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Some results in the study of compact mean labeling

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Abstract

Graph labeling is an essential concept that aims to provide a unique illustration for labeled graphs so that complex networks are easier to grasp for a real-time application. This study deals with theoretical backgrounds for a highly compact mean labeling; where under this approach, vertices are assigned labels from 0 to 2q – p. This approach is applicable for showcasing the potency of compact mean labeling over different categories of graphs, including complete, bipartite, snake, cycle, and wheel graphs. By using this technique, there is more enhanced visual access to the structure highlighted by graphs, hence allowing useful contributions to their properties and applications for solving problems in different areas. This shows the main importance of appropriate labeling in addressing real-life problems.

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1. Introduction

Graph theory has always played a vital role in discrete mathematics, providing practical tools for studying and understanding the object relationships. One of its strongest ideas is graph labeling, which uses symbols or integers to identify a graph's edges, vertices or both, based on specific criteria. Because the approach is theoretically attractive and has many applications in domains like data organization, scheduling, cryptography, and communication networks, it has been extensively studied [1].

Rosa [2] first put forward the concept of graph labeling in 1967 while he was working on the Kotzig-Ringel conjecture. Since then, several labeling techniques have been contributed to use, each of which could be used for a particular kind of graph or issue. These techniques include cordial, mean, magic, harmonious, graceful, and cordial labeling. Odd vertex and even edge root square mean labelling graphs is one of these that has drawn interest due to its unique labeling technique that requires vertex or edge labels to satisfy specific criteria [3]. Maheswari, Ramesh, and Balaji [4], [5], [6] defend the results of skolem and relaxed mean labeling. Shruthy and Maheswari [7] investigates double encryption and decryption process for enhanced vigenere cipher using graph labeling in the paper. Deepa, Maheswari, and Balaji [8] used graph labeling technique to create ciphertext and then decipher it. Possibilities of repeating edges using double mean labeling is studied by Nandhini, Maheswari, and Balaji [9]. A new method of secure encoding for graph based cryptographic applications, difference divisor labeling has been explored by Janaki, Maheswari, and Balaji [10].

Compact Mean Labeling is an upcoming study in this area that generalizes standard mean labeling by efficiently and compactly labeling graph elements with a series pattern of integers. In regular graph classes such as cycles, complete graphs, and bipartite graphs, this type of labeling is most beneficial Because of its structure, it can be used for analysis as well as theoretical research [11].

2. Preliminaries:

Definition 2.1: Relaxed Mean Labeling [12]: "If there exists a function Γ from the vertex set of G to $\{0, 1, 2, 3, \dots, q-1, q+1\}$ and the vertex label q relax such that the given form points to Γ^* from the edge, then the graph $G = (V, E)$ to $\{1, 2, 3, \dots, q\}$ with p vertices and q edges is said to be a relaxed mean graph.

The edge set is defined by $\Gamma^*(\mathrm{e}=\mathrm{uv}) = \begin{cases} \frac{\Gamma(\mathrm{u})+\Gamma(\mathrm{v})}{2} & \text{if } \Gamma(\mathrm{u})+\Gamma(\mathrm{v}) \text{ is even} \\ \frac{\Gamma(\mathrm{u})+\Gamma(v)+1}{2} & \text{if } \Gamma(u)+\Gamma(\mathrm{v}) \text{ is odd} \end{cases}$

the resulting labels are distinct."

Definition 2.2:Compact Mean Labeling [13]: Consider a graph G with p vertices and q edges. A labeling Γof the vertices of G is called a compact mean labeling if

$$\Gamma\colon \mathrm{V(G)} \to \text{of } \{0, 1, 2, \dots .2q-p\}$$

Assign distinct integers to the vertices and for every edge uvϵ $E(G)$, the edge is labeled by

$$\Gamma(uv) = \begin{cases} \frac{\Gamma(u)+\Gamma(v)}{2}; \text{ if } \Gamma(u)+\Gamma(v) \text{ is even} \\ \frac{\Gamma(u)+\Gamma(v)+1}{2}; \text{ if } \Gamma(u)+\Gamma(v) \text{ is odd} \end{cases},$$

Γ of a graph G is said to have a Compact Mean Labeling (CML) when the resulting edge label is exactly $\{1, 2, \ldots . q\}$.

3. Main Results:

Theorem 3.1: *A graph $\boldsymbol{K_n}$ is said to be compact mean labeled graph for $n = 3$.*

Proof: Let K_n be a complete graph with distinct vertices u_1, u_2, u_3 and edges e_1, e_2, e_3. Define a mapping $\Gamma: V \ (K_n) \rightarrow \{0, 1, 2, \ldots, 2q - p\}$

by $$\Gamma(u_i) = \begin{cases} i - 1 \text{ for } i = 1, 3 \text{ (odd)} \\ 2i - 1 \text{ for } i = 2 \text{ (even)} \end{cases}$$

The vertex labels are,

When $i = 1, f\ (u_1) = 0$; $i = 2, f\ (u_2) = 3$; $i = 3, f\ (u_3) = 2$,

The label of the edge $u_1u_3 = 1$ and the label of the edge $u_1u_2 = 2$.

The label of the edge $u_2u_3 = 3$ and the label of the edge $u_{n-2}u_n = 1$.

The label of the edge $u_{n-2}u_{n-1} = 2$ and the label of the edge $u_{n-1}u_n = 3$.

All the edge values are distinct. Hence K_3 is a complete graph [13].

Theorem 3.2: *A graph K_n admits compact mean labeling if $n = 4$.*

Proof: Let K_4 denote the complete graph. Its vertex is denoted as $V(K_4)=\{u_1, u_2, u_3, u_4\}$ and edges set as $E(K_4) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.

Define mapping $\Gamma: V \ (K_n) \rightarrow \{0, 1, 2, \ldots, 2q - p\}$

Such that $\Gamma(u_i) = \begin{cases} 2i - 2 \ \forall\, i = 1, 2, 3 \\ i + 4 \text{ when } i = 4 \end{cases}$

From the mapping $\Gamma: V \ (K_n) \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

The vertex label is defined as

When $i = 1$, $\Gamma(u_1) = 0$,

$i = 2$, $\Gamma(u_2) = 2$; $i = 3$, $\Gamma(u_3) = 4$; $i = 4$, $\Gamma(u_4) = 8$.

The induced edge labels are,

$u_{n-3}u_{n-2} = u_1u_2 = 1$; $u_{n-3}u_{n-1} = u_1u_3 = 2$

$u_{n-2}u_4 = u_2u_4 = 5$; $u_{n-3}u_n = u_1u_4 = 4$

$u_{n-2}u_{n-1} = u_2u_3 = 3$; $u_{n-1}u_4 = u_3u_4 = 6$.

∴ With this labeling, the resulting edges labels are distinct and form the set $\{1, 2, \ldots q\}$.

∴ The complete graph K_4 admits compact mean labeling [13].

Theorem 3.3: *The complete bipartite graph $\boldsymbol{K_{2,2}}$ possesses a compact mean labeling.*

Proof: Consider the bipartition of $K_{2,2}$ given by

$$\nu_1 = \{u, \nu\}, \nu_2 = \{u_1, u_2\}.$$

Define a labeling function

$$\Gamma: V(K_{2,2}) \rightarrow \{0,1,2,3,4\}$$

by

$$\Gamma(u_i) = \begin{cases} 2i - 2, i = 1,2,3, \\ i - 1, i = 4. \end{cases}$$

By the mapping the vertex of $K_{2,2}$ receive distinct labels from the set $\{0,1, \ldots, 4\}$.

When $i = 1$ $\Gamma(u_1) = 2i - 2 = 0$
$i = 2$ $\Gamma(u_2) = 2i - 2 = 2$
$i = 3$ $\Gamma(u_3) = 2i - 2 = 4$
$i = 4$ $\Gamma(u_4) = i - 1 = 3$
The following are the corresponding edges

$$u_1 u_2 = 1\ (ie)\ 1 \leq i \leq 3$$

$u_{n-3} u_{n-2} = 1; u_{n-3} u_n = 2; u_{n-2} u_{n-1} = 3; u_{n-1} u_n = 4$

∴ Consequently, all the edge labels are distinct and form the set $\{1, 2, \ldots q\}$.

∴ Therefore, the graph $K_{2,2}$ admits a compact mean labeling [13].

Theorem 3.4: *If* $\boldsymbol{m = 2}$ *and* $\boldsymbol{n = 3}$ *then the graph* $\boldsymbol{K_{2,3}}$ *is CML.*

Proof: Consider ν_1, ν_2 as the partition of $K_{2,3}$ where $\nu_1 = \{u, \nu\}$, $\nu_2 = \{u_1, u_2, u_3\}$

The labeling function can be defined as $\Gamma: V(K_{2,3}) \rightarrow \{0,1,2, \ldots, 2q - p\}$

$$\Gamma: V(K_{2,3}) \rightarrow \{0,1,2,3,4,5,6,7\}$$

$$\text{If } \Gamma(u) = 0, \Gamma(\nu) = n + 3$$

The label of vertex ν_2 is

$$\Gamma(u_i) = \{2i\} \text{ for } 1 \leq i \leq n - 2.$$

Then values of the vertices u_1, u_2, u_3 are assigned as follows:
When $i = 1$, we get $\Gamma(u_1) = 2$
if $i = 2$ we get $\Gamma(u_2) = 4$
if $i = 3$ we get $\Gamma(u_3) = 6$
The following are the corresponding edge labels
Let's consider uu_1 as 1

$$u\nu_{i+1} \text{ is } i + 1 \text{ when } i = 1, \ldots n - 1$$

$$\nu\nu_1 = 4, \nu\nu_2 = 5, \nu\nu_3 = 6$$

Hence $K_{2,3}$ is a compact mean labeled graph.

Theorem 3.5: *If* $\boldsymbol{m = 2, n = 4}$ *then the graph* $\boldsymbol{K_{m,n}}$ *is compact mean labeled.*

Proof: Consider V_1, V_2 be the partition of $K_{\mathrm{m,n}}$ if $\mathrm{m} = 2, \mathrm{n} = 4$, where $V_1 = \{u, \mathrm{v}\}, V_2 = \{u_1, u_2, u_3, u_4\}$

The labeling function can be defined as $\Gamma: V(K_{2,4}) \rightarrow \{0,1,2, \dots..2q - p\}$

$$\Gamma: V(K_{2,4}) \rightarrow \{0,1,2,3,4,5,6,7,8,9,10\}$$

Let $\Gamma(u) = 0$; $\Gamma(\mathrm{v}) = 3n + 2$

The vertex labeling of each vertex of u_i is

$$\Gamma(u_i) = 2i - 1 \text{ for } 1 \leq \mathrm{i} \leq \mathrm{n}.$$

For instance, when i = 1, the assigned label is $\Gamma(u_1) = 2(1) - 1 = 1$.

$$\Gamma(u_2) = 3, \Gamma(u_3) = 5, \Gamma(u_4) = 7 \text{ (or)}$$

$$\Gamma(u_i) = 1, \Gamma(u_{i+1}) = 3, \Gamma(u_{i+2}) = 5, \Gamma(u_{i+3}) = 7$$

The following are the corresponding edge labels

$$\mathrm{v}u_1 = n + 1$$
$$\mathrm{v}u_2 = n + 2$$
$$\mathrm{v}u_3 = n + 3$$
$$\mathrm{v}u_4 = n + 4$$

$\mathrm{v}u_i = n + i$, where i = 1, ... n

$$u_{n-5}u_{n-8} = u_5u_2 = 8$$
$$u_{n-7}u_{n-5} = u_3u_5 = 9$$
$$u_{n-6}u_{n-5} = u_4u_5 = 10$$

Every edge value isa distinct value from the set $\{1, 2, 3, 4, \dots 8\}$.

∴ The compact mean labeling graph is then admitted by the $K_{2,4}$.

Theorem 3.6: *Consider the cycle graph* $\boldsymbol{C_n = u_1u_2 \dots u_nu_1}$. *Define a graph G with* $\boldsymbol{V(G) = V(C_n) \cup \{w_1, w_2, \dots, w_n\}}$, $\boldsymbol{E}$ *(G)* = $\boldsymbol{E(C_n) \cup u_iw_i\{u_iw_i, u_{i+1}w_i: 1 \leq i \leq n\}}$. *Then G admits a CML. graph.*

Proof: By defining $\Gamma: \mathrm{V(C_n)} \rightarrow \{0, 1, 2, \dots, 2\mathrm{q}\text{-}\mathrm{p}\}$

$$\Gamma(u_1) = 0, \Gamma(u_2) = 2, \Gamma(u_3) = 1, \Gamma(u_\mathrm{n}) = \mathrm{n} + 2.$$

If n is odd, then for the remaining vertices we set

$\Gamma(u_\mathrm{i}) = \mathrm{i} + 1$ whenever I range from $\frac{\mathrm{n}+1}{2}$ up to $\mathrm{n} - 1$,

$\Gamma(u_\mathrm{j}) = \mathrm{j} + 1$ for every vertex $\mathrm{j} \neq \mathrm{i}$.

If n is even, then the labeling is adjusted accordingly,

The labeling function can be defined as $\Gamma: \mathrm{V(C_n)} \rightarrow \{0, 1, 2, \dots, 2\mathrm{q}\text{-}\mathrm{p}\}$

$$\Gamma(u_1) = 0; \Gamma(u_2) = 2;$$
$$\Gamma(u_3) = 1; \Gamma(u_\mathrm{n}) = \mathrm{n} + 2.$$

$\Gamma(u_\mathrm{i}) = \mathrm{i} + 1$ whenever i ranges from $\frac{\mathrm{n}}{2} \leq i \leq \mathrm{n} - 1$.

$\Gamma(u_\mathrm{j}) = \mathrm{j} + 1$ such that $\mathrm{i} \neq \mathrm{j}$.

Hence $\mathrm{C_n}$ is a CML graph.

Theorem 3.7: *The graph Quadrilateral snake Q_n is a compact mean labeled graph.*

Proof: Consider the quadrilateral snake Q_n.

The labeling function can be defined as $\Gamma: V(Q_n) \to \{0, 1, 2, \dots, 2q - p\}$

Assign the vertex labels as follows:

For vertices u_i where $i = 1,2, \dots n - 1$, let $\Gamma(u_i) = 4(i - 1)$

For vertices v_i where $i = 1, 2, \dots, n - 2$, let $\Gamma(v_i) = 2(2i - 1)$

Let $\Gamma(v_{n-1}) = 4n - 7$

For vertices w_i where $i = 1, 2, \dots, n - 2$, let $\Gamma(w_i) = 4i - 1$ and $\Gamma(w_{n-1}) = 2(2n - 3)$.

The Corresponding edge labels are:

$$\Gamma(u_{i-1}u_i) = 4i - 6 \ (1 \le i \le n - 1), \Gamma(u_{n-1}u_n) = 4n - 5,$$

$$\Gamma(u_i v_i) = 4i - 3 \ (1 \le i \le n - 2), \Gamma(u_{n-1}v_{n-1}) = 4n - 7$$

$$\Gamma(u_{i+1}w_i) = 4i \ (1 \le i \le n - 2), \Gamma(u_n w_{n-1}) = 4(n-1)$$

$$\Gamma(u_{i+1}w_i) = 4i \ (1 \le i \le n - 2), \Gamma(u_n w_{n-1}) = 2(2n - 3)$$

$$\Gamma(v_i w_i) = 4i - 1 \ (1 \le i \le n - 2), \Gamma(v_{n-1}w_{n-1}) = 4n - 6$$

Hence the quadrilateral snake Q_n satisfy the condition of CML graph.

Theorem 3.8: *$L_n \Theta K_1$ is a graph which is compact mean label.*

Proof: Consider $V(L_n) = \{a_i, b_i: 1 \le i \le n\}$,

$E(L_n) = \{a_i b_i: 1 \le i \le n - 1\} \cup \{a_i a_{i+1}: 1 \le i \le n - 1\} \cup \{b_i b_{i+1}: 1 \le i \le n - 1\}$.

Let a_i be joined to a pendent vertex denoted by C_i and let b_i be joined to a pendent vertex denoted by d_i.

The labeling function can be defined as $\Gamma: V(L_n \Theta K_1) \to \{0,1, \dots 2q - p\}$

$$\Gamma(a_i) = 5i - 4 \text{ where } i = 1, \dots, n;$$

$$\Gamma(b_i) = 5i - 3 \text{ where } i = 1, \dots n;$$

$$\Gamma(c_i) = 5i - 5 \text{ where } i = 1, \dots n;$$

$$\Gamma(d_i) = 5i - 2 \text{ where } i = 1, \dots n - 1 \text{ and}$$

$$\Gamma(d_n) = 5n - 1.$$

The edge labels of $c_i a_i$ is $5i - 4$ where $i = 1, \dots, n$; for $a_i b_i = 5i - 3$ where $i = 1, \dots, n$; $b_i d_i = 5i - 2$ where $i = 1, \dots n$; $a_i a_{i+1} = 5i - 1$ where $i = 1, \dots n - 1$ and $b_i b_{i+1} = 5i$ where $i = 1, \dots n - 1$.

∴ Hence, the graph $L_n \Theta K_1$ satisfies the conditions for compact mean labeling.

Theorem 3.9: *For all n then $K_n^c + 2K_2$ graph is said to be compact mean labeled.*

Proof: Consider $V(K_n) = \{u_1, u_2, \dots, u_n\}$.

Let $2K_2$ have $V(2K_2) = \{u, v, w, z\}$ and $E(2K_2) = \{uv, wz\}$.
Consider the graph G= $K_n^c + 2K_n$
Define

$$\Gamma: V(G) \to \{0,1, \dots 2q - p\}$$

By $\Gamma(u) = 2, \Gamma(v) = 0, \Gamma(w) = 4n + 3, \Gamma(z) = 4n, \Gamma(u_i) = 4i - 1$ where $1 \leq i \leq n$.

Under compact mean labeling rule, the edge labels satisfy
$\Gamma(uv) = 1$and $\Gamma(wz) = 4n + 2$ where $i = 1, \dots n$.

The edge labels are: $uu_i = 2i + 1; vu_i = 2i; wu_i = 2(n + i) + 1$ and $zu_i = 2(n + i)$ in which $i = 1, \dots n$.

Hence $K_n^c + 2K_n$ is a compact mean labeled graph.

Theorem 3.10: *If* $\boldsymbol{n = 4}$ *then* $\boldsymbol{W_n}$ *is a compact mean labeled graph.*

Proof: Consider $W_4 = C_4 + K_1$, whereas C_4 is the cycle $u_1u_2u_3u_4u_1$ and $V(K_1) = \{u\}$.

By defining, $\Gamma: V(W_n) \to \{0,1, \dots 2q - p\}$.

Then $\Gamma(u) = 6$; $\Gamma(u_i) = 2(i - 1)$ where $i = 1, \dots n - 1$ and $\Gamma(u_i) = 2i + 1$ where $i = n$.

The corresponding edge label is {1,2,3,4}

Hence, W_4 is a compact mean labeled graph.

4. Graph Labeling in News and Media Analysis

Graph labeling is a highly specialized technique and it aids in the detection of fake news as well. It is common for fake news to be shared on social networks and other websites and the activities there can be viewed in terms of a network. In this case vertices represent users, articles or sources while edges represent sharing, commentary or hyper linking. All these activities can be interspersed with fake activities such as sharing and labeling., Hence the tying of a label to these vertices and edges allows the study of patterns and activities of people who engage in or come across the fake news. One of the key benefits of graph labeling is the ability to uncover the propagation patterns of said news. Far from trustworthy information, fake news usually has a more 'unorthodox' way of dissemination, in such a way it is spread or shared with no verification at all, making it viral in minutes or within a few hours. With the aid of subject nodes (spreader, for example) and all their interlinks or connections, we are able to pinpoint peculiar trends in how information is shared. These unusual patterns can also suggest that the information being shared is fake news. Another benefit is that the nodes can be classified or sorted out, like a news article or a user could be labeled/found out to be from a fake or a real source. Supervised graphs help in training the machine learning model, bringing about more accuracy and determinacy even on nodes that are not tagged. Further, the clusters can also help in identifying users or even groups that are prone to sharing fake news.

Advanced techniques such as Graph Neural Networks (GNNs) use labeled graphs as input to learn network structure and predict the occurrence of fake news more precisely. These systems also use graph labeling toward distinguishing between rumor and the actual news items, following content and diffusion patterns.

For example, let’s take two news sources: Website X and Website Y. Website X has been a known spreader of misinformation and is labeled as “non-credible”, while Website Y is verified and labeled as “Credible”. A news article, Article A, published by Website X, is labeled as “Fake News”, whereas Article B from Website Y is labeled as “Reliable”. Users interacting with these articles also receive labels based on their behaviour. For example, if Jason often reposts stuff from Website X, he would then be categorized as a “Spreader” and if Carol usually reposts fact-checking website links, then her classification would be that of a “Fact-checker”.

The graph displays a pattern of rapid spreading, specifically involving a particular cluster of users in the case of Article A. Those users are categorized as being part of a “High-Risk Community” for spreading misinformation. Article B, however, is a slower, more measured spread in which many users link the article to sources such as Website Y, which have a verified existence.

With this labeled graph, the machine learning algorithm studies the patterns of relationships and propagation. It sees that Article A originates from a “Non-credible” source and propagates rapidly among a network of “Spreaders”. In contrast, Article B does not spread since it only disseminates to credible nodes and fact-checkers. Therefore, based on these observations, the algorithm concludes that Article A is likely fake news.

With this understanding, the platform acts. It disallows the sharing of Article A and informs users of the suspicious nature of the post. It also provides sources for fact-checking for the education of users to know about the misinformation. This example demonstrates how graph labeling allows for a structured approach to fake news detection and mitigation. It does this by analyzing relationships and propagation behaviours to establish differences between reliable and unreliable information.

In summary, graph labeling improves fake news detection with a structured approach to the study of relationships, credibility, and propagation behaviors within a network. The method enables pinpointing of misinformation in a more efficient manner to support the fight against fake news in digital environments.

5. Conclusion

The concept of compact mean labeling, which expands the familiar framework of mean labeling to a higher class of standard graphs, has been widely studied in this paper. Compact mean labeling can be used effectively for a varies types of graph types, including complete graphs, cycle graphs, bipartite graphs, and many more, as the study shows. This study shows a significant advancement in the study of graph labeling, which showcase the theoretical structure that show the labeling technique’s adaptability and tenacity. Graph labeling is a highly specialized technique and it helps in the identification of fake news.

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