan et al. [11] studied EPQ incentive inventory model involving partially backlogged shortages. Ravithammal et al. [12] studied a deterministic production model for buyer- manufacturer with quantity discount and completely backlogged shortages. Ravithammal et al. [13] developed a production model for perishable items with fixed and linear backorders.

## II. Notations and Assumptions

The model use the following notations and assumptions.

***Notations***

| | |
|---|---|
| D | Demand rate / time unit |
| $R_1$ | Buyer's unit ordering cost / order |

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|---|---|
| $R_2$ | Vendor’s unit setup cost / order |
| s | Buyer’s unit shortage cost / order |
| p | Buyer’s unit purchase cost / order |
| $Q_1$ | Backorder level |
| $Q^*$ | Optimum Order quantity for non coordination scheme |
| $Q_c^*$ | Optimum Order quantity for coordination scheme |
| $H_b$ | Buyer’s unit holding cost / order / unit |
| $H_v$ | Vendor’s unit holding cost / order / unit |
| $s_c$ | Vendor’s unit screening cost / order / unit |
| n | Vendor’s multiples of order without coordination |
| m | Vendor’s multiples of order with coordination |
| k | Buyer’s multiples of order with coordination |
| d(k) | Discount factor |
| $W$ | Maximum inventory level |

***Assumptions***

1. Demand rate is known and constant.
2. In non coordination shortages are occurring and in coordination no shortages are occurring for buyer.
3. In non coordination vendor screened the damaged items and in coordination buyer screened the damaged items for resale. Also the vendor offers quantity discount for bulk purchase.
4. Integrated system cost is formulated for both situations.
5. The lot size $Q$ satisfies the inventory level constraint. Mathematically, the constraint will be taken as $\frac{Q}{2} \leq W$ where $W$ denotes maximum available inventory.

## III. Model Formulation

In this section, integrated system cost is determined for coordination and non coordination. In non coordination situation, vendor screened the damaged items and buyer having shortages. In coordination situation, vendor offers quantity discount to the buyer and buyer having no shortage and he himself screened the damaged items. In this situation buyer order size is larger than regular quantity. In addition, inventory level constrain is also satisfies for both situations.

***Case i: Integrated System Cost Model for Non Coordination***

The total cost for buyer contains ordering cost, holding cost and shortage cost.

$$\text{i.e., } TC_b = \frac{R_1 D}{Q} + \frac{H_b Q_1^2}{2Q} + \frac{s(Q-Q_1)^2}{2Q}$$

The total cost for vendor contains setup cost, holding cost and the screening cost.

$$\text{i.e., } TC_v = \frac{R_2 D}{nQ} + \frac{H_v nQ}{2} + \frac{s_c nQ}{2}$$

Now, the integrated system cost is written as

$$TC_s = TC_b + TC_v$$

$$= \frac{R_1 D}{Q} + \frac{H_b Q_1^2}{2Q} + \frac{s(Q-Q_1)^2}{2Q} + \frac{R_2 D}{nQ} + \frac{H_v nQ}{2} + \frac{s_c nQ}{2} \quad (1)$$

$$\text{s.t } \frac{Q}{2} \leq W$$

Here, we consider the buyer’s inventory level constraint. Now, Lagrange multiplier function , $0 \leq \lambda, \leq 1$ is added on system cost and it can be written as follows:

$$TC_s = TC_b + TC_v + \lambda\left(\frac{Q}{2} - W\right)$$

$$= \frac{R_1 D}{Q} + \frac{H_b Q_1^2}{2Q} + \frac{s(Q-Q_1)^2}{2Q} + \frac{R_2 D}{nQ} + \frac{H_v nQ}{2} + \frac{s_c nQ}{2} + \lambda\left(\frac{Q}{2} - W\right) \quad (2)$$

Equation (2) will be written as

$$TC_s = \left(\frac{H_b + s}{2Q}\right) Q_1^2 - sQ_1 + \frac{R_1 D}{Q} + \frac{R_2 D}{nQ} + \frac{H_v nQ}{2} + \frac{s_c nQ}{2} + \frac{sQ}{2} + \lambda\left(\frac{Q}{2} - W\right)$$

It is of the form $a_1 Q_1^2 + a_2 Q_1 + a_3$.

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B will be taken as, $Q_1 = \frac{-a_2}{2a_1}$ (Cardenas Barron (2011))

$$\text{Now, } Q_1^* = \frac{sQ}{H_b+s} \qquad (3)$$

Also, equation (2) will be written as

$$TC_s = \{\frac{sH_b+n(H_v+s_c)(H_b+s)}{2(H_b+s)} + \frac{\lambda}{2}\} Q + D\,(R_1 + \frac{R_2}{n})\frac{1}{Q} + -\lambda W$$

It is of the form $a_1 Q + \frac{a_2}{Q} + a_3$.

$Q$ will be taken as, $Q = \sqrt{\frac{a_2}{a_1}}$ (Cardenas Barron (2011))

$$\text{Now, } Q^* = \sqrt{\frac{2D(H_b+s)(R_1+\frac{R_2}{n})}{sH_b+(H_b+s)[n(H_v+s_c)+\lambda]}} \qquad (4)$$

$$\text{where } \lambda = \frac{D(H_b+s)\left(R_1+\frac{R_2}{n}\right)-2W^2\{sH_b+n(H_v+s_c)(H_b+s)\}}{2W^2(H_b+s)}$$

***Case ii: Integrated System Cost Model for Coordination***

The total cost for buyer contains, ordering cost, holding cost and screening cost.

$$\text{i.e., } TC_{b1} = \frac{R_1 D}{Q_c} + \frac{H_b Q_c}{2} + \frac{s_c Q_c}{2}$$

The total cost for vendor contains, setup cost, holding cost and buyer's quantity discount.

$$\text{i.e., } TC_{v1} = \frac{R_2 D}{kmQ_c} + \frac{H_v kmQ_c}{2} + pDd(k)$$

Now, the integrated system cost is written as

$$TC_{s1} = TC_{b1} + TC_{v1}$$
$$= \frac{R_1 D}{Q_c} + \frac{H_b Q_c}{2} + \frac{s_c Q_c}{2} + \frac{R_2 D}{kmQ_c} + \frac{H_v kmQ_c}{2} + pDd(k) \qquad (5)$$
$$\text{s.t } \frac{Q_c}{2} \le W$$

Here, we consider the buyer's inventory level constraint. Now, Lagrange multiplier function , $0 \le \lambda \le 1$ is added on system cost and it can be written as follows:

$$TC_{s1} = TC_{b1} + TC_{v1} + \lambda\left(\frac{Q}{2} - W\right)$$
$$= \frac{R_1 D}{Q_c} + \frac{H_b Q_c}{2} + \frac{s_c Q_c}{2} + \frac{R_2 D}{kmQ_c} + \frac{H_v kmQ_c}{2} + pDd(k) + \lambda\,(\frac{Q}{2} - W) \qquad (6)$$

Also, equation (6) will be written as

$$TC_{s1} = \{\frac{H_b+s_c+kmH_v+\lambda}{2}\} Q_c + D\,(R_1 + \frac{R_2}{km})\frac{1}{Q_c} + pDd(k) - \lambda W$$

It is of the form $a_1 Q_c + \frac{a_2}{Q_c} + a_3$.

$Q_c$ will be taken as, $Q_c = \sqrt{\frac{a_2}{a_1}}$ (Cardenas Barron (2011))

$$\text{Now, } Q_c^* = \sqrt{\frac{2D(R_1+\frac{R_2}{km})}{H_b+s_c+kmH_v+\lambda}} \qquad (7)$$

$$\text{where } \lambda = \frac{D\left(R_1+\frac{R_2}{km}\right)-2W^2\{H_b+s_c+kmH_v\}}{2W^2}$$

## IV. Numerical Examples

***Example 1***

Let $R_1$ = 500 per order, $R_2$ = 800 per order, D = 3000 units per year, $H_v$ = 0.04\$, $H_b$ = 0.025\$, $s_c$ = 0.02\$, s = 0.025\$, p = 0.5\$, n = 2, m = 3, k= 4, d(k) = 10%, W = 400.

The optimal solutions are

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$Q^* = 800, Q_1^* = 400, TC_s(Q^*, Q_1^*) = 3.4280\ X\ 10^3, Q_c^* = 800, TC_{s1}(Q_c^*) = 2.6350\ X\ 10^3$

***Example 2***

Let $R_1 = 600$ per order, $R_2 = 1000$ per order, D = 2000 units per year, $H_v = 0.04\$$, $H_b = 0.025\$$, $s_c = 0.2\$$, s = 0.25$, p = 0.2$, n = 3, m = 4, k= 3, d(k) = 20%, W = 800.

The optimal solutions are

$Q^* = 1600, Q_1^* = 1454.5, TC_s(Q^*, Q_1^*) = 1.7608\ X\ 10^3, Q_c^* = 1600, TC_{s1}(Q_c^*) = 1.4982\ X\ 10^3$

## V. Conclusion

Integrated inventory model for buyer – vendor under two part coordination is considered for this study. For system optimization integrated system cost is developed and it also satisfies the inventory level constraint. Analytical geometry and algebraic method is used to find optimal values. Also, Lagrange's multiplier technique is used to solve this type of problems. The model revels that coordination situation is the best situation to minimize the integrated system cost. To illustrate the developed model, numerical examples are also given. For the further researches, the model can be extended in credit period, temporary discount, prize discount etc.,

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