Economic production model for manufacturer – buyer with quantity discount and inventory level constraint

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Abstract - This paper focuses on production inventory model with quantity discount, screening processes and shortages. The inventory model investigates the manufacturer - buyer comfort of supply chain developed under coordinate situations. In addition to the model satisfies the inventory level constraint and Lagrange multiplier technique is used to solve this type of difficulties. Numerical examples are included in detail to develop and understand the proposed method.

Keywords: Production, Inventory, Lagrange's Techniques, Shortages, Quantity discount.

I. INTRODUCTION

In industries, the demand is almost not predictable and suitable. In particular, ordering goods should be in time and in correct quantity. Production is the raw material formed from the process of finished products. But manufacturing is the one of major functions of all kind of business involves in production. The total inventory cost is to minimize in unique intention of traditional inventory models.

S. Hemamalini et al. [4] proposed EOQ inventory model for buyer-vendor with screening disposed cost and controllable lead time. A. Mari Selvi et al. [7] obtained Vendor-Buyer integrated inventory model for controllable lead time with screening and disposed cost. M. Babu et al. [1] found out integrated EPQ shortage inventory model with quantity discount. M. Ravithammal et al. [9] discussed about EOQ inventory model using algebraic method with inventory level constraint. M. K. Vediappan et al. [11] investigated integrated coordination inventory model for buyer – vendor using Lagrange multiplier technique. A. Khanna et al. [6] developed strategic production modeling for defective items with imperfect inspection process, rework, and sales return under two-level trade credit. L.F. Hsu and J.T. Hsu [5] analyzed economic production quantity (EPQ) models under an imperfect production process with shortages backordered. C. H. Glock and T. Kim [3] studied a joint economic lot size model with returnable transport items. A. Taleizadeh et al. [10] developed optimal price, replenishment lot size and number of shipments for an EPQ model with rework and multiple shipments. P. Muniappan et al [8] discussed an EPQ incentive inventory model for deteriorating products involving partially backlogged shortages. M. Babu et al. [2] formulated centralized production inventory model for buyer – vendor with quantity discount for fixed life time products.

II. NOTATIONS AND ASSUMPTIONS

The model use the following notations which is same as M. Babu et al. [1]

Notations

D	Demand rate
r_1	Buyer's unit ordering cost
r_2	Vendor's unit setup cost
P	Production cost

Q	Economic Order Quantity
h_b	Buyer's unit holding cost
h_v	Vendor's unit holding cost
C_s	Vendor's unit screening cost
C_d	Disposed cost
n	Vendor's multiples of order
s	Shortage cost
k	Buyer's multiples of order
d(k)	Discount factor
p	Purchase cost
u	Percentage of defecting items
v	Percentage of scrap items

Assumptions

(i) Demand rate is known and constant.

(ii) Buyer having shortage and manufacturer manufacture the products and provide quantity discount to the buyer.

(iii) For system optimization integrated system cost is formulated and satisfies the inventory level constraint.

(iv) Mathematically, the constraint will be taken as $\frac{Q}{2} \leq Y$ where Y denotes maximum available inventory.

III. MODEL FORMULATION

The total cost for buyer contains following cost

i.e., ordering cost $\frac{r_1 D}{Q}$, holding cost $\frac{h_b Q_1^2}{2Q}$, shortage cost $\frac{s(Q-Q_1)^2}{2Q}$and screening cost $\frac{C_s Q}{2}$.

The buyer cost will be written as

TC_b = Ordering cost + Holding cost + Shortage Cost+ Screening Cost

i.e., $TC_b = \frac{r_1 D}{Q} + \frac{h_b Q_1^2}{2Q} + \frac{s(Q-Q_1)^2}{2Q} + \frac{C_s Q}{2}$ (1)

The total cost for vendor contains following costs

i.e., setup cost $\frac{r_2 D}{nkQ}$, holding cost $\frac{h_v k(n-1)Q}{2}\left(\frac{P-D}{P}\right)$, discount factor $pDd(k)$ and disposed cost $\frac{uvC_d nQ}{2}$.

The manufacturer cost will be written as

TC_m = Setup cost + Holding cost + Discount factor + Disposed Cost

i.e., $TC_m = \frac{r_2 D}{nkQ} + \frac{k(n-1)h_v Q}{2}\left(\frac{P-D}{P}\right) + pDd(k) + \frac{uv\,C_d Q}{2}$ (2)

Now, the integrated system cost is written as

$TC_s = TC_b + TC_m$

subject to the constrain $\frac{Q}{2} \leq Y$.

Here, we consider the buyer's inventory level constraint. Now, Lagrange multiplier function $\lambda, 0 \leq \lambda \leq 1$ is added on system cost can be written as follows:

$TC_s = TC_b + TC_m + \lambda(\frac{Q}{2} - Y)$

$= \frac{r_1 D}{Q} + \frac{h_b Q_1^2}{2Q} + \frac{s(Q-Q_1)^2}{2Q} + \frac{C_s Q}{2} + \frac{r_2 D}{nkQ} + \frac{h_v k(n-1)Q}{2}\left(\frac{P-D}{P}\right) + pDd(k) + \frac{uvC_d nQ}{2} + \lambda(\frac{Q}{2} - Y)$ (3)

For optimality $\frac{\partial TC_s}{\partial Q_1} = 0$ and $\frac{\partial^2 TC_s}{\partial Q_1^2} > 0$ and $\frac{\partial TC_s}{\partial Q} = 0$ and $\frac{\partial^2 TC_s}{\partial Q^2} > 0$ we get,

$Q_1^* = \frac{sQ}{h_b + s}$ and

$$Q^* = \sqrt{\frac{2(h_b+s)\left[Dr_1 + \frac{Dr_2}{nk}\right]}{sh_b + (h_b+s)\left[C_s + (n-1)kh_v\left(\frac{P-D}{P}\right) + nuvC_d + \lambda\right]}} \qquad (4)$$

where $\lambda = \frac{2(h_b+s)\left[Dr_1 + \frac{Dr_2}{nk}\right] - Y^2\left[sh_b + (h_b+s)\left[C_s + (n-1)kh_v\left(\frac{P-D}{P}\right) + nuvC_d\right]\right]}{Y^2(h_b+s)}$

IV. NUMERICAL EXAMPLE

Let $R_1 = 100$ per order, $R_2 = 300$ per order, D = 2000 units per year, P = 3000, H_v = 0.6\$, H_b = 0.3\$, s_c = 0.2\$, s = 0.25\$, $C_d = 0.5$, u = 0.4, v = 0.3, p = 1.2\$, n = 2, k= 3, d(k) = 20%, Y = 800.

The optimal solutions are

$Q^* = 800$, $Q_1^* = 235$, $TC_s(Q^*, Q_1^*) = 1.1172X\ 10^3$ satisfies the constraint $\frac{Q}{2} \leq 800$.

V. CONCLUSION

In this paper, a manufacturer – buyer model is developed with quantity discount under coordination situation. The manufacturer provides quantity discount to the buyer for bulk purchase. Integrated system cost is developed for equal gain of buyer and manufacturer. Also, this integrated system cost satisfies inventory level constraint. For the further researches, our proposed model can be extended into multilevel products, multi constraints etc.,

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