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AN ANALYTICAL APPROACH TO DECISION MAKING USING DETERMINANT AND ADJOINT OF SOFT SQUARE MATRICES

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Abstract

This paper determines fundamental operations of a soft matrices and examines the properties of their multiplication. We define and examine the concepts of transpose and symmetric soft square matrices, as well as the notions of determinant and adjoint, illustrating their properties with examples. Additionally, we explore the singular and non-singular forms of soft square matrices and demonstrate them through examples. Finally, we determine the applications of adjoint of square soft matrices in decision making problems.

Keywords: Soft Matrix, Transpose and Symmetric soft matrix, Multiplication of soft matrix, and Determinant and Adjoint of Soft Matrix.

Subject Classification: 03E20

1. Introduction

Real-world problems often involve uncertainty and vagueness. Traditional methods usually struggle with complex issues in fields like economics, engineering, and environmental studies due to inherent uncertainties. Various mathematical frameworks, such as probability theory, rough set theory, etc., have been created to tackle these challenges. However, each of these methods has its own limitations, primarily because their parameterization techniques are insufficient. To address these shortcomings, the origin of theory of soft sets was developed by Molodtsov[4] (1999) is a completely a new mathematical tool for modelling uncertainties. This soft set approach sidesteps many of the issues found in traditional methods. Later, Maji et al. [2] (2003) built on this framework by defined operations on soft sets and its properties, showing their usefulness in decision analysis.

Çağman and Enginoğlu [5] (2010) defined the concept of soft matrix with their operations, offering a more effective framework for theoretical studies within soft set theory. Singh and Onyeozili [9] (2012) showed that a defined soft set operations are equal to those done on their corresponding soft matrices. Later, Singh and Onyeozili [1] (2013) expanded on this by presenting operations on soft matrices, establishing their basic properties, and applying them to decision-making. In further studies, Çağman and Enginoğlu (2012) introduced fuzzy soft (fs) matrices and examined their operations to push forward theoretical investigations in fs-set theory. N. Sarala and M. Prabhavathi [6] introduce fuzzy adjoint ordering with fuzzy soft matrices. Dhar [3] (2013) contributed by exploring the expansion of determinants in fuzzy matrices and looking into several of their properties. Madhumangal Pal [8] (2024) explore the concept of planarity within a generalized m-polar fuzzy environment.

The structure of paper is outlined as; Section 2 shows operations and types of soft matrix. Section 3 explores the transpose and symmetric of soft square matrix and their properties in detail. Section 4 focuses on the multiplication of soft matrices are examines some of their properties. In Section 5 introduced the notation of determinant and adjoint of soft matrix and examples are provided to illustrate some of the properties of determinant and adjoint of the soft matrix. In addition, the singular and non-singular form of soft square matrix are examined and demonstrated using examples. Finally, In Section 6 gave an application of soft matrix's determinant and adjoint with a case study in decision making problems.

2. Preliminaries

Definition 2.1 [4]

Let $\mathbb{U}$ denote the initial universal set and $\mathbb{E}$ be set of attributes (parameters) related to $\mathbb{U}$. The power set of $\mathbb{U}$ is denoted by $\mathcal{P}(\mathbb{U})$. For a non-empty subset $\mathcal{A} \subseteq \mathbb{E}$, a soft set over $\mathbb{U}$ is defined as an ordered pair $(\tau_A, \mathbb{E})$ where τ denoted a mapping from $\mathcal{A}$ to $\mathcal{P}(\mathbb{U})$.

Definition 2.2 [7]

Let $\mathbb{U}$ be a universe, $\mathbb{E}$ be a parameter set with related to $\mathbb{U}$. A soft set be $(\tau_A, \mathbb{E})$ can expressed a relation form as a subset $\mathcal{R}_\mathcal{A}$ of $\mathbb{U} \times \mathbb{E}$ where $\mathcal{R}_\mathcal{A} = \{(u, e): e \in \mathcal{A},\ u \in \tau_\mathcal{A}(e)\}$.

$\delta_{\mathcal{R}_\mathcal{A}}$ represent the characteristic function of $\mathcal{R}_\mathcal{A}$ which expressed as $\delta_{\mathcal{R}_\mathcal{A}} : \mathbb{U} \times \mathbb{E} \to \{0,1\}$, with

$\delta_{\mathcal{R}_\mathcal{A}}(u, e) = \begin{cases} 1, & (u, e) \in \mathcal{R}_\mathcal{A} \\ 0, & (u, e) \notin \mathcal{R}_\mathcal{A} \end{cases}$ then the soft set $(\tau_\mathcal{A}, \mathbb{E})$ can be expressed in a matrix $[\alpha_{ij}]$ known as $m \times n$ "**Soft Matrix**" as

$$[\omega_{ij}]_{m\times n} = \begin{bmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1n} \\ \omega_{21} & \omega_{22} & \cdots & \omega_{2n} \\ \vdots & \vdots & & \vdots \\ \omega_{m1} & \omega_{m2} & \cdots & \omega_{mn} \end{bmatrix} \text{ where } \omega_{ij} = \delta_{\mathcal{R}_\mathcal{A}}(u_i, e_i)$$

A soft matrix having $\mathfrak{m}$ rows and $\mathfrak{n}$ columns is known as soft matrix of order $\mathfrak{m}\ x\ \mathfrak{n}$.

Example 2.1

Assume that $\mathbb{U} = \{\mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3, \mathfrak{r}_4, \mathfrak{r}_5\}$ is a universal set and $\mathbb{E} = \{\wp_1, \wp_2, \wp_3, \wp_4\}$ be set of attributes. If $Q \subseteq E = \{\wp_1, \wp_3, \wp_4\}$, the soft set describe the parameters as $\tau_Q(\wp_1) = \{\mathfrak{r}_3, \mathfrak{r}_4\}$, $\tau_Q(\wp_3) = \{\mathfrak{r}_1, \mathfrak{r}_5\}, and\ \tau_Q(\wp_4) = \{\mathfrak{r}_1, \mathfrak{r}_3, \mathfrak{r}_5\}$ then soft set (τ_Q, E) is $(\tau_Q, E) = \{\ (\wp_1, \{\mathfrak{r}_3, \mathfrak{r}_4\}), (\wp_3, \{\mathfrak{r}_1, \mathfrak{r}_5\}), (\wp_4, \{\mathfrak{r}_1, \mathfrak{r}_3, \mathfrak{r}_5\})\}$

The soft matrix is given by

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$$[\alpha_{ij}] = \begin{matrix} \\ \mathfrak{r}_1 \\ \mathfrak{r}_2 \\ \mathfrak{r}_3 \\ \mathfrak{r}_4 \\ \mathfrak{r}_5 \end{matrix} \begin{matrix} \mathcal{p}_1 & \mathcal{p}_2 & \mathcal{p}_3 & \mathcal{p}_4 \\ \end{matrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad i = 1,2,\ldots,5; j = 1,2,\ldots,4.$$

Definition 2.2

Let $\tilde{\mathcal{A}} = [\omega_{ij}^{\tilde{\mathcal{A}}}] \in SM_{\mathfrak{m} \, x \, \mathfrak{n}}$, then $\tilde{\mathcal{A}}$ is called

(i) A **zero soft matrix** if $\omega_{ij}^{\tilde{\mathcal{A}}} = 0 \; \forall \; \mathfrak{i} \; and \; \mathfrak{j}$

(ii) A **universal soft matrix** if $\omega_{ij}^{\tilde{\mathcal{A}}} = 1 \; \forall \; \mathfrak{i} \; and \; \mathfrak{j}$

Definition 2.3[TYPES OF SOFT MATRIX][1]

Let $\tilde{\mathcal{A}} = [\omega_{ij}^{\tilde{\mathcal{A}}}] \in SM_{m \times n}$, then $\tilde{\tilde{\mathcal{A}}}$ is

(i) A **rectangular soft matrix**, if $\mathfrak{m} \neq \mathfrak{n}$ (i.e.,) no. of row is not equal to no. of column
No. of universal set is not equal to no. of parameter set
(ii) A **square soft matrix** if $\mathfrak{m} = \mathfrak{n}$ (i.e.,) no. of row is equal to no. of column
No. of universal set is equal to no. of parameter set
(iii) A **row soft matrix** if $\mathfrak{m} = 1$ (i.e.,) it has order 1 x $\mathfrak{n}$ that it contains only one row. Only one object exists in universal set.
(iv) A **column soft matrix** if $\mathfrak{n} = 1$(i.e.,) it has order $\mathfrak{m}$ x 1 that it contains only one column. Only one parameter exists in parameter set.
(v) A **diagonal soft matrix** of order $\mathfrak{m}$ x $\mathfrak{n}$ that every non-diagonal is zero.
(vi) Two soft matrices $\tilde{\tilde{\mathcal{A}}} = [\omega_{ij}^{\tilde{\mathcal{A}}}]$, $\tilde{\mathcal{B}} = [\omega_{ij}^{\tilde{\mathcal{B}}}] \in SM_{m \times n}$ are **equal soft matrix** if and only if the elements are identical. (i.e.,) $[\omega_{ij}^{\tilde{\mathcal{A}}}] = [\omega_{ij}^{\tilde{\mathcal{B}}}] \; \forall \; \mathfrak{i} \; and \; \mathfrak{j}$

Definition 2.4 [1]

Let $\tilde{\mathcal{A}} = [\omega_{ij}^{\tilde{\mathcal{A}}}] \in SM_{m \times n}$, $\tilde{\mathcal{B}} = [\omega_{ij}^{\tilde{\mathcal{B}}}] \in SM_{m \times n}$. Then

(i) the **union** is defined as $(\tilde{\mathcal{A}} \cup \tilde{\mathcal{B}})_{ij} = \max \{\omega_{ij}^{\tilde{\tilde{\mathcal{A}}}}, \omega_{ij}^{\tilde{\mathcal{B}}}\}$ and denoted by $\tilde{\mathcal{A}} \cup \tilde{\mathcal{B}}$

(ii) the **intersection** is $(\tilde{\mathcal{A}} \cap \tilde{\mathcal{B}})_{ij} = \min \{\omega_{ij}^{\tilde{\mathcal{A}}}, \omega_{ij}^{\tilde{\mathcal{B}}}\}$ and denoted by $\tilde{\mathcal{A}} \cap \tilde{\mathcal{B}}$

(iii) the **complement** of $\tilde{\mathcal{A}}$ is $(\widetilde{\mathcal{A}^o})_{ij} = [\omega_{ij}^{\tilde{\mathcal{A}}}] = [1 - \omega_{ij}^{\tilde{\mathcal{A}}}]$ and denoted by $\widetilde{\mathcal{A}^o}$

Definition 2.5

(i) Let $\tilde{\mathcal{A}} = [\omega_{ij}^{\tilde{\mathcal{A}}}] \; and \; \tilde{\mathcal{B}} = [\omega_{ij}^{\tilde{\mathcal{B}}}] \in SM_{m \times n}$. The **addition** of soft matrices is
$(\tilde{\mathcal{A}} + \tilde{\mathcal{B}})_{ij} = [\omega_{ij}^{\tilde{c}}]_{m \times n} = \max \{\omega_{ij}^{\tilde{\tilde{\mathcal{A}}}}, \omega_{ij}^{\tilde{\mathcal{B}}}\}$ and denoted by $\tilde{\mathcal{A}} + \tilde{\mathcal{B}}$
(ii) Let $\tilde{\mathcal{A}} = [\omega_{ij}^{\tilde{\mathcal{A}}}]$ and $\tilde{\mathcal{B}} = [\omega_{ij}^{\tilde{\mathcal{B}}}] \in SM_{m \times n}$. The **subtraction** of soft matrices is
$(\tilde{\mathcal{A}} - \tilde{\mathcal{B}})_{ij} = [\omega_{ij}^{\tilde{c}}]_{m \times n} = min\{\omega_{ij}^{\tilde{\mathcal{A}}}, \omega_{ij}^{\tilde{\mathcal{B}}}\}$ and denoted by $\tilde{\mathcal{A}} - \tilde{\mathcal{B}}$

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Example 2.2

Let $\tilde{\mathcal{A}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $\tilde{\mathcal{B}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ are two soft matrices, then

(i) the addition is $\tilde{\mathcal{A}} + \tilde{\mathcal{B}} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{3\times 3}$

(ii) the subtraction is $\tilde{\mathcal{A}} - \tilde{\mathcal{B}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}_{3\times 3}$

3. TRANSPOSE AND SYMMETRIC OF SOFT MATRIX

This section shows the transpose of soft matrix and symmetric soft matrix with their properties with example

Definition 3.1

If $\left[\omega_{ij}^{\tilde{\mathcal{A}}}\right]$ is a square soft matrix of size m x n, then its **transpose**, as $\left[\omega_{ij}^{\tilde{\mathcal{A}}}\right]^T$, is defined by interchanging rows with columns, give a matrix of order n x m. The soft set with $\left[\omega_{ij}^{\tilde{\mathcal{A}}}\right]^T$ is a modified soft set formulated on common universe and parameter set.

Example 3.1

Let $\mathbb{U}$ be set of five hats $\{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5\}$ and the parameter set $\{s_1, s_2, s_3, s_4, s_5\}$. Let $Q = \{s_1, s_3, s_4, s_5\}$

Thus, soft set $(\tau_Q, \mathbb{E})$ is shown as

$(\tau_Q, \mathbb{E}) = \{(s_1, \{\hbar_3, \hbar_4, \hbar_5\}), (s_3, \{\hbar_1, \hbar_3, \hbar_4\}), (s_4, \{\hbar_2, \hbar_4, \hbar_5\}), (s_5, \{\hbar_1, \hbar_3\}$ which associated soft matrix is

$$\left[\omega_{ij}^{\tilde{Q}}\right] = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Now, its transpose soft matrix is given by

$$\left[\omega_{ij}^{\tilde{Q}}\right]^T = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Therefore, soft set associated with $\left[\omega_{ij}^{\tilde{Q}}\right]^T$ is $(\varphi_Q, E) = \{(s_1,\{h_3,h_5\}),(s_2,\{h_4\}),(s_3,\{h_1,h_3,h_5\}),(s_4,\{h_1,h_3,h_4\}),(s_5,\{h_1,h_4\})\}$

Proposition 3.1

For any soft matrices $\left[\omega_{ij}^{\tilde{A}}\right]$ *and* $\left[\omega_{ij}^{\tilde{B}}\right]$ of suitable orders, we have

(i) the transpose of a transpose, $\left[\left[\omega_{ij}^{\tilde{A}}\right]^T\right]^T = \left[\omega_{ij}^{\tilde{A}}\right]$

(ii) the transpose over union, $\left[\left[\omega_{ij}^{\tilde{A}}\right] \cup \left[\omega_{ij}^{\tilde{B}}\right]\right]^T = \left[\omega_{ij}^{\tilde{A}}\right]^T \cup \left[\omega_{ij}^{\tilde{B}}\right]^T$

(iii) the transpose over intersection, $\left[\left[\omega_{ij}^{\tilde{A}}\right] \cap \left[\omega_{ij}^{\tilde{B}}\right]\right]^T = \left[\omega_{ij}^{\tilde{A}}\right]^T \cap \left[\omega_{ij}^{\tilde{B}}\right]^T$

Proof:

For any soft matrices $\left[\omega_{ij}^{\tilde{A}}\right]$ *and* $\left[\omega_{ij}^{\tilde{B}}\right]$

Let $\left[\omega_{ij}^{\tilde{A}}\right] = \begin{bmatrix}1 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix}$ and $\left[\omega_{ij}^{\tilde{B}}\right] = \begin{bmatrix}1 & 0 & 0\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}$

(i) $\left[\left[\omega_{ij}^{\tilde{A}}\right]^T\right] = \begin{bmatrix}1 & 0 & 1\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix} \Rightarrow \left[\left[\omega_{ij}^{\tilde{A}}\right]^T\right]^T = \begin{bmatrix}1 & 1 & 0\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix} = \left[\omega_{ij}^{\tilde{A}}\right]$

(ii) $\left[\omega_{ij}^{\tilde{A}}\right] \cup \left[\omega_{ij}^{\tilde{B}}\right] = \begin{bmatrix}1 & 1 & 0\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix} \Longrightarrow \left[\left[\omega_{ij}^{\tilde{A}}\right] \cup \left[\omega_{ij}^{\tilde{B}}\right]\right]^T = \begin{bmatrix}1 & 1 & 1\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}$

$\left[\omega_{ij}^{\tilde{A}}\right]^T = \begin{bmatrix}1 & 0 & 1\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}$ and $\left[\omega_{ij}^{\tilde{B}}\right]^T = \begin{bmatrix}1 & 1 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$, $\left[\omega_{ij}^{\tilde{A}}\right]^T \cup \left[\omega_{ij}^{\tilde{B}}\right]^T = \begin{bmatrix}1 & 1 & 1\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}$

$\therefore \left[\left[\omega_{ij}^{\tilde{A}}\right] \cup \left[\omega_{ij}^{\tilde{B}}\right]\right]^T = \left[\omega_{ij}^{\tilde{A}}\right]^T \cup \left[\omega_{ij}^{\tilde{B}}\right]^T$

(iii) Similarly, $\left[\left[\omega_{ij}^{\tilde{A}}\right] \cap \left[\omega_{ij}^{\tilde{B}}\right]\right]^T = \left[\omega_{ij}^{\tilde{A}}\right]^T \cap \left[\omega_{ij}^{\tilde{B}}\right]^T$

Definition 3.2

Let $\left[\omega_{ij}^{\tilde{A}}\right] \in SM_{m\times n}$ is **a symmetric soft matrix** if it equals its transpose (i.e.,) $\left[\omega_{ij}^{\tilde{A}}\right]^T = \left[\omega_{ij}^{\tilde{A}}\right]$. Equivalently, it requires that $\left[\omega_{ij}^{\tilde{A}}\right] = \left[\omega_{ji}^{\tilde{A}}\right]$ *for every i and j* . Then soft set with both $\left[\omega_{ij}^{\tilde{A}}\right]$& $\left[\omega_{ij}^{\tilde{A}}\right]^T$ represent the same soft set.

Note: In general, the concept of a skew-symmetric soft matrix is not applicable in soft matrix theory

Example 3.2

Let the universal set $U = \{t_1, t_2, t_3, t_4, t_5\}$ and the parameter set $\mathbb{E} = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5\}$. Let P = $\{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5\}$ then soft set is

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$$(f_P, E) = \{(\text{п}_1, \{t_1, t_3, t_4\}), (\text{п}_2, \{t_2, t_4, t_5\}), (\text{п}_3, \{t_1, t_4\}), (\text{п}_4, \{t_1, t_2, t_3\}), (\text{п}_5, \{t_2, t_5\})\}$$

$[\omega_{ij}^{\tilde{P}}] = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ here, $[\omega_{ij}^{\tilde{P}}] = [\omega_{ji}^{\tilde{P}}]$ $\forall\ i\ and\ j$

$$[\omega_{ij}^{\tilde{P}}]^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, $[\omega_{ij}^{\tilde{P}}]^T = [\omega_{ij}^{\tilde{P}}]$. Thus, $[\omega_{ij}^{\tilde{P}}]$ is a symmetric soft matrix.

Theorem 3.1

For every square soft matrix $[\omega_{ij}^{\tilde{P}}]$

(i) $[\omega_{ij}^{\tilde{P}}] \cup [\omega_{ij}^{\tilde{P}}]^T$ is symmetric soft matrix

(ii) $[\omega_{ij}^{\tilde{P}}] \cap [\omega_{ij}^{\tilde{P}}]^T$ is also symmetric soft matrix

Proof:

(i) Let us consider Q = $[\omega_{ij}^{\tilde{Q}}] = [\omega_{ij}^{\tilde{P}}] \cup [\omega_{ij}^{\tilde{P}}]^T \in SM_{m\times n}$

$Q^T = [\omega_{ij}^{\tilde{Q}}]^T = \left[[\omega_{ij}^{\tilde{P}}] \cup [\omega_{ij}^{\tilde{P}}]^T\right]^T = [\omega_{ij}^{\tilde{P}}]^T \cup [\omega_{ij}^{\tilde{P}}] = [\omega_{ij}^{\tilde{P}}] \cup [\omega_{ij}^{\tilde{P}}]^T = [\omega_{ij}^{\tilde{Q}}] = Q.$

Therefore, $[\omega_{ij}^{\tilde{P}}] \cup [\omega_{ij}^{\tilde{P}}]^T$ is symmetric soft matrix

(ii) Let us consider $R = [\omega_{ij}^{\tilde{R}}] = [\omega_{ij}^{\tilde{P}}] \cap [\omega_{ij}^{\tilde{P}}]^T \in SM_{m\times n}$

$R^T = [\omega_{ij}^{\tilde{R}}]^T = \left[[\omega_{ij}^{\tilde{P}}] \cap [\omega_{ij}^{\tilde{P}}]^T\right]^T = [\omega_{ij}^{\tilde{P}}]^T \cap [\omega_{ij}^{\tilde{P}}] = [\omega_{ij}^{\tilde{P}}] \cap [\omega_{ij}^{\tilde{P}}]^T = [\omega_{ij}^{\tilde{R}}] = R$

Therefore, $[\omega_{ij}^{\tilde{P}}] \cap [\omega_{ij}^{\tilde{P}}]^T$ is also symmetric soft matrix

Example 3.3

Let us consider the $\tilde{\mathcal{P}} = [\omega_{ij}^{\tilde{\mathcal{P}}}] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $[\omega_{ij}^{\tilde{P}}]^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$[\omega_{ij}^{\tilde{P}}] \cup [\omega_{ij}^{\tilde{P}}]^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is a symmetric soft matrix.

$[\omega_{ij}^{\tilde{P}}] \cap [\omega_{ij}^{\tilde{P}}]^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is also a symmetric soft matrix.

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4. MULTIPLICATION OF SOFT MATRIX

This section represents the multiplication of soft matrix and its fundamental properties are discussed.

Definition 4.1

Let $\tilde{\mathcal{P}} = [\omega_{ij}^{\tilde{\mathcal{P}}}] \in SM_{mxn}$ *and* $\tilde{\mathcal{Q}} = [\omega_{jk}^{\tilde{\mathcal{Q}}}] \in SM_{n\times p}$, the **multiplication** of soft matrices, denoted as $\tilde{\mathcal{P}} \times \tilde{\mathcal{Q}}$ as

$$(\tilde{\mathcal{P}} \times \tilde{\mathcal{Q}})_{ik} = [\omega_{ik}^{\tilde{C}}]_{m\times p} = \max_j \{min\{\omega_{ij}^{\tilde{\mathcal{P}}}, \omega_{jk}^{\tilde{\mathcal{Q}}}\}\}$$

Example 4.1

Consider $\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ and $\tilde{\mathcal{Q}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ are two soft matrices, then the multiplication of these two matrices is

$$\tilde{\mathcal{P}} \times \tilde{\mathcal{Q}} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Proposition 4.1

Let $[\omega_{ij}^{\tilde{\mathcal{P}}}]$, $[\omega_{jk}^{\tilde{\mathcal{Q}}}]$and $[\omega_{kl}^{\tilde{\mathcal{R}}}]$ be soft matrices, then

1. Multiplication of soft matrix does not satisfy law of commutative.

$[\omega_{ij}^{\tilde{\mathcal{P}}}] \times [\omega_{jk}^{\tilde{\mathcal{Q}}}] \neq [\omega_{jk}^{\tilde{\mathcal{Q}}}] \times [\omega_{ij}^{\tilde{\mathcal{P}}}]$

2. Multiplication of soft matrix satisfies law of association.

$[\omega_{ij}^{\tilde{\mathcal{P}}}] \times ([\omega_{jk}^{\tilde{\mathcal{Q}}}] \times [\omega_{kl}^{\tilde{\mathcal{R}}}]) = ([\omega_{ij}^{\tilde{\mathcal{P}}}] \times [\omega_{jk}^{\tilde{\mathcal{Q}}}]) \times [\omega_{kl}^{\tilde{\mathcal{R}}}]$

3. Multiplication of soft matrix satisfies law of distribution.

$[\omega_{ij}^{\tilde{\mathcal{P}}}] \times ([\omega_{jk}^{\tilde{\mathcal{Q}}}] + [\omega_{kl}^{\tilde{\mathcal{R}}}]) = ([\omega_{ij}^{\tilde{\mathcal{P}}}] \times [\omega_{jk}^{\tilde{\mathcal{Q}}}]) + ([\omega_{jk}^{\tilde{\mathcal{Q}}}] \times [\omega_{kl}^{\tilde{\mathcal{R}}}])$

4. Multiplication of soft matric satisfies identity law.

$[\omega_{ij}^{\tilde{\mathcal{P}}}] \times I_n = [\omega_{ij}^{\tilde{\mathcal{P}}}]$ or $I_m \times [\omega_{ij}^{\tilde{\mathcal{P}}}] = [\omega_{ij}^{\tilde{\mathcal{P}}}]$

5. Soft matrix multiplication satisfies the law of transportation.

$([\omega_{ij}^{\tilde{\mathcal{P}}}] \times [\omega_{jk}^{\tilde{\mathcal{Q}}}])^T = [\omega_{jk}^{\tilde{\mathcal{Q}}}]^T \times [\omega_{ij}^{\tilde{\mathcal{P}}}]^T$

Proof:

Thus, the proof is followed by given example

1. $[\omega_{ij}^{\tilde{\mathcal{P}}}] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $[\omega_{jk}^{\tilde{\mathcal{Q}}}] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$[\omega_{ij}^{\tilde{\mathcal{P}}}] \times [\omega_{jk}^{\tilde{\mathcal{Q}}}] = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad [\omega_{jk}^{\tilde{\mathcal{Q}}}] \times [\omega_{ij}^{\tilde{\mathcal{P}}}] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

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$\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] \neq \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] \times \left[\omega_{ij}^{\tilde{\mathcal{P}}}\right]$

Note: In general, soft matrices does not satisfy commutative law

2. $\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $\left[\omega_{kl}^{\tilde{\mathcal{R}}}\right] = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times (\left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] \times \left[\omega_{kl}^{\tilde{\mathcal{R}}}\right]) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad (\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right]) \times \left[\omega_{kl}^{\tilde{\mathcal{R}}}\right] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times (\left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] \times \left[\omega_{kl}^{\tilde{\mathcal{R}}}\right]) = (\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right]) \times \left[\omega_{kl}^{\tilde{\mathcal{R}}}\right]$

3. $\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $\left[\omega_{kl}^{\tilde{\mathcal{R}}}\right] = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times (\left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] + \left[\omega_{kl}^{\tilde{\mathcal{R}}}\right]) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $\quad \left(\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right]\right) + \left(\left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] \times \left[\omega_{kl}^{\tilde{\mathcal{R}}}\right]\right) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

$\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times (\left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] + \left[\omega_{kl}^{\tilde{\mathcal{R}}}\right]) = (\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right]) + (\left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] \times \left[\omega_{kl}^{\tilde{\mathcal{R}}}\right])$

4. $\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, I = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times I_n = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = I_m \times \left[\omega_{ij}^{\tilde{\mathcal{P}}}\right]$

$\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times I_n = \left[\omega_{ij}^{\tilde{\mathcal{P}}}\right]$ or $I_m \times \left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] = \left[\omega_{ij}^{\tilde{\mathcal{P}}}\right]$

5. $\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right] = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$(\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right])^T = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ $\quad \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right]^T \times \left[\omega_{ij}^{\tilde{\mathcal{P}}}\right]^T = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$(\left[\omega_{ij}^{\tilde{\mathcal{P}}}\right] \times \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right])^T = \left[\omega_{jk}^{\tilde{\mathcal{Q}}}\right]^T \times \left[\omega_{ij}^{\tilde{\mathcal{P}}}\right]^T$

5. DETERMINANT AND ADJOINT OF SOFT MATRIX

This section starts by introducing the notation of the determinant and adjoint of soft matrices and proved some properties.

Let S_n be a symmetric group on {1, 2, 3, …, n }

Definition 5.1

Let $\tilde{\mathcal{P}} = \left[\omega_{ij}^{\tilde{\mathcal{P}}}\right]_{n\times n}$ *of size* $n \times n$. The **determinant** of $\tilde{\mathcal{P}}$ is formulated as

$|\tilde{\mathcal{P}}| = \prod_{\pi\in S_n}(\alpha_{1\pi(1)} + \alpha_{2\pi(2)} + \cdots + \alpha_{n\pi(n)})$ (or)

$$|\tilde{\mathcal{P}}| = \prod_{\pi\in S_n}\left(\sum_{i=1}^{n} \alpha_{i,\pi(i)}^{\tilde{\mathcal{P}}}\right)$$

where the product is taken over all of S_n. It is denoted as $|\tilde{\mathcal{P}}|$.

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Determinant of soft square matrix of order 2

If $\tilde{\mathcal{P}} = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix}$, then $|\tilde{P}| = \begin{vmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{vmatrix} = \min(\max(\omega_{11}, \omega_{22}), \max(\omega_{21}, \omega_{12}))$

Example 5.1

Let $\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $|\tilde{\mathcal{P}}| = 0$

Determinant of soft square matrix of order 3

If $\tilde{\mathcal{P}} = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{bmatrix}$, then $|\tilde{\mathcal{P}}| = \begin{vmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{vmatrix}$

$$= min\,(max[\omega_{11}, \min(\max(\omega_{22}, \omega_{33}), max(\omega_{32}, \omega_{23}))],$$

$$\max[\omega_{12}, \min(\max(\omega_{21}, \omega_{33}), \max(\omega_{31}, \omega_{23}))],$$

$$\max[\omega_{13}, \min(\max(\omega_{21}, \omega_{32}), \max(\omega_{31}, \omega_{22}))])$$

Example 5.2

Let $\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, then $|\tilde{\mathcal{P}}| = 1$

Determinant of soft square matrix of order n

If $\tilde{\mathcal{P}} = \begin{bmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1n} \\ \omega_{21} & \omega_{22} & \cdots & \omega_{2n} \\ \vdots & \vdots & & \vdots \\ \omega_{n1} & \omega_{n2} & \cdots & \omega_{1n} \end{bmatrix}$, then $|\tilde{\mathcal{P}}| = \begin{vmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1n} \\ \omega_{21} & \omega_{22} & \cdots & \omega_{2n} \\ \vdots & \vdots & & \vdots \\ \omega_{n1} & \omega_{n2} & \cdots & \omega_{1n} \end{vmatrix}$

$= \min(\max(\omega_{1j,} |\tilde{\mathcal{P}}_{1j}|)\ for\ j = 1,2,\ldots,N)$

Proposition 5.1 Properties of determinants

Let a soft square matrix be $\tilde{\mathcal{P}}$.

(i) If two rows of $\tilde{\mathcal{P}}$ are interchanged to obtain $\tilde{\mathcal{Q}}$, then $|\tilde{\mathcal{P}}| = |\tilde{\mathcal{Q}}|$

(ii) If two columns of $\tilde{\mathcal{P}}$ are interchanged to obtain $\tilde{\mathcal{Q}}$, then $|\tilde{\mathcal{P}}| = |\tilde{\mathcal{Q}}|$

Proof:

(i) If $\tilde{\mathcal{P}} = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix}$, then $|\tilde{\mathcal{P}}| = \min(\max(\omega_{11}, \omega_{22}), \max(\omega_{21}, \omega_{12}))$

Interchanging rows, we get

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If $\tilde{\mathcal{Q}} = \begin{bmatrix} \omega_{21} & \omega_{22} \\ \omega_{11} & \omega_{12} \end{bmatrix}$, then $|\tilde{\mathcal{Q}}| = \min(\max(\omega_{21}, \omega_{12}), \max(\omega_{11}, \omega_{22}))$

$\therefore |\tilde{\mathcal{P}}| = |\tilde{\mathcal{Q}}|$

(ii) If $\tilde{\mathcal{P}} = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix}$, then $|\tilde{\mathcal{P}}| = \min(\max(\omega_{11}, \omega_{22}), \max(\omega_{21}, \omega_{12}))$

Interchanging columns, we get

If $\tilde{\mathcal{Q}} = \begin{bmatrix} \omega_{12} & \omega_{11} \\ \omega_{22} & \omega_{21} \end{bmatrix}$, then $|\tilde{\mathcal{Q}}| = \min(\max(\omega_{12}, \omega_{21}), \max(\omega_{11}, \omega_{22}))$

$\therefore |\tilde{\mathcal{P}}| = |\tilde{\mathcal{Q}}|$

Proposition 5.2

Let $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$ be two soft square matrices,

(i) $|\tilde{\mathcal{P}}| = |\tilde{\mathcal{P}}^T|$

(ii) $|\tilde{\mathcal{P}} \cdot \tilde{\mathcal{Q}}| = |\tilde{\mathcal{P}}| \,.\, |\tilde{\mathcal{Q}}|$

(iii) $|\tilde{\mathcal{P}} + \tilde{\mathcal{Q}}| = |\tilde{\mathcal{P}}| + |\tilde{\mathcal{Q}}|$

Proof:

(i) Let $\widetilde{\mathcal{P}} = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix}$, then $|\tilde{\mathcal{P}}| = \min(\max(\omega_{11}, \omega_{22}), \max(\omega_{21}, \omega_{12}))$

and $\tilde{\mathcal{P}}^{\,T} = \begin{bmatrix} \omega_{11} & \omega_{21} \\ \omega_{12} & \omega_{22} \end{bmatrix}$, and $|\tilde{\mathcal{P}}^{\,T}| = \min(\max(\omega_{11}, \omega_{22}), \max(\omega_{21}, \omega_{12}))$

$\therefore |\tilde{\mathcal{P}}| = |\tilde{\mathcal{P}}^T|$

(ii) Let $\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\tilde{\mathcal{Q}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, then $\tilde{\mathcal{P}} \cdot \tilde{\mathcal{Q}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

$|\tilde{\mathcal{P}}| = 1$, $|\tilde{\mathcal{Q}}| = 0$ and $|\tilde{\mathcal{P}} \cdot \tilde{\mathcal{Q}}| = 0$

$\therefore |\tilde{\mathcal{P}} \cdot \tilde{\mathcal{Q}}| = |\tilde{\mathcal{P}}| \cdot |\tilde{\mathcal{Q}}|$

(iii) Let $\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\tilde{\mathcal{Q}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, then $\tilde{\mathcal{P}} + \tilde{\mathcal{Q}} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

$|\tilde{\mathcal{P}}| = 1$, $|\tilde{\mathcal{Q}}| = 0$ and $|\tilde{\mathcal{P}} + \tilde{\mathcal{Q}}| = 1$

$\therefore |\tilde{\mathcal{P}} + \tilde{\mathcal{Q}}| = |\tilde{\mathcal{P}}| + |\tilde{\mathcal{Q}}|$

Definition 5.2

Let $\tilde{\mathcal{P}} = \left[\omega_{ij}^{\tilde{\mathcal{P}}}\right]$ be any soft matrix. The **adjoint** soft matrix of size $\text{n}\ x\ \text{n}$ soft matrix is a transpose of its cofactor $\tilde{\mathcal{Q}}$, $where\ \tilde{\mathcal{Q}} = \left[\beta_{ij}^{\tilde{\mathcal{Q}}}\right]$ where $\beta_{ij} = |\tilde{\mathcal{P}}_{ij}|$ and $|\tilde{\mathcal{P}}_{ij}|$ denotes determinant of $(\text{n}\ x\ \text{n}) - (\text{n}\ x\ \text{n})$ minor obtained from $\tilde{\mathcal{P}}$ by removing j^th^ row and i^th^ column.

$\tilde{\mathcal{P}} = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{bmatrix}$, $\tilde{\mathcal{Q}} = \left[\beta_{ij}^{\tilde{\mathcal{Q}}}\right]$ where the cofactor is $C_{ij}^{Soft} = \beta_{ij} = |\tilde{\mathcal{P}}_{ij}|$

$\beta_{11} = |\tilde{\mathcal{P}}_{11}| = \begin{vmatrix} \omega_{22} & \omega_{23} \\ \omega_{32} & \omega_{33} \end{vmatrix}$ $\beta_{12} = |\tilde{\mathcal{P}}_{12}| = \begin{vmatrix} \omega_{21} & \omega_{23} \\ \omega_{31} & \omega_{33} \end{vmatrix}$ $\beta_{13} = |\tilde{\mathcal{P}}_{13}| = \begin{vmatrix} \omega_{21} & \omega_{22} \\ \omega_{31} & \omega_{32} \end{vmatrix}$

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$\beta_{21} = |\tilde{\mathcal{P}}_{21}| = \begin{vmatrix} \omega_{12} & \omega_{13} \\ \omega_{32} & \omega_{33} \end{vmatrix}$ $\beta_{22} = |\tilde{\mathcal{P}}_{22}| = \begin{vmatrix} \omega_{11} & \omega_{13} \\ \omega_{31} & \omega_{33} \end{vmatrix}$ $\beta_{23} = |\tilde{\mathcal{P}}_{23}| = \begin{vmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{vmatrix}$

$\beta_{31} = |\tilde{\mathcal{P}}_{31}| = \begin{vmatrix} \omega_{12} & \omega_{13} \\ \omega_{22} & \omega_{23} \end{vmatrix}$ $\beta_{32} = |\tilde{\mathcal{P}}_{32}| = \begin{vmatrix} \omega_{11} & \omega_{13} \\ \omega_{21} & \omega_{23} \end{vmatrix}$ $\beta_{33} = |\tilde{\mathcal{P}}_{33}| = \begin{vmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{vmatrix}$

The cofactor $\tilde{\mathcal{Q}} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix}$

Then the adjoint of the matrix $\tilde{\mathcal{P}}$ would symbolically be represented as

$adj\ \tilde{\mathcal{P}} = \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \\ \beta_{13} & \beta_{23} & \beta_{33} \end{bmatrix}$

Note: In soft matrix, we ignore the (-) sign and use the determinant of the minor as the cofactor.

Example 5.3

Let $\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

here,

$\beta_{11} = |\tilde{\mathcal{P}}_{11}| = 0$ $\beta_{12} = |\tilde{\mathcal{P}}_{12}| = 1$ $\beta_{13} = |\tilde{\mathcal{P}}_{13}| = 1$ $\beta_{21} = |\tilde{\mathcal{P}}_{21}| = 1$

$\beta_{22} = |\tilde{\mathcal{P}}_{22}| = 1$ $\beta_{23} = |\tilde{\mathcal{P}}_{23}| = 1$ $\beta_{31} = |\tilde{\mathcal{P}}_{31}| = 0$ $\beta_{32} = |\tilde{\mathcal{P}}_{32}| = 1$

$\beta_{33} = |\tilde{\mathcal{P}}_{33}| = 1$

$\therefore adj\ \tilde{\mathcal{Q}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Proposition 5.3

Let $\tilde{\mathcal{P}}$ be square soft matrix.

(i) $adj\ (\tilde{\mathcal{P}}^{\,T}) = (adj\ \tilde{\mathcal{P}})^{\,T}$

Proof:

Consider $\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $\tilde{\mathcal{P}}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Here, $adj(\tilde{\mathcal{P}}) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $adj(\tilde{\mathcal{P}}^T) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

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$$\left(adj\tilde{\mathcal{P}}\right)^{T} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\therefore adj(\tilde{\mathcal{P}}^{T}) = \left(adj\tilde{\mathcal{P}}\right)^{T}$$

Definition 5.3

(i) A soft square matrix $\tilde{\mathcal{P}}$ is **singular** whenever its determinant equals zero.
i.e., $|\tilde{\mathcal{P}}| = 0$
(ii) A soft square matrix $\tilde{\mathcal{P}}$ is **non-singular** whenever its determinant not equals zero. i.e., $|\tilde{\mathcal{P}}| \neq 0$

Example 5.4

(i) A soft square matrix $\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ is said to be singular, because $\left|\tilde{\mathcal{P}}\right| = 0$

(ii) A soft square matrix $\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is said to be non-singular, because $\left|\tilde{\mathcal{P}}\right| = 1$

Definition 5.4

Let $\tilde{\mathcal{P}} = \left[\alpha_{ij}^{\tilde{\mathcal{P}}}\right]_{\mathfrak{n}\times\mathfrak{n}}$ be a soft matrix. Then the **inverse** of $\tilde{\mathcal{P}}$, as $\tilde{\mathcal{P}}^{-1}$, is defined

$$\tilde{\mathcal{P}}^{-1} = \begin{cases} adj\tilde{\mathcal{P}} & if \det\left(\tilde{\mathcal{P}}\right) = 1 \\ 0 & if \det\left(\tilde{\mathcal{P}}\right) = 0 \end{cases}$$

Note: No inverse exists if $\det\left(\tilde{\mathcal{P}}\right) = 0$

6. APPLICATIONS OF SOFT MATRIX IN DECISION ANALYSIS

This section, we present a problem that involves the concept of determinant and adjoint of Soft Matrices

6.1 Decision Analysis Algorithm based on Determinant and adjoint of soft matrices

Let the set of places be $\mathbb{U}$ and set of attributes be $\mathbb{E}$. We construct a soft set $(\tau, \mathbb{E})$ over $\mathbb{U}$ to describe the places with green surrounding at time t where τ from a mapping $\tau : \mathbb{E} \longrightarrow \mathcal{P}(\mathbb{U})$. similarly, another soft set $(\varphi, \mathbb{E})$ over universal set to describe the places with green surrounding at time t_1. The soft set $(\tau_{\mathcal{P}}, \mathbb{E})$ and $(\varphi_{\mathcal{Q}}, \mathbb{E})$ associate with $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. We determine the complements $(\tau_{\mathcal{P}}, \mathbb{E})^{o}$ and $(\varphi_{\mathcal{Q}}, \mathbb{E})^{o}$ associate with $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$ respectively.

Algorithm

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1. Input the soft matrices associate with $(\tau_{\mathcal{P}}, \mathbb{E})$ and $(\varphi_{Q}, \mathbb{E})$. Determine the addition of $\tilde{\mathcal{P}}$ *and* $\tilde{Q}$.

2. Determine soft matrices $\widetilde{\mathcal{P}^o}$ *and* $\widetilde{Q^o}$. Also find the addition of $\widetilde{\mathcal{P}^o}$ and $\widetilde{Q^o}$.

3. Compute adj $\left(\tilde{\mathcal{P}} + \tilde{Q}\right)$

4. Compute adj $(\widetilde{\mathcal{P}^o} + \widetilde{Q^o})$

5. From the score matrix $S_{\text{adj}\,(\tilde{\mathcal{P}} + \tilde{Q}).\text{adj}\,(\widetilde{\mathcal{P}^o} + \widetilde{Q^o})}$.

6. For each place $\mathcal{P}_i$ in $\mathbb{U}$, calculate the total score S_i

7. Identify $S_K = \max(S_i)$. Then the value concludes that the places $\mathcal{P}_K$ has the highest green surrounding between time t and t_1 respectively.

Case Study

Consider $(\tau_{\mathcal{P}}, \mathbb{E})$ and $(\varphi_{Q}, \mathbb{E})$ be the soft set describe the green surrounding of three places $\mathbb{U} = \{ \mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3\}$ at time t and t_1 respectively. Let $\mathbb{E} = \{e_1$ (Clean air), e_2 (Eco-friendly), e_3 (Green Coverage)$\}$ be the set of attributes where each attribute may vary with time.

$$(\tau_{\mathcal{P}}, \mathbb{E}) = \{ (\mathfrak{e}_1, \{\mathfrak{p}_1, \mathfrak{p}_2\}), (\mathfrak{e}_2, \{\mathfrak{p}_2\}), (\mathfrak{e}_3, \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3\})\}$$

$$(\varphi_{Q}, \mathbb{E}) = \{(\mathfrak{e}_1, \{\mathfrak{p}_1, \mathfrak{p}_2\}), (\mathfrak{e}_2, \{\mathfrak{p}_2, \mathfrak{p}_3\}), (\mathfrak{e}_3, \{\mathfrak{p}_1, \mathfrak{p}_3\})\}$$

The soft matrices from the above parameterized soft set respectively,

$$\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \tilde{Q} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

The soft matrices at time t and t_1 describe the green surrounding by

$$\widetilde{\mathcal{P}^o} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } \widetilde{Q^o} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Then the soft matrix adj $\left(\tilde{\mathcal{P}} + \tilde{Q}\right)$

$$(\tilde{\mathcal{P}} + \tilde{Q}) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \text{ } and \text{ adj} \left(\tilde{\mathcal{P}} + \tilde{Q}\right) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Again, the soft matrix adj $(\widetilde{P^o} + \widetilde{Q^o})$

$$(\widetilde{\mathcal{P}^o} + \widetilde{Q^o}) == \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } adj\ (\widetilde{\mathcal{P}^o} + \widetilde{Q^o}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

We now calculate the score matrix $S_{\text{adj}\,(\tilde{\mathcal{P}} + \tilde{Q}).\text{adj}\,(\widetilde{P^o} + \widetilde{Q^o})}$

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$$S_{\text{adj}\,(\tilde{\mathcal{P}}+\tilde{\mathcal{Q}})\,.\,\text{adj}\,(\widetilde{P^{o}}+\widetilde{Q^{o}})}=\begin{bmatrix}1&1&1\\1&1&1\\1&1&0\end{bmatrix}$$

Total score for healthy environment $S_i = \begin{vmatrix}3\\3\\2\end{vmatrix}$

The evaluation shows that S_i has the maximum value corresponds to the city c_1 and c_2 has got the high score and thus, these places having the green surrounding at time t and t_1

6.2 An approach for dress selection under multiple attribute choices.

Let $\mathbb{U} = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ denotes the set of four dresses. Consider the attribute set, $\mathbb{E} = \{cheap, colourful, fitable, beautiful\} = \{e_1, e_2, e_3, e_4\}$. The friends Mr.$\mathcal{A}$, Mr.$\mathcal{B}$ and Mr.$\mathcal{C}$ wish to select a dress for Mr.$\mathcal{D}$. Their parameter choice are $\tilde{\mathcal{P}} = \{e_1, e_2, e_3\}$ $\tilde{\mathcal{Q}} = \{e_2, e_3, e_4\}$ and $\tilde{\mathcal{R}} = \{e_1, e_2, e_4\}$. Accordingly, the soft sets $(\tau_{\mathcal{P}}, \mathbb{E}), (\varphi_{\mathcal{Q}}, \mathbb{E})$ and $(\omega_{\mathcal{R}}, \mathbb{E})$ describe “the attractiveness of the dresses” based on the choice of Mr.$\mathcal{A}$, Mr.$\mathcal{B}$ and Mr.$\mathcal{C}$.

6.2 Algorithm

1. Initialize soft sets $(\tau_{\mathcal{P}}, \mathbb{E}), (\varphi_{\mathcal{Q}}, \mathbb{E})$ and $(\omega_{\mathcal{R}}, \mathbb{E})$

2. Construct the corresponding soft matrices $\tilde{\mathcal{P}}$, $\tilde{\mathcal{Q}}$ and $\tilde{\mathcal{R}}$.

3. Compute the adjoint $adj(\tilde{\mathcal{P}}), adj(\tilde{\mathcal{Q}})$ and $adj(\tilde{\mathcal{R}})$

4. Form the score matrix $S_{\text{adj}\,(\tilde{\mathcal{P}})+\text{adj}\,(\tilde{\mathcal{Q}})+\text{adj}\,(\tilde{\mathcal{R}})}$

5. For each $\delta_i \in \mathbb{U}$, calculate the total score S_i

6. Identify $S_K = \max(S_i)$. Then the dress δ_k has the most attractive dress.

Case Study

Let the soft sets $(\tau_{\mathcal{P}}, \mathbb{E}), (\varphi_{\mathcal{Q}}, \mathbb{E})$ and $(\omega_{\mathcal{R}}, \mathbb{E})$which describe “the attractiveness of the dresses” from the choice of Mr.$\mathcal{A}$, Mr.$\mathcal{B}$ and Mr.$\mathcal{C}$.

$$(\tau_{\mathcal{P}}, \mathbb{E}) = \{\, (e_1, \{\delta_1, \delta_3, \delta_4\}), (e_2, \{\delta_1, \delta_2, \delta_4\}), (e_3, \{\delta_3, \delta_4\})\}$$

$$(\varphi_{\mathcal{Q}}, \mathbb{E}) = \{\, (e_2, \{\delta_2, \delta_4\}), (e_3, \{\delta_1, \delta_3, \delta_4\}), (e_4, \{\delta_1, \delta_3\})\}$$

$$(\omega_{\mathcal{R}}, \mathbb{E}) = \{\, (e_1, \{\delta_2, \delta_3, \delta_4\}), (e_2, \{\delta_1, \delta_3\}), (e_4, \{\delta_1, \delta_2, \delta_4\})\}$$

Then the soft matrices are

$$\tilde{\mathcal{P}} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad \tilde{\mathcal{Q}} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \; and \; \tilde{\mathcal{R}} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Then the adjoint of soft matrices are

$$adj(\tilde{\mathcal{P}}) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad adj(\tilde{\mathcal{Q}}) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad adj(\tilde{\mathcal{R}}) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

We now calculate the score matrix $S_{\text{adj }(\tilde{\mathcal{P}})+\text{adj }(\tilde{\mathcal{Q}})+\text{adj }(\tilde{\mathcal{R}})}$

$$S_{\text{adj }(\tilde{\mathcal{P}})+\text{adj }(\tilde{\mathcal{Q}})+\text{adj }(\tilde{\mathcal{R}})} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Total score $S_i = \begin{vmatrix} 3 \\ 4 \\ 3 \\ 3 \end{vmatrix}$

The computation show S_i has the maximum value corresponds to dress δ_2 and hence δ_2 has the attractiveness of the dresses among these four dresses for Mr. $\mathcal{D}$

7. CONCLUSION

This study establishes and contributes by introducing soft square matrices and examined the multiplication with their properties. We also defined the determinant and adjoint and studies some of the properties. Then we explored the notation of singular and non-singular soft square matrices. Finally, we demonstrated the applications of the determinant and adjoint in solving decision analysis problems.

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