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Generation of prime graphs using corona product of cycles and wheels with complete graphs

M. Thamothara Pandian[1], S. Senthil[2], and K. Rajendran[3][1]Research Scholar

Department of Mathematics

Vels Institute of Science Technology and Advanced StudiesChennai, India

[2]Assistant Professor

Department of Mathematics

Vels Institute of Science Technology and Advanced StudiesChennai, India

[3]Assistant Professor

Department of Mathematics

Vels Institute of Science Technology and Advanced StudiesChennai, India

Abstract

A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, . . . , |V|$ such that for each edge xy the labels assigned to x and y are relatively prime. In this paper, we prove that graphs$C_n \odot K_1$, for any $n \geq 3$ and $W_n \odot K_1$ are prime when n is even.

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1 Introduction

Graphs we considered here are finite, simple and undirected graphs. For the graph theoretic terminologies, we refer the book [7]. Rosa [5] introduced various graph labeling and after that many researchers introduced various graph labeling for to decompose graphs. One such graph labeling is prime labeling introduced by Entringer. A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, . . . , |V|$ such that for each edge xy the labels assigned to x and y arerelatively prime. In the year 1980, Entringer conjectured that all trees have a prime labeling. Seoud et.al., [6] proved the necessary and sufficient conditions for a graph to be prime. They also gave a procedure to determine whether or not a graph is prime. Deretsky et al., [1] proved that cycles and disjoint union of cycles are prime. Lee et. al. [4] proved that complete graph does not have a prime labeling for $n \geq 4$ and wheel graphs W_n are prime if and only if n is even. For an exhaustive survey on Graceful Tree Conjecture, refer the excellent survey by Gallian [3]. In this paper, we prove that graphs

$C_n \odot K_1$, for any $n \geq 3$ and $W_n \odot K_1$ are prime when n is even.

2 Prime labeling of $C_n \odot K_1$ and $W_n \odot K_1$

From the literature of prime labelings, we know that cycle graphs C_n are prime graphs. In this section, we prove that the corona product of cycles with the complete graph on one vertex allow prime labeling.

Theorem 1. *$C_n \odot K_1$ admit prime labeling for any $n \geq 3$.*

Proof. Consider a cycle C_n with vertices as $v_1, v_2, \ldots, v_n$ along with its prime labeling given by the function $f(v_i) = i$ for any $1 \leq i \leq n$. It is clear that, the function f gives the prime labeling for the cycle C_n. In the generation of graph $C_n \odot K_1$, let u_i be the corresponding vertex copy of the vertex $v_i \in V(C_n)$. By the definition of corona product of graphs, $C_n \odot K_1$ has $2n$ vertices and $2n$ edges. Assume that those $2n$ verticesare $v_1, v_2, \ldots v_n, u_1, u_2, \ldots, u_n$. Now, let us define the prime labeling $g : V(C_n \odot K_1) \rightarrow$

$\{1, 2, 3, \ldots, 2n\}$ for $C_n \odot K_1$ using the prime labeling of cycles C_n. Define $g(v_i) = 2f(v_i) - 1$for $1 \leq i \leq n$ and $g(u_i) = 2f(v_i)$ for $1 \leq i \leq n$. It is clear that from the definition of g, labels of the vertices of $C_n \odot K_1$ are distinct and from the set $\{1, 2, 3, \ldots, 2n\}$. Since the labels of the vertices v_i are odd by the definition of the function g, their vertex labels are

relatively prime. Since the labels of the vertices u_i are even and their unique adjacent vertex v_i receive an odd label by the definition of the function g, which proves the prime labeling of the graph $C_n \odot K_1$ for any $n \geq 3$. □

Theorem 2. *$W_n \odot K_1$ admit prime labeling for any even n.*

Proof. Consider a wheel W_n with unique central vertex as u and spokes vertices as$v_1, v_2, \ldots, v_n$ along with its prime labeling given by the function $f(v) = 1$ and $f(v_i) = i+1$for any $1 \leq i \leq n$. It is clear that, the function f gives the prime labeling for the wheel W_n when n is even. In the generation of graph $W_n \odot K_1$, let u_i be the corresponding vertex copy of the vertex $v_i \in V(W_n)$. Similarly, let u be the corresponding vertex copy of the vertex v for the corona product between W_n and K_1. By the definition of corona product of graphs, $W_n \odot K_1$ has $2n + 2$ vertices and $3n + 1$ edges. Assume that those$2n + 2$ vertices are $v, v_1, v_2, \ldots v_n, u, u_1, u_2, \ldots, u_n$. Now, let us define the prime labeling$g : V(W_n \odot K_1) \rightarrow \{1, 2, 3, \ldots, 2n + 2\}$ for $W_n \odot K_1$ using the prime labeling of wheel W_n.Define $g(v_i) = f(v_i)$ for $1 \leq i \leq n$, $g(v) = f(v) = 1$, $g(u_i) = f(v_i)+n+1$ for $1 \leq i \leq n-1$, $g(u_n) = f(v_n) + 1$ and $g(u) = 2n + 1$. It is clear that from the definition of g, labels of the vertices of $W_n \odot K_1$ are distinct and from the set $\{1, 2, 3, \ldots, 2n + 2\}$. Since the labelsof the vertices v_i are same as the labels of the vertices of the corresponding wheel graph, their vertex labels are relatively prime. Further, wheel graphs are prime graph and the labeling of g are defined such that the labels of adjacent vertices in $W_n \odot K_1$ are relative prime, it is clear that the graph $W_n \odot K_1$ admits the prime labeling for any even n. □

3 Conclusion

In this paper, we prove that graphs $C_n \odot K_1$, for any $n \geq 3$ and $W_n \odot K_1$ are prime when n is even. Main objective of the paper is to generate prime graphs using the corona product of graphs. In this direction, we raise a question of how to generate prime graphs using some other binary products apart from corona product of two graphs.

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