Centralized Production Inventory Model for Buyer – Vendor with Quantity Discount for Fixed Life Time Products

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Abstract--- The study deals with the production inventory model for buyer-vendor with quantity discount for fixed life time product. The centralized model is analyzing the benefit of both the buyer and vendor to coordinating the supply chain. The quantity discount mechanism is implemented for vendor to the buyer for bulk purchase. The model includes a detailed numerical example for more understanding of the proposed method.

Keywords--- Production, Inventory, Order Quantity, Quantity Discount.

I. Introduction

The two major functions of every business are production and marketing which involves in selling and manufacturing. One of the parts of this major factor is inventory. The raw materials are converted into finished products and this process is called as production. An aggregate of functions that involves a process of moving goods from producer to consumer is marketing. The raw materials in stock, finished goods, semi finished goods and spares to alleviate marketing and production are known as Inventory.

Biswajit Sarkar and Ilkyenog Moon [1] developed an production model with inflation in an imperfect production system. Gede Agus Widyadana and Hui Ming Wee [3] studied production inventory models for deteriorating items with random machine break down and stochastic repair time. Mandal et al. [5] analyzed multi item multi period production problem for Fuzzy stochastic environment. Muniappan et al. [9] considered multi-echelon multi-constraints inventory model with quantity discount. Jawla and Singh [4] studied multi-item production model for imperfect products with multiple production setups and rework.

Cardenas-Barron [2] developed the derivation of EOQ/EPQ inventory models using analytic geometry and algebra. Sugirtha et al. [15] developed buyer – vendor inventory model for deteriorating items involving back orders, screening process and transportation cost. Mari Selvi et al. [6] developed vendor-buyer integrated inventory model for controllable lead time with screening and disposed cost.

Ravithammal et al. [12] studied an production model for perishable items with Fixed and Linear Backorders. Muniappan et al. [8] developed a production model for vendor–buyer with quantity discount, backordering and rework for fixed life time products. Ravithammal et al. [11] analyzed production inventory model for buyer-manufacturer with quantity discount and completely backlogged shortages for fixed life time product. Muniappan et al. [10] developed production inventory model for deteriorating products involving partially backlogged shortages. Ravithammal et al. [13] concentrated inventory model for price discount with shortage, back ordering and rework. Mohamad Y. Jaber and Ahmed M.A El Saadany [7] studied production and remanufacturing model with learning effects. Ravithammal et al. [14] considered an optimal pricing inventory model for deteriorating items with positive exponential function of price discount rate of demand.

II. Notations and Assumptions

The model use the following notations and assumptions

Notations

D	Demand rate per unit
r_1	Ordering cost for buyer

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r_2	Setup cost for vendor
V	Vendor unit variable cost for order handling and receiving
P	Production cost per order
Q	Economic Order Quantity
F	Fixed transportation cost per order
h_b	Holding cost for buyer
h_v	Holding cost for vendor
C_s	Screening cost for buyer
C_d	Disposed cost for buyer
n	Order's multiples for vendor
k	Order's multiples for buyer
$d(k)$	Discount factor
p	Purchase cost per order
u	Percentage of defecting items
v	Percentage of scrap items

Assumptions

(i) Demand rate is constant and known.
(ii) Buyer having ordering cost, holding cost, screening cost and disposed cost.
(iii) Vendor having setup cost, holding cost, transportation cost.
(iv) Vendor having production and provides quantity discount to the buyer for bulk purchase.
(v) For system optimization integrated system cost is developed.

III. Model Formulation

3.1 Centralized model with Quantity Discount

The total cost for buyer contains ordering cost, holding cost, screening cost and disposed cost.

TC_b = Ordering cost + Holding cost + Screening cost + Disposed cost

$$\text{i.e., } TC_b = \frac{r_1 D}{nQ} + \frac{h_b nQ}{2} + \frac{C_s nQ}{2} + \frac{uvC_d nQ}{2} \qquad (1)$$

The total cost for vendor contains setup cost, holding cost, discount factor and transportation cost.

TC_v = Setup cost + Holding cost + Discount factor + Transportation cost

$$\text{i.e., } TC_v = \frac{r_2 D}{nkQ} + \frac{h_v k(n-1)Q\left(1-\frac{D}{P}\right)}{2} + pDd(k) + F + VQ \qquad (2)$$

Now, the integrated system cost is written as

$$\mathrm{TC_s} = \mathrm{TC_b} + \mathrm{TC_v}$$

$$= \frac{r_1 D}{nQ} + \frac{h_b nQ}{2} + \frac{C_s nQ}{2} + \frac{uvC_d nQ}{2} + \frac{r_2 D}{nkQ} + \frac{h_v k(n-1)Q\left(1-\frac{D}{P}\right)}{2} + pDd(k) + F + VQ \qquad (3)$$

It can be written as

$$\mathrm{TC_S} = Q\left[\frac{n(h_b + C_s + uvC_d) + h_v k(n-1)Q\left(1-\frac{D}{P}\right) + 2V}{2}\right] + \frac{1}{Q}\left[\frac{r_1 D}{n} + \frac{r_2 D}{kn}\right] + pDd(k) + F \quad (4)$$

It is of the form $a_1 Q + \frac{a_2}{Q} + a_3$.

Q will be taken as, $Q = \sqrt{\frac{a_2}{a_1}}$ [Cardenas Barron (2011)]

$$\text{Now, } Q = \sqrt{\frac{2D\left[\frac{r_1}{n} + \frac{r_2}{kn}\right]}{n(h_b + C_s + uvC_d) + h_v k(n-1)\left(1-\frac{D}{P}\right) + 2V}} \qquad (5)$$

3.2 Profit of Buyer and Vendor

Profit of the buyer is calculated as $\mathrm{B_p} = \frac{\mathrm{TC_b}}{\mathrm{TC_S}} \times 100$

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Profit of the vendor is calculated as $V_p = \frac{TC_v}{TC_S} \times 100$

IV. Numerical Examples

Example 1

Let $r_1 = 100$ per order, $r_2 = 400$ per order, D = 5000 units, P = 8000 units, h_v = 0.03\$, h_b = 0.02\$, C_s = 0.5\$, C_d = 0.3\$, p = 0.4\$, n = 2, k = 2, d(k) = 10%, F = 10, V = 0.2, u = 0.1, v = 0.2.

The optimal solutions are

$Q = 1251.6$, $TC_b = 869.36$, $TC_V = 873.89$, $TC_S = 1743.2$, $B_p = 49.87\%$, $V_p = 50.13\%$

4.1 Sensitivity Analysis

We now study the effects of changes in the value of parameters D, r_1, r_2, P, $d(k)$ on Q, TC_b, TC_v and TC_s of the Example 1. i.e., the sensitivity analysis is performed by changing one parameter by +50%, +25%, -25% and -50% and keeping the remaining parameters unchanged. The results are shown in Table 1.

Table 1: Effect of changes in value of parameters

Parameters		**Q**	TC_b	TC_v	TC_S	B_p	V_p
r_1	+50						
	+25						
	-25						
	-50						
r_2	+50						
	+25						
	-25						
	-50						
D	+50						
	+25						
	-25						
	-50						
P	+50						
	+25						
	-25						
	-50						
$d(k)$	+50						
	+25						
	-25						
	-50						

The profit of both vendor (P_V) and buyer (P_B) are slightly sensitive if changing the value of D, r_1, r_2, P, $d(k)$.

V. Conclusion

This study analyze production inventory model for buyer-vendor with quantity discount for fixed life time product. Centralized model is provided for benefits of buyer and vendor to coordinating the supply chain. For bulk purchase, the vendor provides quantity discount to the buyer. Hence the buyer order is larger than regular quantity. Basic algebraic method is implemented for finding optimal value. Numerical examples are also proposed for developed model. For the further researches, the model can be extended in multi products, prize discount, temporary discount, credit period etc.,

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