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A note on soft set theory

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Abstract

After the introduction of the concept of soft set theory by Molodstov [2], many researchers developed the concepts and applied in various fields. One such paper on soft set theory is by P.K. Maji, R. Biswas and A.R. Roy [1]. In [1], the proposition 2.5 result (iii) is one of the distributive law which is stated to be true without proof. In this paper, we disprove the distributive law of union over intersection for soft sets.

1 Soft set theory

Definition 1. Let U be an universal set and E be a set of parameters. $P(U)$ be the power set of U and $A \subset E$. Let $F : A \to P(U)$ be a mapping, then the pair (F, A) is called a soft set over U.

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In other words, a soft set over U is a parametrized family of subsets of U. For $e \in A$, the set $F(e)$ may be considered as the set of e-elements of the soft set (F, A) or the set of e-approximate elements of the soft set (F, A).
So, a soft set (F,A) can be written as $(F, A) = \{F(e) : e \in A\}$.

Definition 2. Let (F, A), (G, B) be two soft sets over U. We say that (F, A) is a soft subset of (G, B) if

(i) $A \subset B$ and

(ii) $\forall e \in A$, $F(e)$ and $G(e)$ are identical approximations.

We then write $(F, A)\overline{\subset}(G, B)$.

Definition 3. Two soft sets (F, A) and (G, B) over U are said to be soft equal if $(F, A)\overline{\subset}(G, B)$ and $(G, B)\overline{\subset}(F, A)$ and it is written as $(F, A) = (G, B)$.

Definition 4. The union of two soft sets (F, A) and (G, B) over U is the soft set (H, C) where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A)\overline{\cup}(G, B) = (H, A \cup B)$.

Definition 5. The intersection of two soft sets (F, A) and (G, B) over U is the soft set (H, C) where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \text{or} G(e) = F(e) \cup G(e)$. We write $(F, A)\overline{\cap}(G, B) = (H, A \cap B)$.

Consider the example. Let $X = \{h_1, h_2, \cdots, h_6\}$ be the universal set, where $h_1, h_2, \cdots, h_6$ are houses of different nature which a person is planning to purchase and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the parameter set, where e_1 is the parameter expensive, e_2 is the parameter beautiful, e_3 is the parameter wooden, e_4 is the parameter cheap, e_5 is the parameter in the green surroundings.
Let $E_1 = \{e_1, e_3, e_5\}$, $E_2 = \{e_1, e_2, e_3\}$ and $E_3 = \{e_1, e_4\}$.
Let $(F, E_1) = \{\{h_2, h_4\}, \{h_1\}, \{h_3, h_4, h_5\}\}$,
$(G, E_2) = \{\{h_1, h_3\}, \{h_3\}, \{h_1, h_4\}\}$ and $(H, E_3) = \{\{h_2, h_5\}, \{h_1, h_4\}\}$

be the soft subsets.
The distributive law of union over intersection is

$(F,E_1)\overline{\cup}((G,E_2)\overline{\cap}(H,E_3)) = ((F,E_1)\overline{\cup}(G,E_2))\overline{\cap}((F,E_1)\overline{\cup}(H,E_3)).$

$$LHS = (F,E_1)\overline{\cup}((G,E_2)\overline{\cap}(H,E_3))$$

Let $(G,E_2)\overline{\cap}(H,E_3) = (K, E_2 \cap E_3)$, where $\forall e \in E_2 \cap E_3$, $K(e) = G(e) \cup H(e)$. But $E_2 \cap E_3 = \{e_1\}$.
Therefore, $K(e_1) = G(e_1) \cup H(e_1) = \{h_1, h_3\} \cup \{h_2, h_5\} = \{h_1, h_2, h_3, h_5\}$.
Therefore, $LHS = (F,E_1) \cup (K, E_2 \overline{\cap} E_3) = (L, E_1 \cup (E_2 \cap E_3))$
where $\forall e \in E_1 \cup (E_2 \cap E_3)$.

$$L(e) = \begin{cases} F(e) & \text{if } e \in E_1 - (E_2 \cap E_3) \\ G(e) & \text{if } e \in (E_2 \cap E_3) - E_1 \\ F(e) \cup G(e) & \text{if } e \in (E_2 \cap E_3) \cap E_1 \end{cases}$$

But, $E_1 \cup (E_2 \cap E_3) = \{e_1, e_3, e_5\}$, $E_1 - (E_2 \cap E_3) = \{e_3, e_5\}$, $(E_2 \cap E_3) - E_1 = \phi$ and $E_1 \cap (E_2 \cap E_3) = \{e_1\}$.
Therefore, $L(e_3) = F(e_3) = \{h_1\}$, $L(e_5) = F(e_5) = \{h_3, h_4, h_5\}$.
$L(e_1) = F(e_1) \cup K(e_1) = \{h_2, h_4\} \cup \{h_1, h_2, h_3, h_5\} = \{h_1, h_2, h_3, h_4, h_5\}$
and $RHS = ((F,E_1)\overline{\cup}(G,E_2))\overline{\cap}((F,E_1)\overline{\cup}(H,E_3))$.
Let $(F,E_1)\overline{\cup}(G,E_2) = (M, E_1 \cup E_2)$ where $\forall e \in E_1 \cup E_2$.

$$M(e) = \begin{cases} F(e) & \text{if } e \in E_1 - E_2 \\ G(e) & \text{if } e \in E_2 - E_1 \\ F(e) \cup G(e) & \text{if } e \in E_1 \cap E_2 \end{cases}$$

But $E_1 \cup E_2 = \{e_1, e_2, e_3, e_5\}$, $E_1 - E_2 = \{e_5\}$, $E_2 - E_1 = \{e_2\}$, $E_1 \cap E_2 = \{e_1, e_3\}$. Therefore, $M(e_5) = F(e_5) = \{h_3, h_4, h_5\}$, $M(e_2) = G(e_2) = \{h_3\}$, $M(e_1) = F(e_1) \cup G(e_1) = \{h_2, h_4\} \cup \{h_1, h_3\} = \{h_1, h_2, h_3, h_4\}$, $M(e_3) = F(e_3) \cup G(e_3) = \{h_1\} \cup \{h_1, h_4\} = \{h_1, h_4\}$.
Let $(F,E_1)\overline{\cup}(H,E_3) = (N, E_1 \cup E_3$, where $\forall e \in E_1 \cup E_3$,

$$N(e) = \begin{cases} F(e) & \text{if } e \in E_1 - E_3 \\ H(e) & \text{if } e \in E_3 - E_1 \\ F(e) \cup H(e) & \text{if } e \in E_1 \cap E_3 \end{cases}$$

But, $E_1 \cup E_3 = \{e_1, e_3, e_4, e_5\}$, $E_1 - E_3 = \{e_3, e_5\}$, $E_3 - E_1 = \{e_4\}$, $E_1 \cap E_3 = \{e_1\}$.

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Therefore, $N(e_3) = F(e_3) = \{h_1\}$, $N(e_5) = F(e_5) = \{h_3, h_4, h_5\}$, $N(e_4) = H(e_4) = \{h_1, h_4\}$. $N(e_1) = F(e_1) \cup H(e_1) = \{h_2, h_4\} \cup \{h_2, h_5\} = \{h_2, h_4, h_5\}$. Therefore, $RHS = (M, E_1 \cup E_2)\overline{\cap}(N, E_1 \cup E_3) = (R, (E_1 \cup E_2) \cap (E_1 \cup E_3))$, where $\forall e \in (E_1 \cup E_2) \cap (E_1 \cup E_3)$. $R(e) = M(e) \cup N(e)$, but $(E_1 \cup E_2) \cap (E_1 \cap E_3) = \{e_1, e_2, e_3, e_5\} \cap \{e_1, e_3, e_4, e_5\} \implies E_1 \cup (E_2 \cap E_3) = \{e_1, e_3, e_5\}$. Therefore, $R(e_1) = M(e_1) \cup N(e_1) = \{h_1, h_2, h_3, h_4\} \cup \{h_2, h_4, h_5\} = \{h_1, h_2, h_3, h_4, h_5\}$. $R(e_3) = M(e_3) \cup N(e_3) = \{h_1, h_4\} \cup \{h_1\} = \{h_1, h_4\}$. $R(e_5) = M(e_5) \cup N(e_5) = \{h_3, h_4, h_5\} \cup \{h_3, h_4, h_5\} = \{h_3, h_4, h_5\}$. Therefore, $L(e_1) = R(e_1)$, $L(e_3) \neq R(e_3)$, $L(e_5) = R(e_5)$. Therefore, $\forall e \in E \cup (E_2 \cap E_3), L \neq R$. Hence $LHS \neq RHS$. So the distributive law of union over intersection for soft sets is not true.

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