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A STUDY OF THE ENERGY OF SOFT GRAPHS

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Abstract

The concept of energy in soft graphs are established and also present various findings regarding the energy operations of soft graphs. We demonstrate that some of these results are accompanied by illustrative examples.

Keywords: Energy of soft graphs, Operations involving the energy of soft graphs such as union, intersection, AND and OR.

1. Introduction

D. Molodstov[1] introduced soft set theory in 1999, providing us with innovative methods to address uncertainty. Akram[4] and Nawaz defined various operations on soft graph. Thenge[4] has defined the adjacency matrix of a soft graph and derived several related results. This framework offers a parameterized perspective for uncertainty and soft computing. Let $\mathbb{U}$ denote the universal set, and let Erepresent the collection of all possible parameters associated with the elements of $\mathbb{U}$. These parameters may take the form of either words or sentences. Generally, these parameters are considered as attributes, features, or properties of the elements in $\mathbb{U}$. The pair $(\mathbb{U}, \mathbb{E})$ is known as a soft universe. The power set of $\mathbb{U}$ is denoted as $\mathcal{P}(\mathbb{U})$.

In this article, we introduce the concept of energy of soft graphs and provide various examples. Additionally, we explore several operations related to the energy of soft graphs and demonstrate these with examples.

2 Preliminaries

We will now explore some basic ideas related to soft set theory.

Definition 2.1[1] Let E be a set of parameters and $\mathbb{U}$ be an initial universe set. Let $\mathcal{P}(\mathbb{U})$ represent the power set of U and $\mathbb{A} \subset$ E. A soft set over $\mathbb{U}$ is a pair (F, $\mathbb{A}$), where F is a mapping denoted by F : $\mathbb{A} \rightarrow \mathcal{P}(\mathbb{U})$.

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Definition 2.2[5] Let G^* = (V, E) be a simple graph and $\mathbb{A}$ is non-empty set of parameters of V. A soft set over V is defined as (Ş, $\mathbb{A}$) and a soft set over E as (Ţ, $\mathbb{A}$). Subgraph (Ş (a), Ţ (a)) is therefore represented by F(a) and is referred to as soft graph G.

Another way to express a soft graph is as follows: G = (G^*, Ş, Ţ, $\mathbb{A}$) = {F(x), x ∈ $\mathbb{A}$ }.

This soft graph is referred to in this paper as (F, $\mathbb{A}$).

Definition 2.3[4] Consider G^* = (V, E) be a simple connected graph, $\mathbb{A} \subseteq$ V and (F, $\mathbb{A}$) be a soft graph of G where the function Ş : $\mathbb{A} \to \mathcal{P}$(V) is represented as Ş (x) = { y ∈ V | d(x, y) ≤ 1}, a the function Ţ : $\mathbb{A} \to \mathcal{P}$ (E) is given as Ţ (x) = { xu ∈ E | u∈ Ş (x) } and F(x) = (Ş(x), Ţ (x)). Let $\mathfrak{C}=\cup_{v\in C}$ Ş$(v) = \{v_1, v_2, \ldots \ldots v_n\}$. The adjacency matrix of the soft graph (F, $\mathbb{A}$) is a square matrix of order n x n represented as $\mathcal{A}_{SG}$(F, $\mathbb{A}$) = (a_{ij}), (i; j)th entry a_{ij} is given by

$$a_{ij} = \begin{cases} 1, & if\ v_i\ is\ adjacent\ to\ v_j \\ 0, & if\ v_i\ is\ not\ adjacent\ to\ vj \end{cases}, \text{ i, j =1, 2, 3, …, n}$$

Definition: 2.4 [5] Let $G_1 = \langle G^*, Ş_1, Ţ_1, \mathbb{A}\rangle$ and $G_2 = \langle G^*, Ş_2, Ţ_2, \mathbb{B}\rangle$ be the soft graphs of G^*.Then G_2 is a soft subgraph of G_1 if $\mathbb{B} \subseteq \mathbb{A}$, $\mathbb{H}_2(x)$ is a subgraph of $\mathbb{H}_1(x)$ for all x∈ B.

Definition: 2.5 [5] Let $G_1 = \langle G^*, Ş_1, Ţ_1, \mathbb{A}\rangle$ and $G_2 = \langle G^*, Ş_2, Ţ_2, \mathbb{B}\rangle$ be the soft graphs of G^*. Then

the extended union of G_1 and G_2 is described as $G_1 \sqcup_E G_2 = G = \langle Ş, Ţ, \mathbb{C}\rangle$

where $\mathbb{C} = \mathbb{A} \cup \mathbb{B}$ and for all e∈C,

$$Ţ(e) = \begin{cases} Ţ_1(e) & \text{if } e \in \mathbb{A} - \mathbb{B} \\ Ţ_2(e) & \text{if } e \in \mathbb{B} - \mathbb{A} \\ Ţ_1(e) \cup Ţ_2(e) & \text{if } e \in \mathbb{A} \cap \mathbb{B} \end{cases}$$

$$Ş(e) = \begin{cases} Ş_1(e) & \text{if } e \in \mathbb{A} - \mathbb{B} \\ Ş_2(e) & \text{if } e \in \mathbb{B} - \mathbb{A} \\ Ş_1(e) \cup Ş_2(e) & \text{if } e \in \mathbb{A} \cap \mathbb{B} \end{cases}$$

Therefore, $G_1 \sqcup_E G_2 = \{H(e) = (Ş(e), Ţ(e))/\ e\in\mathbb{C}\}$.

Definition: 2.6 [5] Let $G_1 = \langle G^*, Ş_1, Ţ_1, \mathbb{A}\rangle$ and $G_2 = \langle G^*, Ş_2, Ţ_2, \mathbb{B}\rangle$ be the soft graphs of G^* such that $\mathbb{A} \cap \mathbb{B} \neq \phi$.The restricted union of G_1 and G_2 is defined as $G_1 \sqcup_R G_2 = \langle G^*, Ş, Ţ, \mathbb{C}\rangle$, where $\mathbb{C} = \mathbb{A} \cap \mathbb{B}$ and for all e∈$\mathbb{C}$, $Ş(e) = Ş_1(e) \cup Ş_2(e)$ and $Ţ(e) = Ţ_1(e) \cup Ţ_2(e)$.

3. Energy of Soft Graph

Definition 3.1 Let $G = (G^*, Ş, Ţ, \mathbb{A})$ be a soft graph of a simple connected graph G^* and $\mathcal{A}_{SG}(F, \mathbb{A})$ be the adjacency matrix of soft graph *G.* Then the energy of a soft graph G is defined as the sum of the absolute values of its eigenvalues. That is, $\mathcal{E}_{SG}(G) = \sum_{i=1}^{n} |\lambda_i|$.

Note that, Energy of soft graph is represented as $\mathcal{E}_{SG}(G)$.

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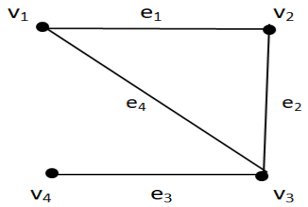
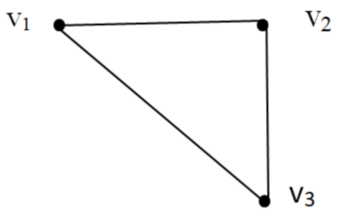
Example 3.2 Let us take a simple graph $G^* = (V, E)$ as shown in below.



Let $\mathbb{A} = \{v_1, v_2\}$. Define $Ţ(x) = \{ xu \in E \mid u \in Ş(x)\}$ and $Ş(x) = \{z \in V \mid d(x, z) \leq 1\}$. Such that, $Ţ(v_1) = \{v_1v_2, v_1v_3\} = \{e_1, e_4\}$, $Ţ(v_2) = \{v_2v_1, v_2v_3\} = \{e_1, e_2\}$ and $Ş(v_1) = \{v_1, v_2, v_3\}$, $Ş(v_2) = \{v_1, v_2, v_3\}$.
Let $\mathfrak{C} = \cup_{v \in C} Ş(v) = \{v_1, v_2, v_3\}$



$$G = (G^*, Ş, Ţ, \mathbb{A})$$

The adjacency matrix of soft graph G is

$$\mathcal{A}_{SG}(G) = \begin{matrix} & v_1 & v_2 & v_3 \\ v_1 & 0 & 1 & 1 \\ v_2 & 1 & 0 & 1 \\ v_3 & 1 & 1 & 0 \end{matrix}$$

To find the eigenvalues of soft graph G:
The characteristic equation is $|\mathcal{A}_{SG}(G) - \lambda I| = 0$
$\therefore \lambda^3 - 3\lambda - 2 = 0$
The eigenvalues of $\mathcal{A}_{SG}(G)$ are -1,-1 and 2.
The energy of a soft graph G is $\mathcal{E}_{SG}(G) = \sum_{i=1}^{n} |\lambda i| = |\lambda_1| + |\lambda_2| + |\lambda_3| = 1+1+2 = 4$
$\therefore$ Energy of soft graph G = 4.

Theorem 3.3 Let $G_1 = \langle G^*, Ş_1, Ţ_1, \mathbb{A}\rangle$ and $G_2 = \langle G^*, Ş_2, Ţ_2, \mathbb{B}\rangle$ be two soft graphs of G^* and G_2 is a soft sub graph of G_1. Then energy of soft sub graph G_2 is less than or equal to the energy of soft graph G_1. That is, $\mathcal{E}_{SG}(G_1) \leq \mathcal{E}_{SG}(G_2)$.

Proof: Given that, G_1 and G_2 are soft graphs with G_2 being a soft subgraph of G_1 and $\mathbb{A}$ is a subset of $\mathbb{B}$.

Let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ and $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_m$ be the eigenvalues of the adjacency matrix of the soft graphs G_1 and G_2 .

According to the definition of a soft subgraph (definition 3.4), the count of eigenvalues of soft graph G_2 is less than or equal to that of soft graph G_1. Since A is a subset of B, the energy

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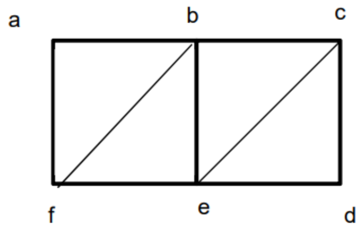
of soft graph G_2 is less than or equal to the energy of soft graph G_1. Therefore $\mathcal{E}_{SG}(G_1) \leq \mathcal{E}_{SG}(G_2)$.

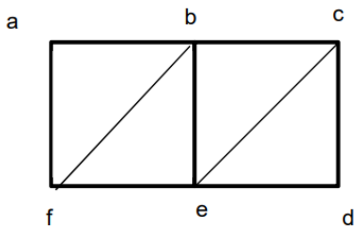
Example 3.4 Consider a simple connected graph $G^* = (V, E)$ as shown in figure.

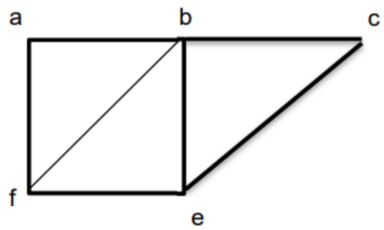


Let $\mathbb{A} = \{b, d, f\}$ and $\mathbb{B} = \{b\}$ be the parameter sets.

We defined $Ş_1 : \mathbb{A} \to \mathcal{P}(\ V)$ by $S_1(x) = \{y \in V / xRy \Leftrightarrow d(x,z) \leq 1\}$ for any $x\epsilon\mathbb{A}$. Such that, $Ş_1(b) = \{a, b, c, e, f\}$, $Ş_1(d) = \{c, d, e\}$ and $Ş_1(f) = \{a, b, e, f\}$. We defined $Ţ_1 : \mathbb{A} \to \mathcal{P}(E)$ by $Ţ_1(x) = \{\ uv \in E / \{\ u, v\} \subseteq S_1(x)\}$ for all $x\epsilon\mathbb{A}$. That is, $Ţ_1(b) = \{ab, bc, bf, be, fe, af\}$, $Ţ_1(d) = \{cd, de, ce\}$, $Ţ_1(f) = \{af, ef, fb, ab\}$. Thus, $\mathbb{H}_1(b) = (Ş_1(b), Ţ_1(b\ ))$, $\mathbb{H}_1(d) = (\ Ş_1(d), Ţ_1(d))$, $\mathbb{H}_1(f) = (Ş_1(\ f), Ţ_1(\ f))$ are subgraphs of G^*.

Here $C = \bigcup_{x\epsilon C} Ş_1(x) = \{a, b, c, d, e, f\}$





The adjacency matrix of soft graph G_1 is $\mathcal{A}_{SG}(G_1) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$

The characteristic equation is $\left| \mathcal{A}_{SG}(G_1) - \lambda I \right| = 0$

$\lambda^6 - 9\lambda^4 - 8\lambda^3 + 10\lambda^2 + 12\lambda + 3 = 0$

The eigenvalues of $\mathcal{A}_{SG}(G)$ are -1.8019, -1.5884, -0.5936, 0.4450, 1.2470, 3.1819.

The energy of a soft graph G_1 is $\mathcal{E}_{SG}\ (G_1) = \sum_{i=1}^{n} \left| \lambda i \right| = 8.8578$.

We defined $Ş_2 : \mathbb{B} \to\ P(V)$ by $Ş_2(\ x) = \{y \in V / xRy \Leftrightarrow\ d(x,z) \leq 1\}$ for any $x\epsilon\mathbb{B}$. That is, $S_2(b) = \{a, b, c, e, f\}$.

We define $Ţ_2 : \mathbb{B} \to\ P(E)$ by $Ţ_2(x) = \{uv \in E / \{u, v\} \subseteq S_2(x)\}$ for each $x\epsilon\mathbb{B}$.

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Such that, $Ţ_2(b) = \{ab, bc, bf, be, fe, af\}$.

Thus, $G_2 = \mathbb{H}_2(b) = (Ş_2(b), Ţ_2(b))$ is subgraph of G^*.

Hence G_2 is a soft graph of G^*.

Here $\mathbb{D} = \cup_{x\epsilon\mathbb{D}} Ş_2(x) = \{a, b, c, e, f\}$

The adjacency matrix of soft graph G_2 is $\mathcal{A}_{SG}\ (G_2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

The characteristic equation is $|\mathcal{A}_{SG}(G_2) - \lambda I| = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_2)$are -1.6180, 1.4728, 0.4626, 0.6180, 2.9354

The energy of a soft graph G_2 is $\mathcal{E}_{SG}\ (G_2) = \sum_{i=1}^{n} |\lambda i| = 7.1068$.

Thus, $\mathcal{E}_{SG}(G_1) \le \mathcal{E}_{SG}(G_2)$.

4. Operations on Energy of soft graphs

Theorem 4.1 Let $G_1 = \langle G^*, S_1, Ţ_1, \mathbb{A}\rangle$ and $G_2 = \langle G^*, S_2, Ţ_2, \mathbb{B}\rangle$ be the soft graphs of G^*. Then $\mathcal{E}_{SG}(G_1 \cup G_2) \le \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(G_2)$.

Proof: Consider $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ and $\mu_1, \mu_2, \ldots, \mu_m$ represent the eigenvalues of the adjacency matrix of the soft graphs $G_1$and G_2 respectively, while $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_p$ denote the eigenvalues of the adjacency matrix of the union of the soft graphs $G_1$and G_2.

According to Theorem 3.2, the union of the soft graphs $G_1$and G_2 is as a subgraph of G^*. Based on theorem 3.3, it follows that $\mathcal{E}_{SG}(G_1) \le \mathcal{E}_{SG}(G_1 \cup G_2)$ and $\mathcal{E}_{SG}(G_2) \le \mathcal{E}_{SG}(G_1 \cup G_2)$

We know that, Hermann Weyl inequalities of eigenvalues, we get, $\sum_{k\epsilon K} \gamma_p \le \sum_{i\epsilon I} \lambda_i + \sum_{j\epsilon J} \mu_j$

Hence, the energy of the union of the soft graphs $G_1$and G_2 is less than or equal to the sum of the energy of the soft graphs $G_1$and G_2.

Therefore, we obtain $\mathcal{E}_{SG}(G_1 \cup G_2) \le \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(G_2)$.

Proposition 4.2 Let $G_1 = \langle G^*, S_1, Ţ_1, \mathbb{A}\rangle$ and $G_2 = \langle G^*, S_2, Ţ_2, \mathbb{B}\rangle$ be two soft graphs of G^* and $A \cap B = \phi$. Then $\mathcal{E}_{SG}(G_1 \sqcup_E G_2) \le \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(G_2)$.

Proof: Let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ and $\mu_1, \mu_2, \ldots, \mu_m$ represent the eigenvalues of the adjacency matrix of the soft graphs $G_1$and G_2 respectively, while $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_p$ denoting the eigenvalues of the adjacency matrix of the soft graph $G_1 \sqcup_E G_2$ of the parameterized set $\mathbb{C} = \mathbb{A} \cap \mathbb{B} = \phi$. According to Theorem 3.2, the extended union of the soft graphs G_1 and G_2 is as a subgraph of G^*.

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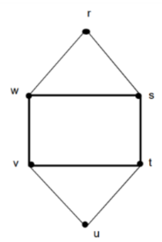
By theorem 3.3, it follows that $\mathcal{E}_{SG}(G_1) \le \mathcal{E}_{SG}(G_1 \sqcup_E G_2)$ and $\mathcal{E}_{SG}(G_2) \le \mathcal{E}_{SG}(G_1 \sqcup_E G_2)$ By Hermann Weyl inequalities of eigenvalues, we get, $\sum_{k\epsilon K} \gamma_p \le \sum_{i\epsilon I} \lambda_i + \sum_{j\epsilon J} \mu_j$ Hence, the energy of the extended union of the soft graphs $G_1$and G_2 is less than or equal to the sum of the energy of the soft graphs $G_1$and G_2. Therefore, we get $\mathcal{E}_{SG}\ (G_1 \sqcup_E G_2) \le \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(\ G_2)$.

Example 4.3 Now we take a simple connected graph $G^* = (V, E)$ as shown in figure:



Let $\mathbb{A} = \{s, v\}$ and $\mathbb{B} = \{r, t\}$ be two parameter sets.

We defined $Ş_1 : \mathbb{A} \to\ P(V)$ by $Ş_1(\ x) = \{y \,\epsilon\, V / xRy\ \Leftrightarrow d(x,z) \le 1\}$ for all $x\epsilon\mathbb{A}$.

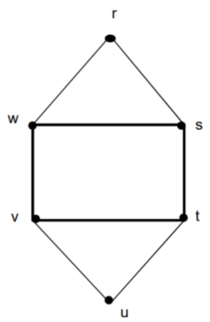
That is, $Ş_1(s) = \{r, s, t, w\}$ and $Ş_1(v) = \{u, v, w, t\}$.

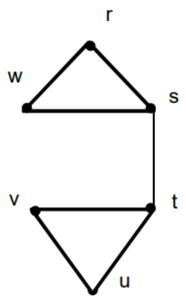
We defined $Ţ_1 : \mathbb{A} \to P(E)$ by $Ţ_1(x) = \{uv \,\epsilon\, E / \{u, v\} \subseteq S_1(x)\}$ for all $x\epsilon A$. Since, $Ţ_1(s) = \{rs, st, sw, rw\}\ and\ Ţ_1(v) = \{vw, uv, vt, ut\}$.

Thus, $H\mathbb{H}_1(s) = \left(Ş_1(s), Ţ_1(s)\right)$, $\mathbb{H}_1(v) = \left(Ş_1(v), Ţ_1(v)\right)$ are subgraphs of G^*.

$\mathbb{H}_2(r) = \left(Ş_2(r), Ţ_2(r)\right)$, $\mathbb{H}_2(t) = \left(Ş_2(t), Ţ_2(t)\right)$ are subgraphs of G^*.

Hence $G_1 = \{\mathbb{H}_1(s), \mathbb{H}(v)\}$ is a soft graph of G^*. Here $\mathbb{C} = \bigcup_{x\epsilon C} Ş_1(x) = \{u, v, w, r, s, t\}$





The adjacency matrix of soft graph G_1 is $\mathcal{A}_{SG}\ (G_1) =$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The characteristic equation is $\left| \mathcal{A}_{SG}(G_1) - \lambda I \right| = 0$

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$\lambda^6 - 8\lambda^4 - 4\lambda^3 + 12\lambda^2 + 8\lambda = 0$

The eigenvalues of A (G) are -2, 1.4142, -0.734, 0, 1.4142, 2.7321

The energy of a soft graph G_1 is $\mathcal{E}_{SG}\ (G_1) = \sum_{i=1}^{n} |\ \lambda i| = 8.3026$.

Hence G_2 is a soft graph of G^*. Here $\mathbb{D} = \bigcup_{x\epsilon\mathbb{D}} Ş_2(x) = \{r, s, t, u, v, w\}$

The adjacency matrix of soft graph G_2 is $\mathcal{A}_{SG}\ (G_2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

The characteristic equation is $|\mathcal{A}_{SG}(G_2) - \lambda I| = 0$.

$\lambda^6 - 7\lambda^4 - 4\lambda^3 + 11\lambda^2 + 12\lambda + 3 = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_2)$ are -1.7321,-1,-1,-0.4142, 1.7321, 2.4142.

The energy of a soft graph G_2 is $\mathcal{E}_{SG}(G_2) = \sum_{i=1}^{n} |\lambda i| = 8.2926$.

The extended union of G_1 *and* G_2 is $G_1 \sqcup_E G_2 = \langle Ş, Ţ, C\rangle$ where $\mathbb{C} = \mathbb{A} \cup \mathbb{B} = \{r, s, t, v\}$ and $Ş(r) = Ş_2(r) = \{r, s, w\}, Ţ(r) = Ţ_2(r) = \{rs, sw, wr\}$, $Ş(s) = Ş_1(s) = \{r, s, t, w\}, T(s) = Ţ_1(s) = \{rs, st, sw, rw\}$, $Ş(t) = Ş_2(t) = \{s, t, u, v\}$, $Ţ(t) = Ţ_2(t) = \{st, tu, tv, uv\}$, $Ş(v) = Ş_1(v) = \{u, v, w, t\}$, $Ţ(v) = Ţ_1(v) = \{vw, uv, vt, ut\}$.

Subgraphs of G^* are $\mathbb{H}(r) = (Ş(r), Ţ(r))$, $\mathbb{H}(s) = (Ş(s), Ţ(s))$, $\mathbb{H}(t) = (Ş(t), Ţ(t))$, $\mathbb{H}(v) = (S(v), Ţ(v))$.

Hence, $G_1 \sqcup_E G_2 = \{\mathbb{H}(r), \mathbb{H}(s), \mathbb{H}(t), \mathbb{H}(v)\}$.

Here, $\mathfrak{C} = \bigcup_{x\epsilon C} Ş_1(x) = \{u, v, w, r, s, t\}$.

The adjacency matrix of soft graph $G_1 \sqcup_E G_2$ is given by $\mathcal{A}_{SG}(G_1 \sqcup_E G_2) =$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The characteristic equation is $|\mathcal{A}_{SG}(G_1 \sqcup_E G_2) - \lambda I| = 0$

$\lambda^6 - 8\lambda^4 - 4\lambda^3 + 12\lambda^2 + 8\lambda = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_1 \sqcup_E G_2)$are -2, 1.4142, -0.734, 0, 1.4142, 2.7321

The energy of a soft graph $(G_1 \sqcup_E G_2)$ is $\mathcal{E}_{SG}(G_1 \sqcup_E G_2)\ ) = \sum_{i=1}^{n} |\lambda i| = 8.3026$.

Thus, $\mathcal{E}_{SG}(G_1 \sqcup_E G_2) \leq \mathcal{E}_{SG}(\ G_1) + \mathcal{E}_{SG}(\ G_2)$.

Proposition 4.4 Consider $G_1 = \langle G^*, S_1, Ţ_1, \mathbb{A}\rangle$ and $G_2 = \langle G^*, S_2, Ţ_2, \mathbb{B}\rangle$ be two soft graphs of G^* with $\mathbb{A} \cap \mathbb{B} \neq \phi$. Then $\mathcal{E}_{SG}(G_1 \sqcup_R G_2) \leq \mathcal{E}_{SG}(\ G_1) + \mathcal{E}_{SG}(\ G_2)$.

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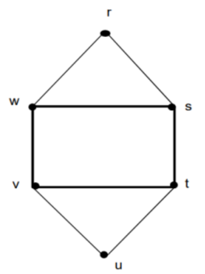
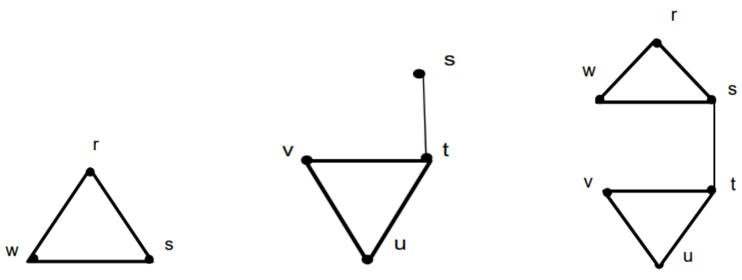
Proof: Assume $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ and $\mu_1, \mu_2, \ldots, \mu_m$ represent the eigenvalues of the adjacency matrix of the soft graphs $G_1 = \langle G^*, S_1, Ţ_1, A\rangle$ and $G_2 = \langle G^*, S_2, T_2, B\rangle$respectively, while $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_p$ denote the eigenvalues of the adjacency matrix of the soft graph $G_1 \sqcup_R G_2$ of the parameterized set $\mathbb{C} = \mathbb{A} \cap \mathbb{B} \neq \phi$. According to Theorem 3.2, the union of the soft graphs $G_1$and G_2 is as a subgraph of G^*. Based on theorem 3.3, it follows that $\mathcal{E}_{SG}(G_1) \leq \mathcal{E}_{SG}(G_1 \sqcup_R G_2)$ and $\mathcal{E}_{SG}(G_2) \leq \mathcal{E}_{SG}(G_1 \sqcup_R G_2)$

By Hermann Weyl inequalities of eigenvalues, we get, $\sum_{k\epsilon K} \gamma_p \leq \sum_{i\epsilon I} \lambda_i + \sum_{j\epsilon J} \mu_j$ Hence, the energy of the restricted union of the soft graphs $G_1$and G_2 is less than or equal to the sum of the energy of the soft graphs $G_1$and G_2. Therefore, we obtain $\mathcal{E}_{SG}(G_1 \sqcup_R G_2) \leq \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(G_2)$.

Example: 4.5 Now take a simple connected graph $G^* = (V, E)$ as shown below**:**



Let $\mathbb{A} = \{r, t\}$ and $\mathbb{B} = \{t, v\}$ be the parameter sets. We represented $Ş_1: \mathbb{A} \rightarrow P(V)$ by $Ş_1(x) = \{y \in V/xRy \Leftrightarrow d(x,z) \leq 1\}$ for all $x\epsilon\mathbb{A}$. That is, $Ş_1(r) = \{r, s, w\}$ and $Ş_1(t) = \{s, t, u, v\}$. We defined $Ţ_1: \mathbb{A} \rightarrow P(E)$ by $Ţ_1(x) = \{uv \in E/\{u, v\} \subseteq Ş_1(x)\}$ for all $x\epsilon A$. Hence $Ţ_1(t) = \{st, tu, uv, tv\}$ and $Ţ_1(r) = \{rs, sw, rw\}$. Thus, $\mathbb{H}_1(s) = (Ş_1(r), Ţ_1(r)), \mathbb{H}_1(t) = (Ş(t), Ţ_1(t))$ are subgraphs of G^* as depicted in figure:



$H_1(r)$ $H_1(t)$ G_1

Hence $G_1 = \{\mathbb{H}_1(r), \mathbb{H}_1(t)\}$ is a soft graph of G^*. Here $\mathfrak{C} = \cup_{x\epsilon C} Ş_1(x) = \{u, v, w, r, s, t\}$

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The adjacency matrix of soft graph G_1 is $\mathcal{A}_{SG}(G_1) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

The characteristic equation is $|\mathcal{A}_{SG}(G_1) - \lambda I| = 0$
$\lambda^6 - 7\lambda^4 - 4\lambda^3 + 11\lambda^2 + 12\lambda + 3 = 0$

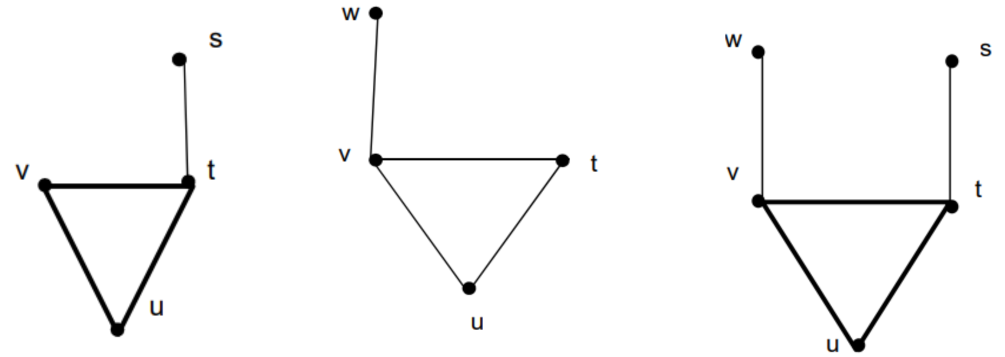
The eigenvalues of A (G_1) are λ_1= - 1.7321, λ_2 = -1, λ_3 = -1, λ_4 = -0.4142, λ_5 =1.7321, λ_6 =2.414 .

The energy of a soft graph G_1 is $\mathcal{E}_{SG}(G_1) = \sum_{i=1}^{n} |\lambda i| = 8.2926$

∴ Energy of soft graph $\mathcal{E}_{SG}(G_1) = 8.2926$

We defined $Ş_2 : \mathbb{B} \to P(V)$ by $Ş_2(x) = \{y \in V / xRy \Leftrightarrow d(x,z) \le 1\}$ for each $x \epsilon \mathbb{B}$. Therefore, $Ş_2(t) = \{s,t,u,v\}$ *and* $Ş_2(v) = \{u,v,w,t\}$. We defined an approximate function $Ţ_2 : \mathbb{B} \to P(E)$ by $Ţ_2(x) = \{uv \in E / \{u,v\} \subseteq Ş_2(x)\}$ for any $x \epsilon \mathbb{B}$. Such that, $Ţ_2(t) = \{st, tu, tv, uv\}$ *and* $Ţ_2(v) = \{vu, vw, vt, tu\}$.

Thus, $\mathbb{H}_2(t) = (Ş_2(t), Ţ_2(t))$ *and* $\mathbb{H}_2(v) = (Ş_2(v), Ţ_2(v))$ is subgraph of G^* as shown in figure:



ℍ 2(t) ℍ 2(v) G2

Hence G_2 is a soft graph of G^*.

Here $\mathbb{D} = \bigcup_{x \epsilon \mathbb{D}} Ş_2(x) = \{s,t,u,v,w\}$

The adjacency matrix of soft graph G_2 is $\mathcal{A}_{SG}(G_2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

The characteristic equation is $|\mathcal{A}_{SG}(G_2) - \lambda I| = 0$
$\lambda^5 - 5\lambda^3 + 2\lambda^2 + 3\lambda = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_2)$are λ_1 = -1.6168, λ_2 = -1.3028, λ_3 = 0, λ_4 = 0.6180, λ_5 = 2.3028.

The energy of a soft graph G_2 is $\mathcal{E}_{SG}(G_2) = \sum_{i=1}^{n} |\lambda i| = 5.8416$

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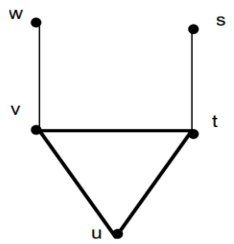
The restricted union of G_1 *and* G_2 is $G_1 \sqcup_R G_2 = \langle S, Ţ, C\rangle$ where $C = A \cap B = \{t\}$ and $Ş(t) = Ş_2(t) = \{s, t, u, v\}$, $Ţ(t) = Ţ_2(t) = \{st, tu, tv, uv\}$
Subgraphs of G^* is $\mathbb{H}(t) = (Ş(t), Ţ(t))$
Hence, $G_1 \sqcup_E G_2 = \{\mathbb{H}(t)\}$.



$G_1 \sqcup_E G_2$

Here $\mathfrak{C} = \cup_{x\epsilon\mathfrak{C}} Ş(x) = \{s, t, u, v\}$

The adjacency matrix of soft graph $G_1 \sqcup_E G_2$ is $\mathcal{A}_{SG}(G_1 \sqcup_E G_2) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

The characteristic equation is $|\mathcal{A}_{SG}(G_1 \sqcup_E G_2)) - \lambda I| = 0$
$\lambda^4 - 3\lambda^2 - 2\lambda + 1 = 0$
The eigenvalues of $\mathcal{A}_{SG}(G_1 \sqcup_E G_2)$are $\lambda_1 = -1.4812$, $\lambda_2 = -1$, $\lambda_3 = 0.3111$, $\lambda_4 = 2.1701$
The energy of a soft graph $\mathcal{E}_{SG}(G_1 \sqcup_E G_2)$ is $\mathcal{E}_{SG}\ (G_1 \sqcup_R G_2) = \sum_{i=1}^{n} |\lambda i| = 4.9624$
Thus, $\mathcal{E}_{SG}(G_1 \sqcup_R G_2) \leq \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(G_2)$.

Corollary: 4.6 Let $G_1 = \langle G^*, Ş_1, Ţ_1, A\rangle$, $G_2 = \langle G^*, Ş_2, Ţ_2, B\rangle$ and $G_3 = \langle G^*, Ş_3, Ţ_3, C\rangle$ be the soft graphs of G^*. Then $\mathcal{E}_{SG}(G_1 \cup G_2 \cup G_3) \leq \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(G_2) + \mathcal{E}_{SG}(G_2)$.

Corollary: 4.7 Assume G_1, G_2, $G_3, \dots G_n$ be the soft graphs. Since $\mathcal{E}_{SG}(G_1 \cup G_2 \cup G_3 \cup \dots G_n) \leq \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(G_2) + \mathcal{E}_{SG}(G_2) + \mathcal{E}_{SG}(G_3) + \cdots \mathcal{E}_{SG}(G_n)$.

Theorem 4.8 Let $G_1 = \langle G^*, Ş_1, Ţ_1, \mathbb{A}\rangle$ and $G_2 = \langle G^*, Ş_2, Ţ_2, \mathbb{B}\rangle$ be two soft graphs of G^*. Then $\mathcal{E}_{SG}(G_1 \cap G_2) \leq \mathcal{E}_{SG}(G_1).\mathcal{E}_{SG}(G_2)$.

Proof: Assume $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ and $\mu_1, \mu_2, \dots, \mu_m$ represent the eigenvalues of the adjacency matrix of the soft graphs $G_1$and G_2 respectively, while $\Upsilon_1, \Upsilon_2, \dots, \Upsilon_p$ denote the eigenvalues of the adjacency matrix of the intersection of soft graphs $G_1$and G_2. According to Theorem 3.2, the intersection of soft graphs $G_1$and G_2 is as a subgraph of G^*. Based on theorem 3.3, it follows that $\mathcal{E}_{SG}(G_1 \cap G_2) \leq \mathcal{E}_{SG}(G_1)$ and $\mathcal{E}_{SG}(G_1 \cap G_2) \leq \mathcal{E}_{SG}(G_2)$

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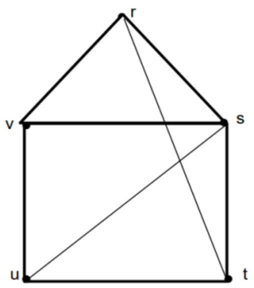
Hence, the energy of the intersection of soft graphs $G_1$and G_2 is less or equal to the product of energy of soft graphs G_1 and G_1. Therefore, $\mathcal{E}_{SG}(G_1 \cap G_2) \leq \mathcal{E}_{SG}(G_1).\mathcal{E}_{SG}(G_2)$.

Proposition 4.9 Let $G_1 = \langle G^*, Ş_1, Ţ_1, \mathbb{A}\rangle$ and $G_2 = \langle G^*, Ş_2, Ţ_2, \mathbb{B}\rangle$ be two soft graphs of G^*. Then $\mathcal{E}_{SG}(G_1 \sqcap_E G_2) \leq \mathcal{E}_{SG}(G_1).\mathcal{E}_{SG}(G_2)$.

Proof: Let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ and $\mu_1, \mu_2, \ldots, \mu_m$ represent the eigenvalues of the adjacency matrix of the soft graphs $G_1$and G_2 respectively, while $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_p$ denote the eigenvalues of the adjacency matrix of the extended intersection of soft graphs $G_1$and G_2 of the parameterized set $\mathbb{C} = \mathbb{A} \cap \mathbb{B} = \phi$
According to Theorem 3.2, the extended intersection of soft graphs $G_1$and G_2 is as a subgraph of G^*.
Based on theorem 3.3, it follows that $\mathcal{E}_{SG}(G_1 \sqcap_E G_2) \leq \mathcal{E}_{SG}(G_1)$ and $\mathcal{E}_{SG}(G_1 \sqcap_E G_2) \leq \mathcal{E}_{SG}(G_2)$

Hence, the energy of the extended intersection of soft graphs $G_1$and G_2 is less or equal to the product of energy of soft graphs G_1 and G_1. Therefore, $\mathcal{E}_{SG}(G_1 \sqcap_E G_2) \leq \mathcal{E}_{SG}(G_1).\mathcal{E}_{SG}(G_2)$.

Example: 4.10 Let a simple connected graph $G^* = (V, E)$ as shown



$G^* = (V, E)$

Let $\mathbb{A} = \{r, t, s\}$ and $\mathbb{B} = \{u, v\}$ be the parameter sets. We defined $Ş_1: \mathbb{A} \to P(V)$ by $Ş_1(x) = \{y \in V / xRy \Leftrightarrow d(x, z) \leq 1\}$ for all $x \epsilon \mathbb{A}$. That is, $Ş_1(r) = \{r, s, t, v\}$, $Ş_1(s) = \{r, s, t, u, v\}$ and $Ş_1(t) = \{s, t, u, v\}$.

We defined $T_1: \mathbb{A} \to P(E)$ by $T_1(x) = \{uv \in E / \{u, v\} \subseteq S_1(x)\}$ for all $x \epsilon \mathbb{A}$. That is, $Ţ_1(r) = \{rv, rs, rt, sv\}, Ţ_1(s) = \{rs, su, sv, st, rv, uv, ut\}\ and\ Ţ_1(t) = \{st, tu, tr, su, rt\}$.
Thus, $\mathbb{H}_1(r) = (Ş_1(r), Ţ_1(r)), \mathbb{H}_1(s) = (Ş_1(s), Ţ_1(s)), \mathbb{H}_1(t) = (Ş_1(t), Ţ_1(t))$ are subgraphs of G^* as shown in figure:

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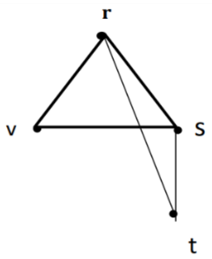
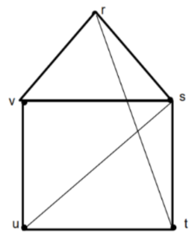
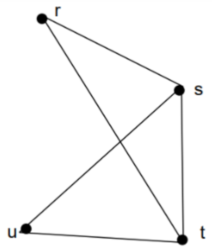


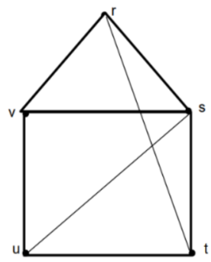
$\mathbb{H}_1(r)$



$\mathbb{H}_1(s)$



$\mathbb{H}_1(t)$



G_1

Hence, the soft graph of G^* is $G_1 = \{\mathbb{H}_1(r),\ \mathbb{H}_1(s), \mathbb{H}_1(t)\}$

Here $\mathfrak{C} = \cup_{x\epsilon C} S_1(x) = \{r, s, t, u, v\}$

The adjacency matrix of soft graph G_1 is $\mathcal{A}_{SG}(G_1) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

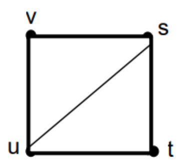
The characteristic equation is $\left|\mathcal{A}_{SG}(G_1) - \lambda I\right| = 0$
$\lambda^5 - 8\lambda^3 - 8\lambda^2 = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_1)$are λ_1= -2, λ_2 = -1.2361, λ_3 = 0, λ_4 = 0, λ_5 = 3.2361
The energy of a soft graph G_1 is $\mathcal{E}_{SG}(G_1) = \sum_{i=1}^{n} |\lambda i| = 6.4722$

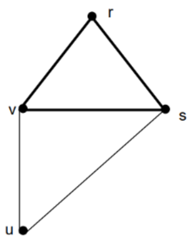
We defined $Ş_2: B \to P(V)$ by $Ş_2(x) = \{y \in V / xRy \Leftrightarrow d(x,z) \le 1\}$ for all $x \epsilon B$. That is, $Ş_2(u) = \{s, t, u, v\}$ and $Ş_2(v) = \{r, s, u, v,\}$

We defined $Ţ_2: B \to P(E)$ by $Ţ_2(x) = \{uv \in E / \{u, v\} \subseteq Ş_2(x)\}$ for all $x \epsilon A$. That is, $Ţ_2(u) = \{uv, ut, us, st, sv\}$ and $Ţ_2(v) = \{vr, uv, us, vs, rs\}$

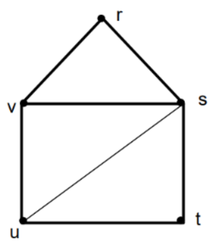
Thus, $\mathbb{H}_2(u) = (Ş_2(u), Ţ_2(u))$ and $\mathbb{H}_2(v) = (Ş_2(v), Ţ_2(v))$ is subgraph of G^* as shown in figure:



$\mathbb{H}_2(u)$



$\mathbb{H}_2(v)$



G_2

Hence G_2 is a soft graph of G^*.
Here $\mathfrak{D} = \cup_{x\epsilon D} Ş_2(x) = \{s, t, u, v, w\}$

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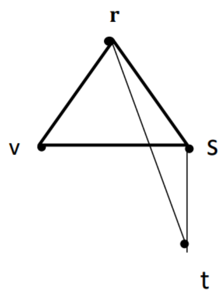
The adjacency matrix of soft graph G_2 is $\mathcal{A}_{SG}(G_2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

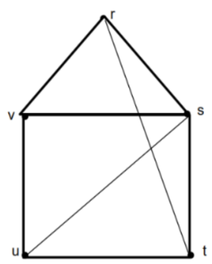
The characteristic equation is $\left| \mathcal{A}_{SG}(G_2) - \lambda I \right| = 0$
$\lambda^5 - 7\lambda^3 - 6\lambda^2 + 3\lambda + 2 = 0$
The eigenvalues of $\mathcal{A}_{SG}(G_2)$ are λ_1= -1.6180, λ_2 = -1.4726, λ_3 = - 0.4626, λ_4 = 0.6180, λ_5 = 2.9354

The extended intersection of G_1 *and* G_2 is $G_1 \sqcap_E G_2 = \langle S, Ţ, \mathbb{C} \rangle$ where $\mathbb{C} = \mathbb{A} \cup \mathbb{B} = \{r, s, t, u, v\}$ and $Ș(r) = \{r, s, t, v\}, Ţ(r) = \{rv, rs, rt, sv\}$, $Ș(s) = \{r, s, t, u, v\}, T(s) = \{rs, su, sv, st, rv, uv, ut\}$, $Ș(t) = \{r, s, t.u\}$, $Ţ(t) = \{st, tu, tr, su, rt\}$, $Ș(u) = \{u, v, s, t\}, Ţ(u) = \{uv, ut, us, st, sv\}$, $Ș(v) = \{r, s, u, v, \}, T(v) = \{vr, uv, us, vs, rs\}$.

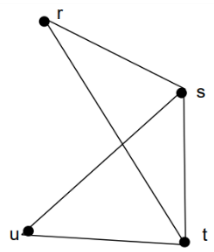
The subgraphs of G^* are $\mathbb{H}(r) = (Ș(r), Ţ(r))$, $\mathbb{H}(s) = (Ș(s), Ţ(s))$, $\mathbb{H}(t) = (Ș(t), Ţ(t))$, $\mathbb{H}(u) = (Ș(u), Ţ(u))$, $\mathbb{H}(v) = (Ș(v), Ţ(v))$.
Hence $G_1 \sqcap_E G_2 = \{\mathbb{H}(r), \mathbb{H}(s), \mathbb{H}(t), \mathbb{H}(u), \mathbb{H}(v)\}$.
Here $\mathcal{R} = \cup_{x \epsilon C} Ș(x) = \{r, s, t, u, v\}$



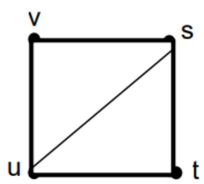
$\mathbb{H}(r)$



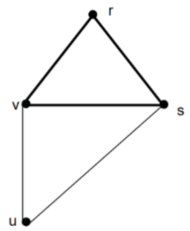
$\mathbb{H}(s)$



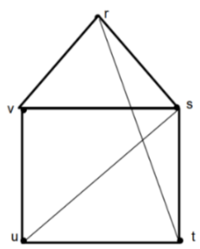
$\mathbb{H}(t)$



$\mathbb{H}(u)$



$\mathbb{H}(v)$



$G_1 \sqcap_E G_2$

The adjacency matrix of soft graph $G_1 \sqcap_E G_2$ is $\mathcal{A}_{SG}(G_1 \sqcap_E G_2) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

The characteristic equation is $\left| \mathcal{A}_{SG}(G_1 \sqcap_E G_2) - \lambda I \right| = 0$

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$\lambda^5 - 8\lambda^3 - 8\lambda^2 = 0$

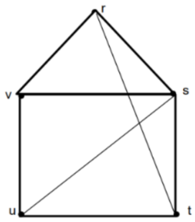
The eigenvalues of $\mathcal{A}_{SG}(G_1 \sqcap_E G_2)$are λ_1= -2, λ_2 = -1.2361, λ_3 = 0, λ_4 = 0, λ_5 = 3.2361 The energy of a soft graph $(G_1 \sqcap_E G_2)$ is $\mathcal{E}_{SG}$ $(G_1 \sqcap_E G_2)$ = $\sum_{i=1}^{n} |\lambda i|$= 6.4722 ∴ Energy of soft graph $\mathcal{E}_{SG}$ $(G_1 \sqcap_E G_2)$ = 6.4722. Hence $\mathcal{E}_{SG}(G_1 \sqcap_E G_2) \leq \mathcal{E}_{SG}(G_1).\mathcal{E}_{SG}(G_2)$.

Proposition 4.11 Let $G_1 = \langle G^*, Ş_1, Ţ_1, A\rangle$ and $G_2 = \langle G^*, Ş_2, Ţ_2, B\rangle$ be the soft graphs of G^* and $\mathbb{A} \cap \mathbb{B} \neq \phi$. Then $\mathcal{E}_{SG}(G_1 \sqcap_R G_2) \leq \mathcal{E}_{SG}(G_1).\mathcal{E}_{SG}(G_2)$.

Proof: Let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ and $\mu_1, \mu_2, \ldots, \mu_m$ represent the eigenvalues of the adjacency matrix of the soft graphs $G_1$and G_2 respectively, while $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_p$ denote the eigenvalues of the adjacency matrix of the restricted intersection of soft graphs $G_1$and G_2 of the parameterized set $\mathbb{C} = \mathbb{A} \cap \mathbb{B} \neq \phi$
According to Theorem 3.2, the restricted intersection of soft graphs $G_1$and G_2 is as a subgraph of G^*.
Based on theorem 3.3, it follows that $\mathcal{E}_{SG}(G_1 \sqcap_R G_2) \leq \mathcal{E}_{SG}(G_1)$ and $\mathcal{E}_{SG}(G_1 \sqcap_R G_2) \leq \mathcal{E}_{SG}(G_2)$

Hence, the energy of the restricted intersection of soft graphs $G_1$and G_2 is less or equal to the product of energy of soft graphs G_1 and G_1. Therefore, $\mathcal{E}_{SG}(G_1 \sqcap_R G_2) \leq \mathcal{E}_{SG}(G_1).\mathcal{E}_{SG}(G_2)$.

Example 4.12 Let $G^* = (V, E)$be a simple connected graph as shown in figure:



Let $\mathbb{A} = \{r, t, s\}$ and $\mathbb{B} = \{u, t\}$ be the parameter sets.

We defined $S_1: \mathbb{A} \to \mathcal{P}(V)$ by $Ş_1(x) = \{y \in V / xRy \Leftrightarrow d(x, z) \leq 1\}$ for all $x \epsilon \mathbb{A}$. That is, $Ş_1(r) = \{r, s, t, v\}$, $Ş_1(s) = \{r, s, t, u, v\}$ and $Ş_1(t) = \{s, t, u, v\}$.

We defined $Ţ_1: \mathbb{A} \to \mathcal{P}(E)$ by $Ţ_1(x) = \{uv \in E / \{u, v\} \subseteq S_1(x)\}$ for all $x \epsilon \mathbb{A}$. That is, $Ţ_1(r) = \{rv, rs, rt, sv\}$, $Ţ_1(s) = \{rs, su, sv, st, rv, uv, ut\}$ and $Ţ_1(t) = \{st, tu, tr, su, rt\}$.
Thus, $\mathbb{H}_1(r) = (Ş_1(r), Ţ_1(r))$, $\mathbb{H}_1(s) = (Ş_1(s), Ţ_1(s))$, $\mathbb{H}_1(t) = (Ş_1(t), Ţ_1(t))$ are subgraphs of G^* as given in figure:

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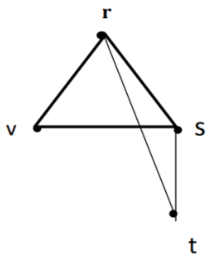
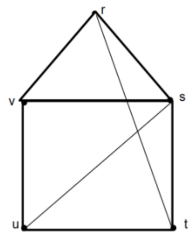
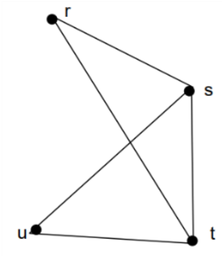


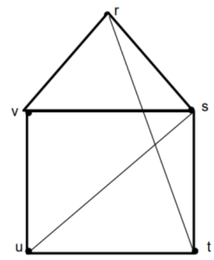
$\mathbb{H}_1(r)$



$\mathbb{H}_1(s)$



$\mathbb{H}_1(t)$



G_1

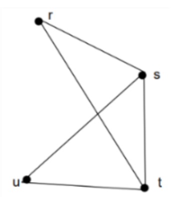
Hence the soft graph of $G^*$is $G_1 = \{\mathbb{H}_1(r), \mathbb{H}_1(t), \mathbb{H}_1(s)\}$. Here $\mathbb{C} = \cup_{x \epsilon C} Ş_1(x) = \{r, s, t, u, v\}$

The adjacency matrix of soft graph $G_1$is $\mathcal{A}_{SG}(G_1) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

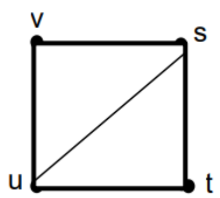
The characteristic equation is $|\mathcal{A}_{SG}(G_1) - \lambda I| = 0$
$\lambda^5 - 8\lambda^3 - 8\lambda^2 = 0$
The eigenvalues of $\mathcal{A}_{SG}(G_1)$ are λ_1= -2, λ_2 = -1.2361, λ_3 = 0, λ_4 = 0, λ_5 = 3.2361
The energy of a soft graph G_1 is $\mathcal{E}_{SG}$ (G_1) = $\sum_{i=1}^{n} |\lambda i|$= 6.4722
Energy of soft graph $\mathcal{E}_{SG}$ (G_1) = 6.4722.
We defined $S_2: \mathbb{B} \to P(V)$ by $Ş_2(x) = \{y \,\epsilon\, V / xRy \iff d(x, z) \leq 1\}$ for all $x \epsilon \mathbb{B}$. That is, $Ş_2(u) = \{s, t, u, v\}$ and $Ş_2(t) = \{r, s, t, u\}$.

We defined $Ţ_2: \mathbb{B} \to P(E)$ by $Ţ_2(x) = \{uv \,\epsilon\, E / \{u, v\} \subseteq Ş_2(x)\}$ for all $x \epsilon \mathbb{B}$. That is, $Ţ_2(u) = \{uv, ut, us, st, sv\}$ and $Ţ_2(t) = \{rs, st, rt, tu, us\}$.

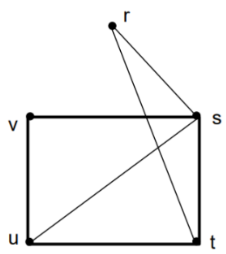
Thus, $\mathbb{H}_2(u) = \left(Ş_2(u), Ţ_2(u)\right)$ and $\mathbb{H}_2(t) = \left(Ş_2(t), T_2(t)\right)$ is subgraph of G^* as shown in figure:



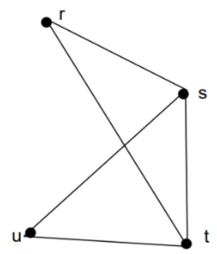
$\mathbb{H}_2(t)$



$\mathbb{H}_2(u)$



G_2



$G_1 \sqcap_R G_2$

Hence G_2 is a soft graph of G^*. Here $\mathbb{D} = \cup_{x \epsilon \mathbb{D}} Ş_2(x) = \{r, s, t, u, v\}$

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The adjacency matrix of soft graph G_2 is $\mathcal{A}_{SG}(G_2) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

The characteristic equation is $|\mathcal{A}_{SG}(G_2) - \lambda I| = 0$

$\lambda^5 - 7\lambda^3 - 6\lambda^2 + 3\lambda + 2 = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_2)$ are λ_1= -1.6180, λ_2 = -1.4728, λ_3 = -0.4626, λ_4 = 0.6180, λ_5 = 2.9354

The energy of a soft graph G_2 is $\mathcal{E}_{SG}\ (G_2) = \sum_{i=1}^{n} |\lambda i| = 7.1068$

The extended intersection of G_1 *and* G_2 is $G_1 \sqcap_R G_2 = \langle Ş, Ţ, C \rangle$ where $\mathbb{C} = \mathbb{A} \cap \mathbb{B} = \{t\}$ and $S(t) = \{r, s, t.u\}$, $Ţ(t) = \{uv, us, ut, vs, st\}$.

Subgraphs of G^* is $\mathbb{H}(t) = (Ş(t), Ţ(t))$.

Hence $G_1 \sqcap_R G_2 = \{\mathbb{H}(r), \mathbb{H}(s), \mathbb{H}(t), \mathbb{H}(u), \mathbb{H}(v)\}$.

Here $\mathfrak{D} = \bigcup_{x \epsilon C} Ş(x) = \{r, s, t, u\}$

The adjacency matrix of soft graph $G_1 \sqcap_E G_2$ is $\mathcal{A}_{SG}(G_1 \sqcap_R G_2) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

The characteristic equation is $|\mathcal{A}_{SG}(G_1 \sqcap_R G_2) - \lambda I| = 0$

$\lambda^4 - 5\lambda^2 - 4\lambda = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_1 \sqcap_R G_2)$are λ_1= -1.5616, λ_2 = -1, λ_3 = 0, λ_4 = 2.5616

The energy of a soft graph $(G_1 \sqcap_R G_2)$ is $\mathcal{E}_{SG}\ (G_1 \sqcap_R G_2) = \sum_{i=1}^{n} |\lambda i| = 5.1232$

Hence $\mathcal{E}_{SG}(G_1 \sqcap_R G_2) \leq \mathcal{E}_{SG}(G_1).\mathcal{E}_{SG}(G_2)$.

Proposition 4.13 Let $G_1 = \langle G^*, Ş_1, Ţ_1, A \rangle$ and $G_2 = \langle G^*, Ş_2, Ţ_2, B \rangle$ be two soft graphs of G^*. Then $\mathcal{E}_{SG}(G_1 \vee G_2) \leq \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(G_2)$.

Proof: Let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ and $\mu_1, \mu_2, \ldots, \mu_m$ represent the eigenvalues of the adjacency matrix of the soft graphs $G_1$and G_2 respectively, while $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_p$ denote the eigenvalues of the adjacency matrix of the OR operations of soft graph $G_1$and G_2 denoted by $G_1 \vee G_2$. According to Theorem 3.2, the soft graph $G_1 \vee G_2$ is as a subgraph of G^*. Based on theorem 3.3, it follows that $\mathcal{E}_{SG}(G_1 \vee G_2) \leq \mathcal{E}_{SG}(G_1)$ and $\mathcal{E}_{SG}(G_1 \vee G_2) \leq \mathcal{E}_{SG}(G_2)$

By, Hermann Weyl inequalities of eigenvalues, we get, $\sum_{k \epsilon K} \gamma_p \leq \sum_{i \epsilon I} \lambda_i + \sum_{j \epsilon J} \mu_j$

Hence, the energy of the to the energy of the soft graph $G_1 \vee G_2$ is less than or equal to the sum of the energy of the soft graphs $G_1$and G_2. Therefore, we have $\mathcal{E}_{SG}(G_1 \vee G_2) \leq \mathcal{E}_{SG}(G_1) + \mathcal{E}_{SG}(G_2)$.

Example: 4.14 Assume a simple connected graph $G^* = (V, E)$ as depicted in figure:

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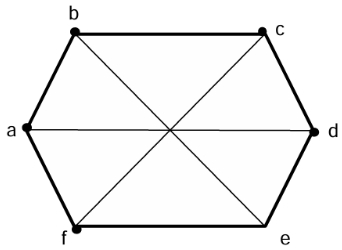


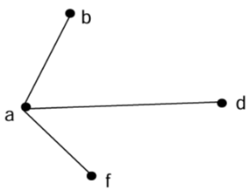
$G^* = (V, E)$

Let $\mathbb{A} = \{a, d\}$ and $B = \{b, e\}$ be the parameter sets.

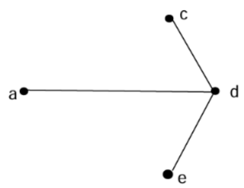
We defined $Ţ_1 : \mathbb{A} \to \mathcal{P}(\ E)$ by $Ţ_1(\ x) = \{\ uv \in E / \{\ u,\ \ v\} \subseteq Ş_1(\ x)\}$ for any $x \epsilon \mathbb{A}$. Such that, $Ţ(a) = \{ab, af, ad\}\ and\ Ţ_1(d) = \{cd, de, ad\}$.

We defined $Ş_1 : \mathbb{A} \to \mathcal{P}(V)$ by $Ş_1(x) = \{y \in V / xRy\ \Leftrightarrow d(x, z) \leq 1\}$ for any $x \epsilon \mathbb{A}$. Therefore, $Ş_1(a) = \{a, b, d, f\}$ and $Ş_1(d) = \{a, c, d, e\}$.

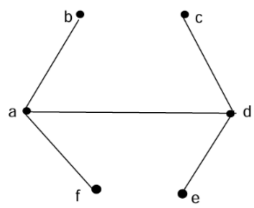
Thus, $\mathbb{H}_1(a) = \left(Ş_1(r), Ţ_1(r)\right)\ and\ \mathbb{H}_1(d) = \left(Ş_1(d), Ţ_1(d)\right)$ are subgraphs of G^* as given in figure:



$\mathbb{H}_1(a)$



$\mathbb{H}_1(d)$



G_1

Hence $G_1 = \{\mathbb{H}_1(a), \mathbb{H}_1(d)\}$ is a soft graph of G^*. Here $\mathbb{C} = \cup_{x \epsilon C} Ş_1(x) = \{a, b, c, d, e, f\}$

The adjacency matrix of soft graph G_1 is $\mathcal{A}_{SG}(G_1) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

The characteristic equation is $\left| \mathcal{A}_{SG}(G_1) - \lambda I \right| = 0$
$\lambda^6 - 4\lambda^4 - \lambda^3 + 2\lambda^2 + 2\lambda = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_1)$are λ_1= -1.9230, λ_2 = -1.7778, λ_3 = 1, λ_4 = - 0.5726 + 0.5071i, λ_5 = - 0.5726 - 0.5071i , λ_6 =0.

The energy of a soft graph G_1 is $\mathcal{E}_{SG}\ (\ G_1) = \sum_{i=1}^{n} \left|\ \lambda i \right| =\ 5.846$.

We defined $Ş_2 : \mathbb{B} \to\ P(V)$ by $Ş_2(\ x) = \{y \in V / xRy\ \Leftrightarrow\ d(x, z) \leq 1\}$ for any $x \epsilon \mathbb{B}$. Since, $Ş_2(b) = \{\ a,\ b,\ c,\ e\}\ Ş_2(e) = \{\ b,\ d,\ e,\ f\}$

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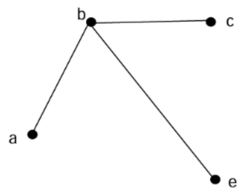
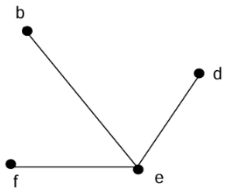
We defined $Ţ_2: \mathbb{B} \to P(E)$ by $Ţ_2(x) = \{ uv \epsilon E / \{u, v\} \subseteq Ş_2(x)\}$ for each $x \epsilon \mathbb{B}$. Such that, $Ţ_2(b) = \{ ab, bc, be\}$, $Ţ_2(e) = \{be, ed, ef\}$

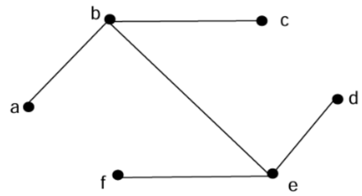
Thus, $\mathbb{H}_2(b) = (Ş_2(e), Ţ_2(e))$, $\mathbb{H}_2(e) = (Ş_2(e), Ţ_2(e))$ is subgraphs of G^*



$H_2(b)$



$H_2(e)$



G_2

Hence G_2 is a soft graph of G^*.

Here $\mathbb{D} = \bigcup_{x\epsilon D} S_2(x) = \{a, b, c, d, e, f\}$

The adjacency matrix of soft graph G_2 is given by

$$\mathcal{A}_{SG}(G_2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The characteristic equation is $|\mathcal{A}_{SG}(G_2) - \lambda I| = 0$

$\lambda^6 - 5\lambda^4 + 4\lambda^2 = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_2)$ are $\lambda_1 = 2$, $\lambda_2 = -1$, $\lambda_3 = 0$, $\lambda_4 = 0$, $\lambda_5 = 1$, $\lambda_6 = 2$.

The energy of a soft graph G_2 is

$$\mathcal{E}_{SG}(G_2) = \sum_{i=1}^{n} |\lambda i| = |\lambda_1| + |\lambda_2| + |\lambda_3| + |\lambda_4| + |\lambda_5| + |\lambda_6| = 6$$

The OR operations of G_1 *and* G_2 is $G_1 \vee G_2 = \langle S, Ţ, \mathbb{A} \times \mathbb{B} \rangle$ where $\mathbb{A} \times \mathbb{B} = \{(a, b), (a, e), (d, b), (d, e)\}$ and

$Ş(a, b) = Ş_1(a) \cup Ş_2(b) = \{a, b, c, d, e, f\}$, $Ţ(a, b) = Ţ_1 \cup Ţ_2 = \{ab, af, ad, ab, bc, be\}$

$Ş(a, e) = Ş_1(a) \cup Ş_2(e) = \{a, b, d, e, f\}$, $Ţ(a, e) = Ţ_1 \cup Ţ_2 = \{ab, af, ad, be, ef, ed\}$

$Ş(d, b) = Ş_1(d) \cup Ş_2(b) = \{a, b, c, d, e\}$, $Ţ(d, b) = Ţ \cup Ţ_2 = \{ab, bc, be, dc, de, ad\}$

$Ş(d, e) = Ş_1(d) \cup Ş_2(e) = \{a, b, d, e, f\}$, $Ţ(d, e) = Ţ_1 \cup Ţ_2 = \{dc, de, ad, be, ef, ed\}$

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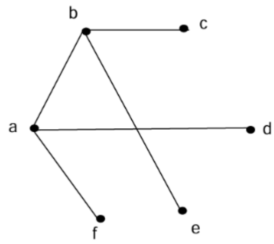
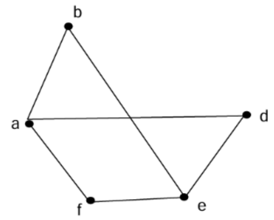
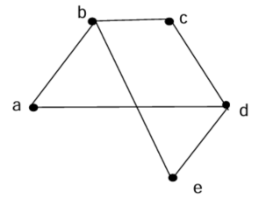
Subgraphs of G^* is $\mathbb{H}\ (a,b) = (Ş\ (a,b), Ţ(a,b))$, $\mathbb{H}(a,e) = (Ş(a,e), Ţ(a,e))$, $\mathbb{H}(d,b) = (Ş(d,b), Ţ(d,b))$, $\mathbb{H}(d,e) = (Ş(d,e), Ţ(d,e))$ as given in figure:



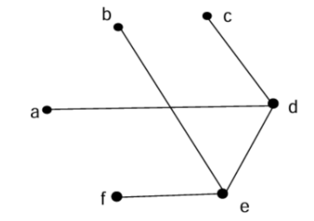
$\mathbb{H}(a,b)$



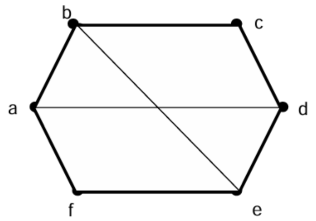
$\mathbb{H}(a,e)$



$\mathbb{H}(d,b)$



$\mathbb{H}(d,e)$



$G_1 \vee G_2$

Hence $G_1 \vee G_2 = \{\ \mathbb{H}(a,e),\ \mathbb{H}(\ c,e),\ \mathbb{H}(\ d,\ b),\ \mathbb{H}(\ d,\ e)\}$.

Here $\mathfrak{D} = \bigcup Ş(x,y) = \{\ a,\ b,\ c,\ d,\ e,\ f\}$

The adjacency matrix of soft graph $G_1 \vee\ G_2$ is given by

$$\mathcal{A}_{SG}(\ G_1 \vee\ G_2) = \ \mathcal{A}_{SG}(\ G_1 \vee\ G_2) - \lambda\mathrm{I}\,| = 0$$

$\lambda^6 - 8\lambda^4 + 4\lambda^2 = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_1 \vee G_2)$ are $\lambda_1 = -\ 2.7321$, $\lambda_2 = -\ 0.7321$, $\lambda_3 = 0$, $\lambda_4 = 0$, $\lambda_5 = 0.7321$, $\lambda_6 = 2.7321$.

The energy of a soft graph $(G_1 \vee G_2)$ is

$\mathcal{E}_{SG}\ (\ G_1 \vee\ G_2) = \sum_{i=1}^{n} |\ \lambda i| = |\ \lambda_1| + |\ \lambda_2| + |\lambda_3| + |\lambda_4| + |\lambda_5| + |\lambda_6| = 6.9284$

Hence $\mathcal{E}_{SG}(\ G_1 \vee G_2) \leq \ \mathcal{E}_{SG}(\ G_1) + \ \mathcal{E}_{SG}(\ G_2)$.

Proposition 4.15 Assume $G_1 = \langle G^*, Ş_1, Ţ_1, A\rangle$ and $G_2 = \langle G^*, Ş_2, Ţ_2,\ B\rangle$ be the soft graphs of G^*. Then $\mathcal{E}_{SG}(\ G_1 \wedge\ G_2) \leq \ \mathcal{E}_{SG}(\ G_1).\ \mathcal{E}_{SG}(\ G_2)$.

Proof:

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Consider $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ and $\mu_1, \mu_2, \ldots, \mu_m$ represent the eigenvalues of the adjacency matrix of the soft graphs $G_1$and G_2 respectively, while $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_p$ denote the eigenvalues of the adjacency matrix of the AND operations of soft graph $G_1$and G_2 represented by $G_1 \wedge G_2$. According to Theorem 3.2, the soft graph $G_1 \wedge G_2$ is as a subgraph of G^*.

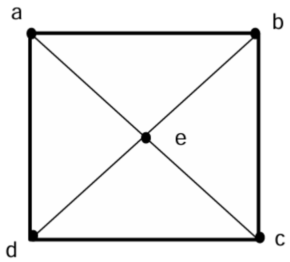
Based on theorem 3.3, it follows that $\mathcal{E}_{SG}(G_1 \wedge G_2) \leq \mathcal{E}_{SG}(G_1)$ and $\mathcal{E}_{SG}(G_1 \wedge G_2) \leq \mathcal{E}_{SG}(G_2)$

Based on by theorem 3.3, it follows that the energy of the soft graph $G_1 \wedge G_2$ is less than or equal to the product of the energy of soft graphs $G_1$and G_2.

Therefore, we have $\mathcal{E}_{SG}(G_1 \wedge G_2) \leq \mathcal{E}_{SG}(G_1).\ \mathcal{E}_{SG}(G_2)$.

Example: 4.16 (AND)

Consider $G^* = (V, E)$ is a simple graph



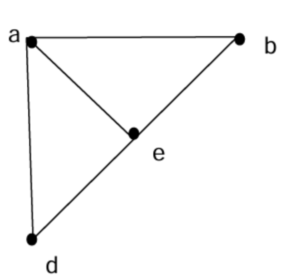
$G^* = (V, E)$

Assume $\mathbb{A} = \{a, c\}$ and $\mathbb{B} = \{e\}$ be the parameter sets.

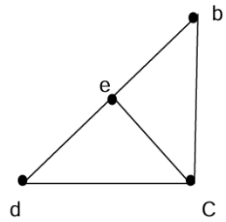
We defined $Ş_1: \mathbb{A} \to \mathcal{P}(V)$ by $Ş_1(x) = \{ y \in V / xRy \Leftrightarrow d(x, z) \leq 1\}$ for any $x \epsilon \mathbb{A}$. Such that, $Ş_1(a) = \{a, b, d, e\}$ and $Ş_1(c) = \{b, c, d, e\}$.

We defined $Ţ_1: \mathbb{A} \to \mathcal{P}(E)$ by $Ţ_1(x) = \{ uv \in E / \{u, v\} \subseteq Ş_1(x)\}$ for any $x \epsilon \mathbb{A}$. Since, $Ţ_1(a) = \{ab, ad, de, eb, ae\}\ and\ Ţ_1(c) = \{bc, cd, ce, de, eb\}$.

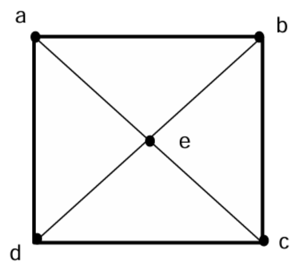
Thus, $\mathbb{H}_1(a) = (Ş_1(r), Ţ_1(r))$, $\mathbb{H}_1(c) = (Ş_1(c), Ţ_1(c))$ are subgraphs of G^*.



$\mathbb{H}_1(a)$



$\mathbb{H}_1(c)$



G_1

Therefore $G_1 = \{ \mathbb{H}_1(a), \mathbb{H}_1(c)\}$ is a soft graph of G^*.

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Since $\mathbb{C} = \bigcup_{x \epsilon C} Ş_1(x\ ) = \{\ a, b, c, d, e\}$

The adjacency matrix of soft graph G_1 $\mathcal{A}_{SG}(\ G_1) = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

The characteristic equation is $|\ \mathcal{A}_{SG}(G_1) - \lambda I| = 0$

$\lambda^5 - 8\lambda^3 - 8\lambda^2 = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_1)$are $\lambda_1 = -2$, $\lambda_2 = -1.2361$, $\lambda_3 = 0$, $\lambda_4 = 0$, $\lambda_5 = 3.2361$.

The energy of a soft graph G_1 is

$$\mathcal{E}_{SG}\ (\ G_1) = \sum_{i=1}^{n} |\ \lambda i| = |\ \lambda_1| + |\ \lambda_2| + |\lambda_3| + |\lambda_4| + |\lambda_5| = 6.4722.$$

We defined $Ţ_2 : \mathbb{B} \to \mathcal{P}(\ E)$ by $Ţ_2(\ x) = \{\ uv\ \epsilon\ E/\{\ u,\ v\} \subseteq Ş_2(x)\}$ for any $x \epsilon \mathbb{B}$. Such that, $Ţ_2(e) = \{\ ab,\ bc,\ cd, de, ae, ec, be, ed\}$

We defined $Ş_2 : \mathbb{B} \to \mathcal{P}(\ V)$ by $Ş_2(\ x) = \{\ y\ \epsilon\ V/xRy\ \Leftrightarrow d(\ x, z) \leq 1\}$ for each $x \epsilon \mathbb{B}$. Therefore, $Ş_2(e) = \{\ a,\ b,\ c,\ d,\ e\}$

Thus, $\mathbb{H}_2(e) = (Ş_2(e), Ţ_2(e))$ is subgraph of G^*.

Hence G_2 is a soft graph of G^*.

Here $\mathbb{D} = \bigcup_{x \epsilon D} Ş_2(x) = \{\ a,\ b,\ c,\ d,\ e\}$

The adjacency matrix of soft graph G_2 is $\mathcal{A}_{SG}(G_2) = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

The characteristic equation is $|\mathcal{A}_{SG}(G_2) - \lambda I| = 0$

$\lambda^5 - 8\lambda^3 - 8\lambda^2 = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_2)$are $\lambda_1 = -2$, $\lambda_2 = -1.2361$, $\lambda_3 = 0$, $\lambda_4 = 0$, $\lambda_5 = 3.2361$.

The energy of a soft graph$\mathcal{E}_{SG}$ G_2 is

$$\mathcal{E}_{SG}\ (G_2) = \sum_{i=1}^{n} |\ \lambda i| = |\ \lambda_1| + |\ \lambda_2| + |\lambda_3| + |\lambda_4| + |\lambda_5| = 6.4722.$$

$\therefore$ Energy of soft graph $(G_2) = 6.4722$.

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The AND operations of G_1 and G_2 is $G_1 \wedge G_2 = \langle S, T, \mathbb{A} \times \mathbb{B} \rangle$ where $\mathbb{A} \times \mathbb{B} = \{(a, e), (c, e)\}$ and

$Ş(a, e) = Ş_1(a) \cap Ş_2(e) = \{a, b, d, e\}$, $Ţ(a, e) = Ţ_1 \cap Ţ_2 = \{ab, ad, de, eb, ae\}$

$Ş(c, e) = Ş_1(c) \cap Ş_2(e) = \{a, b, d, e\}$, $Ţ(c, e) = Ţ_1 \cap Ţ_2 = \{bc, cd, ce, de, eb\}$

Subgraphs of G^* is $\mathbb{H}(a, e) = (Ş(a, e), Ţ(a, e))$ and $\mathbb{H}(c, e) = (Ş(c, e), Ţ(c, e))$ as given in figure:

Hence $G_1 \wedge G_2 = \{\mathbb{H}(a, e), \mathbb{H}(c, e)\}$.

Here $\mathfrak{D} = \cup Ş(x, y) = \{a, b, c, d, e\}$

The adjacency matrix of soft graph $G_1 \wedge G_2$ is $\mathcal{A}_{SG}(G_1 \wedge G_2) = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

The characteristic equation is $|\mathcal{A}_{SG}(G_1 \wedge G_2) - \lambda I| = 0$

$\lambda^5 - 8\lambda^3 - 8\lambda^2 = 0$

The eigenvalues of $\mathcal{A}_{SG}(G_1 \wedge G_2)$ are $\lambda_1 = -2$, $\lambda_2 = -1.2361$, $\lambda_3 = 0$, $\lambda_4 = 0$, $\lambda_5 = 3.2361$.

The energy of a soft graph $(G_1 \wedge G_2)$ is

$$\mathcal{E}_{SG}(G_1 \wedge G_2) = \sum_{i=1}^{n} |\lambda i| = |\lambda_1| + |\lambda_2| + |\lambda_3| + |\lambda_4| + |\lambda_5| = 6.4722.$$

Hence $\mathcal{E}_{SG}(G_1 \wedge G_2) \leq \mathcal{E}_{SG}(G_1)$. $\mathcal{E}_{SG}(G_2)$.

5. Conclusion

In this study, we established the concept as energy in soft graphs and presented various operations related to the energy of soft graphs, such as union, intersection, AND, OR along with examples to clarify these ideas. Lastly, we aim to broaden our research by exploring the complement of the energy of soft graphs.

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