



Manufacturing Model For Deteriorating Products Involving Budget Level And Space Occupied Constraints

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CC License CC-BY-NC-SA 4.0	Abstract The present study develops a manufacturing and non-manufacturing model under shortage and non-shortage situations. In both situation the damaged items are screened by the manufacturer and disposed by the buyer. What's more, the model considers the transportation cost for both manufacturer and buyer. Likewise, the integrated system cost satisfies the budget level and space occupied limit. The fitting circumstances to accomplish the ideal arrangements have been created, and mathematical cases have been given to confirm and assess the outcomes and solution strategy. The valuable techniques to actually decrease the yearly absolute expense are given by the aftereffects of the sensitivity analysis. Keywords: EPQ, Shortages, Transportation cost, Quantity discount.
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1. INTRODUCTION

A significant issue in any deal is that control and keep up with the inventories of decaying things. Merchandise are falling apart inferable from their qualities go down with time. The couple of normal models for decaying things are electronic items, design clothing, drugs, paper-based materials, food sources, vegetables, products of the soil. Along these lines, practically speaking, the misfortune because of disintegration can't be overlooked. Various investigations have been done to resolve the issues of economic production model for deteriorating things.

Muniappan et al. [8] separated a joined monetary solicitation sum model including stock level and item house limit prerequisite. Ravithammal et al. [9] focused on money related demand sum stock model using logarithmic methodology with stock level restriction. Ravithammal et al. [10] cultivated an ideal assessing stock model for rotting things with positive remarkable limit of cost markdown speed of interest. Muniappan et al. [7] explored creation model for turning out badly things including somewhat gathered deficiencies. Ravithammal et al. [11] created planning store network stock model for weakening items.

Ata Allah Taleizadeh [1] fostered an EOQ model with partial backordering and settlements ahead of time for a dissipating thing. Chih-Te Yang et al. [2] created ideal unique exchange credit and conservation innovation assignment for a decaying stock model. Chung-Yuan Dye [3] investigated the impact of protection innovation venture on a non-momentary decaying stock model. Ganesan and Uthayakumar [4] concentrated on EPQ models with bivariate arbitrary defective extents and learning-subordinate creation and request rates. Li et al. [5] examined organizing provider retailer and transporter with cost rebate strategy. Nita Shah and Monika Naik [6] investigated ideal renewal and estimating approaches for decaying things with quadratic interest under exchange credit, amount limits and money limits. Sachin Kumar Verma et al. [12] read up a stock model for variable holding cost with fractional multiplying and steady falling apart under selling cost request rate.

2. NOTATIONS AND ASSUMPTIONS

The model uses the following notations and assumptions.

2.1 Notations

d	Annual Demand rate
P	Production rate
r_1	Ordering cost for Buyer / order
r_2	Ordering cost for Vendor / order
h_1	Buyer's unit holding cost /unit
h_2	Vendor's unit holding cost / unit
b	Shortage Cost
V_1	Buyer unit Variable cost for Ordering handling and receiving
V_2	Vendor unit Variable cost for Ordering handling and receiving
p	Purchase cost for per order
k	Orders of multiples for buyer
$d(k)$	Discount factor
F_1	Buyer fixed transportations cost
F_2	Vendor fixed transportations cost
u	Percentage of defecting items
v	Percentage of scrap items
d_c	Disposed cost
S_c	Vendor's unit screening cost / unit
n	Vendor's multiples of order for without coordination
m	Vendor's multiples of order for with coordination
Q	Economic Order Quantity
Q_1	Back-order level
X	Maximum inventory level
Y	Total available storage space
F_s	Space occupied per product

2.2 Assumptions

- The model acknowledges steady interest.
- For non manufacturing model, buyer having shortage
- For manufacturing model, the manufacturer provide quantity discount to the buyer for mass order and hence buyer have no shortage.
- For both models, damaged products are screened by the manufacturer and disposed by the buyer.
- System cost is developed and it satisfies the budget and space occupied restriction. Mathematically, it will be made as $pQ \leq X$ and $F_s Q \leq Y$.

3. MODEL FORMULATION

In this section, both non-manufacturing and manufacturing model with and without shortage are figured.

Case - I: Non-Manufacturing Model with shortage

The total cost for buyer and manufacturer is contains as following cost

TC_b = Ordering cost + Holding cost + Shortage cost + Disposed cost + Transportation cost

$$\text{i.e., } TC_b = \frac{r_1 d}{Q} + \frac{h_1 Q_1^2}{2Q} + \frac{b(Q-Q_1)^2}{2Q} + \frac{uvd_c Q}{2} + F_1 + V_1 Q$$

TC_M = Setup cost + Holding cost + Screening cost + Transportation cost

$$\text{i.e., } TC_M = \frac{r_2 d}{nQ} + \frac{nh_2 Q}{2} + \frac{S_c Q}{2} + F_2 + nV_2 Q$$

Now, the integrated system cost is written as

$$TC_s = TC_b + TC_M$$

Subject to the constraints, $pQ \leq X$ and $F_s Q \leq Y$

$$TC_s = TC_b + TC_M + \lambda (pQ - X) + \gamma (F_s Q - Y)$$

$$TC_s = \frac{r_1 d}{Q} + \frac{h_1 Q_1^2}{2Q} + \frac{b(Q-Q_1)^2}{2Q} + \frac{uvd_c Q}{2} + F_1 + V_1 Q + \frac{r_2 d}{nQ} + \frac{nh_2 Q}{2} + \frac{S_c Q}{2} + F_2 + nV_2 Q + \lambda (pQ - X) + \gamma (F_s Q - Y)$$

$$TC_s = \left[\frac{h_1 + b}{2Q} \right] Q_1^2 - bQ_1 + \frac{bQ}{2} + \left[\frac{uvd_c + 2V_1 + nh_2 + S_c + 2nV_2 + 2\lambda p + 2\gamma F_s}{2} \right] Q + \frac{1}{Q} \left[r_1 d + \frac{r_2 d}{n} \right] + F_1 + F_2 - \lambda X - \gamma Y$$

For optimality $\frac{\partial TC_s}{\partial Q_1} = 0$ and $\frac{\partial^2 TC_s}{\partial Q_1^2} > 0$ and $\frac{\partial TC_s}{\partial Q} = 0$ and $\frac{\partial^2 TC_s}{\partial Q^2} > 0$ we get,

$$Q_1^* = \frac{bQ}{h_1 + b} \text{ and}$$

$$Q^* = \sqrt{\frac{2(h_1 + b) \left[r_1 d + \frac{r_2 d}{n} \right]}{bh_1 + (h_1 + b)[uvd_c + 2V_1 + nh_2 + S_c + 2nV_2 + 2\lambda p + 2\gamma F_s]}}$$

Where

$$\lambda = \frac{2p^2(h_1 + b) \left[r_1 d + \frac{r_2 d}{n} \right] - X^2[bh_1 + (h_1 + b)[uvd_c + 2V_1 + nh_2 + S_c + 2nV_2 + 2\gamma F_s]]}{2X^2(h_1 + b)p}$$

$$\gamma = \frac{2F_s^2(h_1 + b) \left[r_1 d + \frac{r_2 d}{n} \right] - Y^2[bh_1 + (h_1 + b)[uvd_c + 2V_1 + nh_2 + S_c + 2nV_2 + 2\lambda p]]}{2Y^2(h_1 + b)F_s}$$

Case -II: Manufacturing Model without shortage

The total cost for buyer and vendor is contains as following cost

TC_{b1} = Ordering cost + Holding cost + Disposed cost + Transportation cost

$$\text{i.e., } TC_{b1} = \frac{r_1 d}{kQ_c} + \frac{kh_1 Q_c}{2} + \frac{kuvd_c Q_c}{2} + F_1 + kV_1 Q_c$$

TC_{M1} = Setup cost + Holding cost + Screening cost + Transportation cost + Discount factor

$$\text{i.e., } TC_{M1} = \frac{r_2 d}{mkQ_c} + \frac{mkh_2 Q_c}{2} \left[\frac{P-d}{P} \right] + \frac{mkS_c Q_c}{2} + F_2 + mkV_2 Q_c + dpd(k)$$

Now, the integrated system cost is written as

$$TC_{s1} = TC_{b1} + TC_{M1}$$

Subject to the constraints, $pQ_c \leq X$ and $F_s Q_c \leq Y$

$$TC_{s1} = TC_{b1} + TC_{M1} + \lambda (pQ_c - X) + \gamma (F_s Q_c - Y)$$

$$TC_{s1} = \frac{r_1 d}{k Q_c} + \frac{k h_1 Q_c}{2} + \frac{k u v d_c Q_c}{2} + F_1 + k V_1 Q_c + \frac{r_2 d}{m k Q_c} + \frac{m k h_2 Q_c}{2} \left[\frac{P-d}{P} \right] + \frac{m k S_c Q_c}{2} + F_2 + m k V_2 Q_c + d p d(k) + \lambda (p Q_c - X) + \gamma (F_s Q_c - Y)$$

For optimality $\frac{\partial TC_s}{\partial Q_c} = 0$ and $\frac{\partial^2 TC_s}{\partial Q_c^2} > 0$ we get,

$$Q_c^* = \sqrt{\frac{2 \left[\frac{r_1 d}{k} + \frac{r_2 d}{m k} \right]}{k h_1 + k u v d_c + 2 k V_1 + m k h_2 \left[\frac{P-d}{P} \right] + m k S_c + 2 m k V_2 + 2 \lambda p + 2 \gamma F_s}}$$

Where

$$\lambda = \frac{2 p^2 \left[\frac{r_1 d}{k} + \frac{r_2 d}{m k} \right] - X^2 \left[k h_1 + k u v d_c + 2 k V_1 + m k h_2 \left[\frac{P-d}{P} \right] + m k S_c + 2 m k V_2 + 2 \gamma F_s \right]}{2 X^2 p}$$

$$\gamma = \frac{2 F_s^2 \left[\frac{r_1 d}{k} + \frac{r_2 d}{m k} \right] - Y^2 \left[k h_1 + k u v d_c + 2 k V_1 + m k h_2 \left[\frac{P-d}{P} \right] + m k S_c + 2 m k V_2 + 2 \lambda p \right]}{2 Y^2 F_s}$$

4. NUMERICAL EXAMPLE

Example 1:

Let $P = 4000$, $d = 2500$, $r_1 = 150$, $r_2 = 200$, $h_1 = 2.5$, $h_2 = 1.3$, $b = 0.2$, $n = 6$, $m = 3$, $k = 1.7$, $u = 0.2$, $v = 0.3$, $d_c = 0.35$, $S_c = 0.2$, $V_1 = 0.2$, $V_2 = 0.3$, $F_1 = 1.5$, $F_2 = 2$, $F_s = 2.0$, $X = 400$, $Y = 1700$, $\gamma = 0.2$, $p = 0.25$, $d(k) = 0.2$

The Optimal Solution is

With Shortage: $Q = 680$, $Q_1 = 50$, $TC_s = 1.0738 \times 10^4$ satisfies the constraints $F_s Q \leq 1700$ and $p Q \leq 400$

Without Shortage: $Q_c = 698$, $TC_{s1} = 1.0428 \times 10^4$ satisfies the constraints $F_s Q_c \leq 1700$ and $p Q_c \leq 400$.

Sensitive Analysis

The sensitivity analysis is done with the guide of taking every individual limit and holding the extra limit unaltered. The effects are shown in Table 1.

Table 1: Effects of Changes

Decision Variables	With Shortage				Without Shortage	
	Cost / Unit	Q	Q_1	TC_s	Q_c	TC_{s1}
P	3500	680	50.3704	1.0738×10^4	943.2701	1.0227×10^4
	3750	680	50.3704	1.0738×10^4	785.8369	1.0334×10^4
	4000	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4
	4250	680	50.3704	1.0738×10^4	640.7980	1.0511×10^4
r_1	50	680	50.3704	1.0460×10^4	1279.932	1.0302×10^4
	100	680	50.3704	1.0599×10^4	799.1727	1.0365×10^4
	150	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4
	200	680	50.3704	1.0877×10^4	651.3225	1.0491×10^4
r_2	100	680	50.3704	1.0691×10^4	691.4092	1.0371×10^4
	150	680	50.3704	1.0715×10^4	694.9286	1.0400×10^4
	200	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4
	250	680	50.3704	1.0761×10^4	700.5693	1.0457×10^4
h_1	2.3	680	54.4000	1.0737×10^4	810.8003	1.0312×10^4
	2.4	680	52.3077	1.0737×10^4	748.0718	1.0370×10^4
	2.5	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4
	2.6	680	48.5714	1.0738×10^4	656.7065	1.0486×10^4
h_2	1.2	680	50.3704	1.0258×10^4	609.2294	1.0087×10^4
	1.3	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4

	1.4	680	50.3704	1.1218×10^4	841.7323	1.0769×10^4
	1.5	680	50.3704	1.1698×10^4	1139.6433	1.1110×10^4
S_c	0.1	680	50.3704	1.0658×10^4	842.3179	1.0209×10^4
	0.15	680	50.3704	1.0698×10^4	760.0362	1.0318×10^4
	0.2	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4
	0.25	680	50.3704	1.0778×10^4	648.9511	1.0538×10^4
p	0.15	680	50.3704	1.7818×10^4	697.9489	1.7458×10^4
	0.2	680	50.3704	1.3392×10^4	697.9489	1.3058×10^4
	0.25	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4
	0.3	680	50.3704	8.9732×10^3	697.9489	8.6885×10^3
m	2.5	680	50.3704	1.0738×10^4	1778.832	1.0085×10^4
	2.75	680	50.3704	1.0738×10^4	927.8085	1.0255×10^4
	3	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4
	3.25	680	50.3704	1.0738×10^4	579.0829	1.0603×10^4
n	5.75	680	50.3704	1.0362×10^4	595.1953	1.0208×10^4
	6	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4
	6.25	680	50.3704	1.1114×10^4	882.2239	1.0648×10^4
	6.5	680	50.3704	1.1491×10^4	1388.7753	1.0867×10^4
k	1.6	680	50.3704	1.0738×10^4	1036.8743	1.0227×10^4
	1.65	680	50.3704	1.0738×10^4	823.1464	1.0327×10^4
	1.7	680	50.3704	1.0738×10^4	697.9489	1.0428×10^4
	1.75	680	50.3704	1.0738×10^4	613.0121	1.0530×10^4

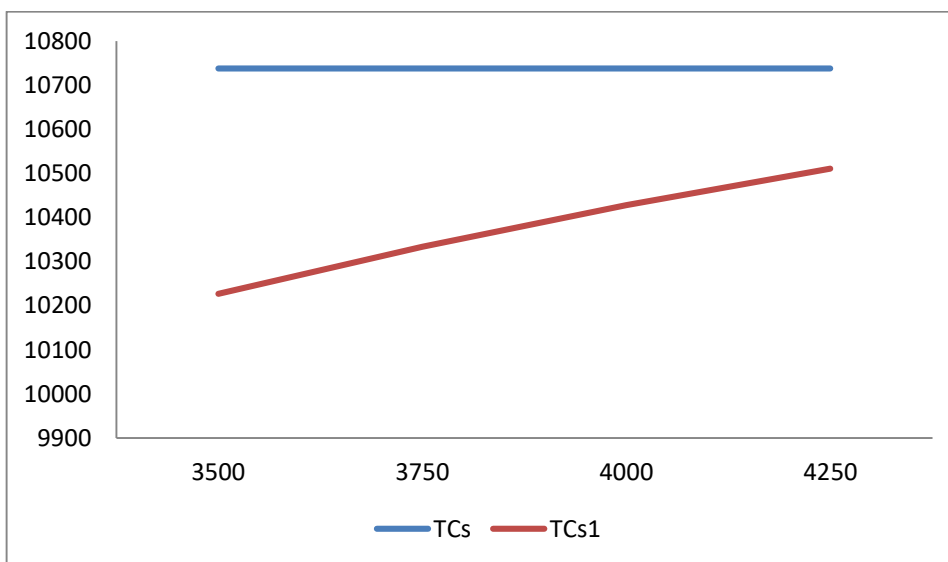


Fig 1: Effect of changes when P increases

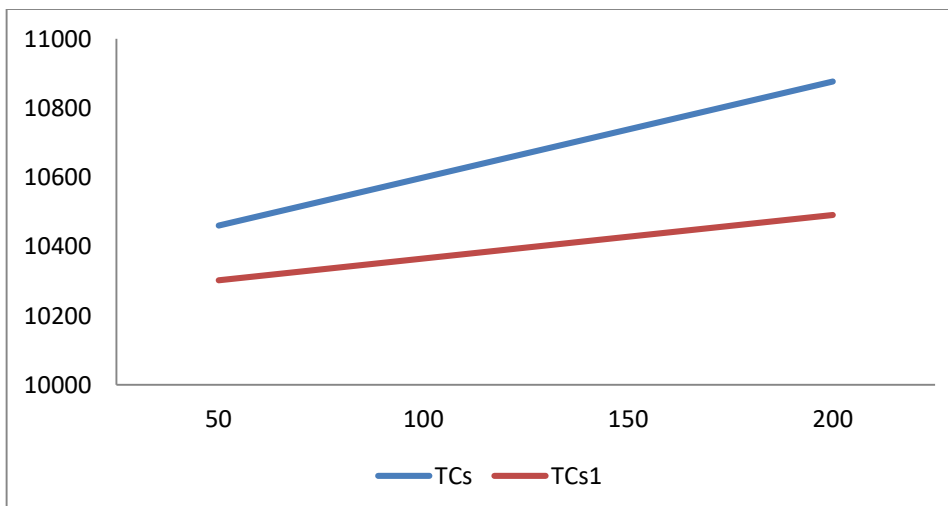


Fig 2: Effect of changes when r_1 increases

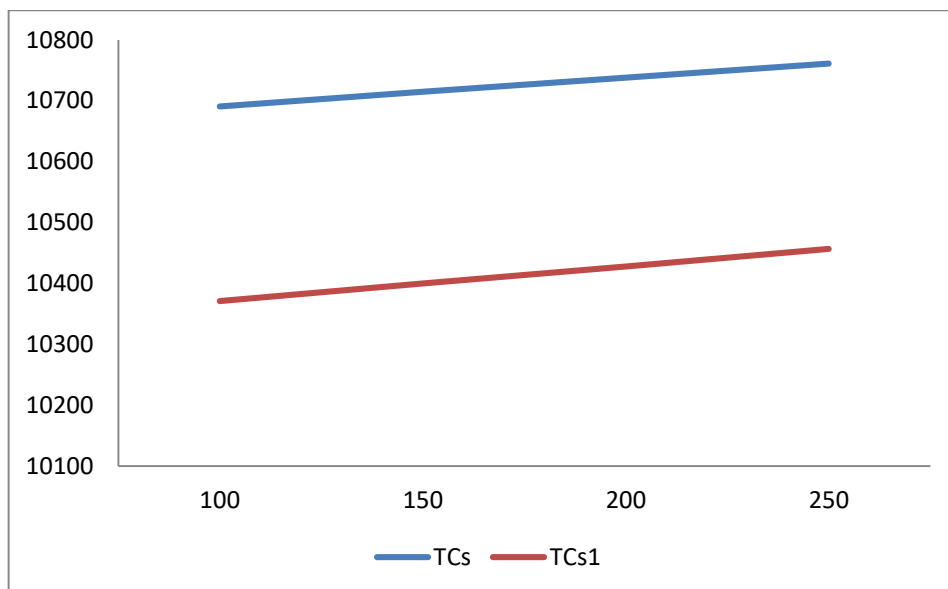


Fig 3: Effect of changes when r_2 increases

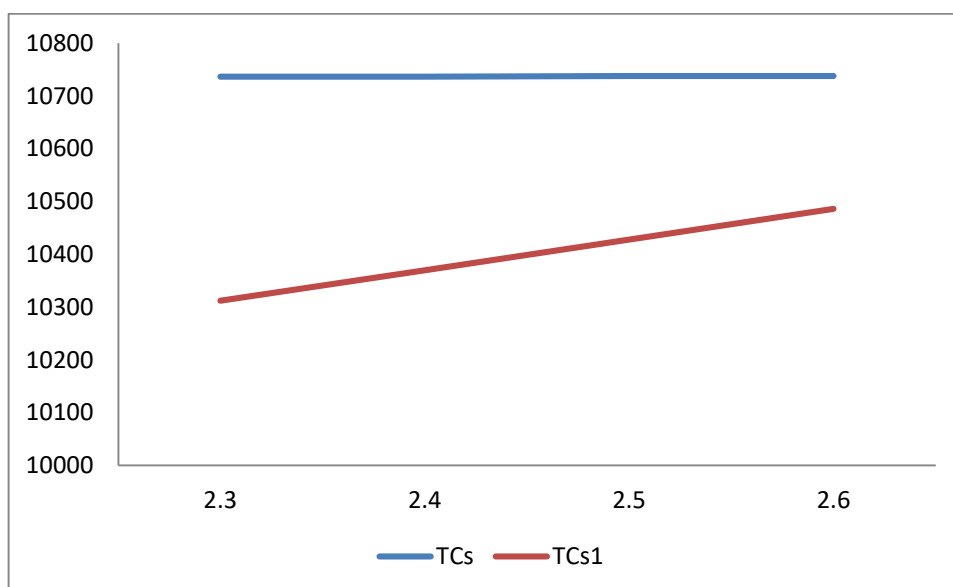


Fig 4: Effect of changes when h_1 increases

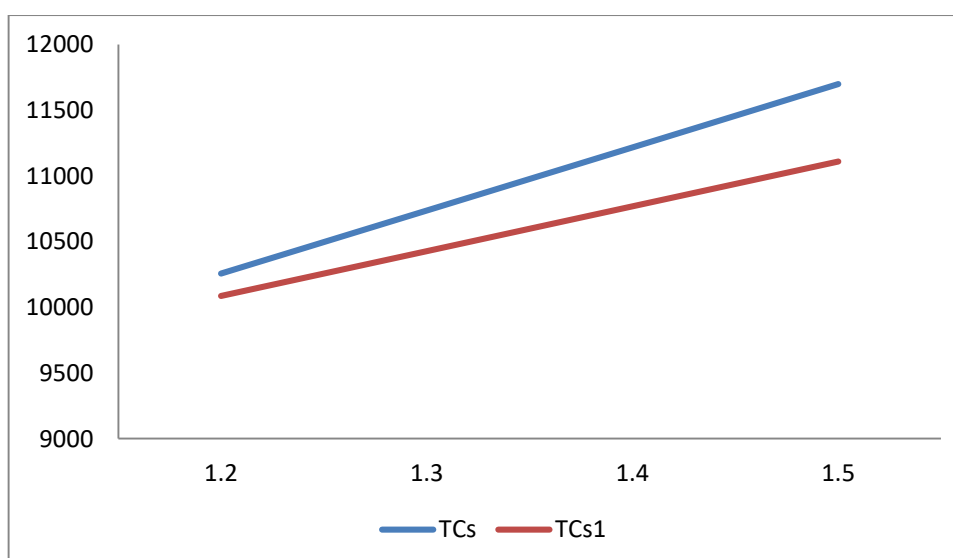


Fig 5: Effect of changes when h_2 increases

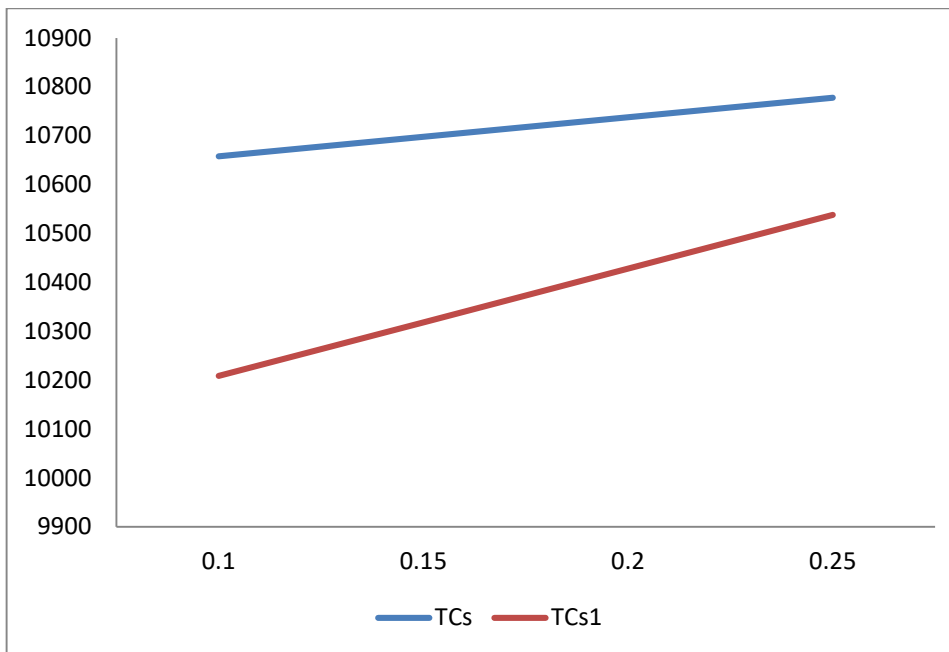


Fig 6: Effect of changes when s_c increases

5. CONCLUSION

In this paper, manufacturer – buyer inventory model is made under manufacturing and non manufacturing situations. Experiencing the same thing buyer has shortage and for manufacturing situation manufacturer created the product and provide quantity discount to the buyer for mass buy. Consequently, buyer has no shortage experiencing the same thing. In both situations system cost is developed and it fulfills budget level and spaced occupied constraints. It is then illustrated with the assistance of mathematical models. For the further investigates, the model can be related in credit period, temporary discount, multi-echelon supply chains, one time discount etc.,

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