

# An EPQ model for deteriorating products with budget constraints

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**Abstract** - This paper investigates manufacturer and buyer production model for deteriorating products with budget constraint. Lagrange's multiplier technique is used to solve this type of problem. Integrated system cost is developed for equal benefits of both buyer and manufacturer. In order to reduce the total inventory cost, optimal order quantity and backorder levels are determined and also budget constraint should be satisfied. Numerical example is also providing to reveal the model.

**Keywords:** Production, Inventory, Order quantity, Budget constraint.

## 1. INTRODUCTION

Inventory is the crucial procedure that requires careful decision to become a production. The decision becomes very critical when the products are fruits, vegetables, pharmaceuticals, volatile liquids (deteriorating products) etc.,

P. Muniappan et al. [6] found out integrated inventory model for multi-echelon multi-constraints with quantity discount and coordination supply chain by using Lagrange multiplier technique. M. Babu et al. [1] developed centralized production inventory model for Buyer – Vendor with quantity discount for fixed life time products. M. Ravithammal et al. [8] deliberated inventory model for price discount with shortage, back ordering and rework. M. Ravithammal et al. [9] developed an optimal pricing inventory model for deteriorating items with positive exponential function of price discount rate of demand. S. Hemamalini et al. [3] developed EOQ inventory model for buyer-vendor with screening, disposed cost and controllable lead time. A. Mari Selvi et al. [4] concentrated vendor-buyer integrated inventory model for controllable lead time with screening and disposed cost.

S. S. Sana [10] investigated an economic production lot size model in an imperfect production system. A. Sugirtha et al. [11] considered Buyer – Vendor inventory model for deteriorating items involving back orders, screening process and transportation cost. C. H. Chen et al. [2] considered modified economic production and raw material model with quality loss for conforming product. A. Mukhopadhyay and A. Goswami [5] developed economic production quantity model for three type imperfect items with rework and learning in setup. M. Ravithammal et al. [7] developed EOQ inventory model using algebraic method with inventory level constraint. M. K. VEDIAPPAN et al. [12] studied integrated coordination inventory model for Buyer – Vendor using Lagrange multiplier technique.

## 2. ASSUMPTIONS AND NOTATIONS

The model use the following notations and assumptions

### Notations

D	Demand rate
P	Production rate
R <sub>1</sub>	Buyer's unit ordering cost
R <sub>2</sub>	Manufacturer unit setup cost
s	Buyer's unit shortage cost
p	Buyer's unit purchase cost

$Q$	Economic Order quantity
$Q_1$	Backorders level
$H_b$	Buyer's unit holding cost
$H_v$	Manufacturer unit holding cost
$s_c$	Manufacturer unit screening cost
$n$	Manufacturer multiples of order for non-coordination
$W$	Maximum available inventory

### Assumptions

- (i) Demand is considered as a constant.
- (ii) Shortages are allowed for buyer and manufacturer screened the damage products.
- (iii) System cost contains both buyer and manufacturer cost and also satisfies the budget constraint. Mathematically, the constraint will be written as  $pQ \leq W$ .

### 3. MODEL FORMULATION

Total cost for buyer, manufacturer and integrated system will write as follows:

The total cost for buyer contains, ordering cost  $\frac{R_1 D}{Q}$ , holding cost  $\frac{H_b Q_1^2}{2Q}$  and shortage cost  $\frac{s(Q-Q_1)^2}{2Q}$  and it can be written as

$$TC_b = \frac{R_1 D}{Q} + \frac{H_b Q_1^2}{2Q} + \frac{s(Q-Q_1)^2}{2Q}$$

The total cost for manufacturer contains the setup cost  $\frac{R_2 D}{nQ}$ , the holding cost  $\frac{H_v nQ}{2} \left(1 - \frac{D}{P}\right)$  and the screening cost  $\frac{s_c nQ}{2}$  and it can be written as

$$TC_m = \frac{R_2 D}{nQ} + \frac{H_v nQ}{2} \left(1 - \frac{D}{P}\right) + \frac{s_c nQ}{2}$$

Integrated system cost can be written as

$$TC_s = TC_b + TC_m$$

$$\text{i.e., } TC_s = \frac{D}{Q} \left(R_1 + \frac{R_2}{n}\right) + \frac{H_b Q_1^2}{2Q} + \frac{s(Q-Q_1)^2}{2Q} + \frac{H_v nQ}{2} \left(1 - \frac{D}{P}\right) + \frac{s_c nQ}{2} \quad (1)$$

subject to the constrain  $pQ \leq W$ .

Here, we consider the buyer's budget constraint. Now, Lagrange multiplier function  $\mu$ ,  $0 \leq \mu \leq 1$  is added on system cost can be written as follows:

$$TC_s = TC_b + TC_m + \mu(pQ - W)$$

$$TC_s = \frac{D}{Q} \left(R_1 + \frac{R_2}{n}\right) + \frac{H_b Q_1^2}{2Q} + \frac{s(Q-Q_1)^2}{2Q} + \frac{H_v nQ}{2} \left(1 - \frac{D}{P}\right) + \frac{s_c nQ}{2} + \mu pQ - \mu W \quad (2)$$

Equation (2) will be written as

$$TC_s = \left(\frac{H_b + s}{2Q}\right) Q_1^2 - sQ_1 + \frac{D}{Q} \left(R_1 + \frac{R_2}{n}\right) + \frac{sQ}{2} + \frac{H_v nQ}{2} \left(1 - \frac{D}{P}\right) + \frac{s_c nQ}{2} + \mu pQ - \mu W$$

It is of the form  $a_1 Q_1^2 + a_2 Q_1 + a_3$ .

$$Q_1 \text{ will be taken as, } Q_1 = \frac{-a_2}{2a_1}$$

$$\text{Now, } Q_1^* = \frac{sQ}{H_b + s} \quad (3)$$

Equation (2) will be written as

$$TC_s = \left\{ \frac{H_b + n \left[ H_v \left( 1 - \frac{D}{P} \right) + s_c \right] (s + H_b) + 2\mu p (s + H_b)}{2(s + H_b)} \right\} Q + \left\{ R_1 D + \frac{R_2 D}{n} \right\} \frac{1}{Q} - \mu W$$

It is of the form of  $a_1 Q + \frac{a_2}{Q} + a_3$ .

$$Q \text{ will be taken as, } Q = \sqrt{\frac{a_2}{a_1}}$$

$$\text{Now, } Q^* = \sqrt{\frac{2D \left( R_1 + \frac{R_2}{n} \right) (H_b + s)}{sH_b + \left( nH_v \left( 1 - \frac{D}{P} \right) + ns_c \right) (H_b + s) + 2\mu p (H_b + s)}} \quad (4)$$

$$\text{Where } \mu = \frac{2Dp^2 \left( R_1 + \frac{R_2}{n} \right) (H_b + s) - W \left\{ sH_b + n \left( H_v \left( 1 - \frac{D}{P} \right) + s_c \right) (s + H_b) \right\}}{2W^2 p (s + H_b)}$$

#### 4. NUMERICAL EXAMPLE

Let  $R_1 = 300$  per order,  $R_2 = 800$  per order,  $D = 1000$  units per year,  $P = 1500$ ,  $H_v = 0.04$ ,  $H_b = 0.015$ ,  $s_c = 0.01$ ,  $s = 0.025$ ,  $p = 1.5$ ,  $n = 3$ ,  $W = 1000$ .

The optimal solution is

$$Q^* = 561.66, Q_1^* = 351.03, TC_s = 1.0419 \times 10^3 \text{ satisfies the constraint } pQ \leq 1000$$

#### 5. CONCLUSION

This model is developed for deteriorating products of buyer – manufacturer with shortage for buyer and screening cost for manufacturer. Integrated system cost is developed for system optimization. Also the model satisfies budget level constraint. This type of problems is solved by using Lagrange's multiplier Technique. Numerical example is also given to illustrate the developed model. Further the proposed model can be extended to consider Quantity discount, inventory level constraint, floor constraint etc.,

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