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Compactness and Connectedness in Beta Weakly Semi – Closed Sets in Topological Spaces

S. Saranya[*1] , V.E. Sasikala*[2]
[*1] Research Scholar,
[*2] Assistant Professor, Research Supervisor Corresponding Author,
Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, (VISTAS), Pallavaram, Chennai, India.
Corresponding author mail id.sasikala.sbs@velsuniv.ac.in

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Abstract:

This research presents an innovative class of Beta weakly semi-CS, namely Compactness and Connectedness in Beta weakly semi-CS in TS. Throughout this paper, βws-Compactness and βws-Connectedness were examined to get the fundamental facts in the Beta weakly semi-CS. In this paper, the notion of countable βws- compact in TS were explored and βws – Connectedness ($C_{\beta ws}$) in TS were also studied to get results. The βws - $C_{\beta ws}$ and βws – Compactness fulfilled most of the connectedness and compactness properties in TS. Here, many characterizations were obtained along with some of their features. The paper concludes on how it relates to other kinds of functions and beta ws-Compactness in TS and its characteristics were studied to obtain results theoretically.

Keywords: Beta weakly semi – closed sets (βws - closed), Beta weakly semi – open sets (βws-open), Beta weakly semi-closed sets – compactness (βws – compactness), Beta weakly semi closed sets – connectedness (βws – connectedness).

1. Introduction:

The fundamental concepts of connectedness and compactness play crucial roles in general topology and various other branches of mathematics. Numerous researchers have investigated into exploring their fundamental properties, leading to inspirations for generalizing these concepts to innovative extents. [1], " On b-open sets, Math Vesnik in topological spaces''. K. Rekha [2], "**b-Compactness and **b-Connectedness in Topological Spaces''. [3], " θ-b-continuous functions, Acta Math. Hungar''. D. Sivaraj and V.E. Sasikala [4-6], "A Study on Soft α–open sets" in topological spaces. "On soft semi weakly generalized closed set'' and "Soft swg Separation Axioms in soft topological spaces''. V. Kavitha, V. E. Sasikala [7], "Beta Generalized CS'' in Topological Spaces. [8], "Minimal weakly open sets and maximal weakly closed sets in topological spaces". [9], "Some properties of contra-γ-continuous functions''. [10], "On generalized continuous maps'' in topological spaces.[11], Studies "On Generalizations of Closed Maps and Homeomorphisms in Topological Spaces".[12],"On gpr-continuous functions'' in topological spaces. A. Pushpalatha,[13] Studies "On Generalizations of Mapping in Topological Spaces''. [14], "A Study on Generalizations of Closed Sets and Continuous Maps'' in Topological and Bitopological Spaces. This research aims to introduce the ideas extending βws - compactness and βws – connectedness ($C_{\beta ws}$) within TS, along with providing characterizations for these concepts.

Review of Literature:

A thorough review of theoretical contexts and terminology that have contributed to the current knowledge of the Beta weakly semi-closed sets in topological spaces were studied.

Whenever A is αg*s-compact of the subspace of X is called αg*s-compact [15]. A topological space X is said to be generalized semi-open connected (briefly gso-connected) if X cannot be written as the union of two non-empty disjoint gso-open sets [16]. A space X is said to be **b-compact if every "**b-open cover $\{A_\alpha\}$ of X contains a finite sub collection that also covers X'' [1-3].

The methods applied for studying these sets, the extensiveness of the research, and the way these concepts are used to better comprehend authentic topological issues. The emergence of beta weakly semi-closed sets in topological spaces, which provide an innovative viewpoint, is anticipated to advance our understanding of the field of topological spaces and its basic features.

2. Preliminary Notes:

All through this study, (X, τ) and (Y, σ) as generic topological spaces were considered unless specified otherwise, without assuming any separation axioms. For the closure of set A is X, we denote it as cl (A), and Int (A) represents the interior of A.

Definition:2.1.[1] "A subset A of X is said to be b-open [1] if A⊆Int(cl(A))∪cl(Int(A)). The complement of b-open set is said to be b-closed. The family of all b-open sets (respectively b-closed sets) of (X, τ) is denoted by bO(X, τ) [respectively bcl(X, τ)]".

Definition 2.2.[1] Let A ⊆ X. Then

(i) " b-interior [1] of A is the union of all b-open sets contained in A".

(ii)"b-closure [1] of A is the intersection of all b-closed sets containing A. The b-interior [respectively b-closure] of A is denoted by b-Int(A) [respectively b-Cl(A)]".

3. βws-Compactness in TS:

This section examines the idea of βws-Compactness other than subsequently explores it's characteristics and also provides its fundamental properties

Definition 3.1.

The collection $\{Ģ_i : i \in I\}$ of βws-O-S in a TS X is known as βws-O-C ⊆ Ģ related to X if $Ģ \subseteq \bigcup_{i\in I} Ģ_i$

Definition 3.2.

The TS X were named as βws-compact with each and every one βws-O-C of X include a finite sub cover.

Definition 3.3.

A ⊆ Ģ of the TS X were named as βws-compact related to X, ∀ collection $\{Ģ_i : i \in I\}$ of βws – O ⊆ X ∃ $Ģ \subseteq \bigcup_{i\in I} A_i$ ∃ finite ⊆ I_0 of I ∃ $Ģ \subseteq \bigcup_{i\in I} A_i$.

Definition 3.4.

A ⊆ Ģ of a TS X were named as βws - Compact if Ģ is βws-Compact ⊂ X.

Theorem 3.5.

A βws-C ⊆ βws -compact space were considered as βws-compact relative to X.

Proof.

Assume that Ģ be a βws-C ⊆ TS in X. Thence $Ģ^c$ is βws-O in X, $\mathcal{S}$ = {$Ģ_i$:i∊I} exist a βws-O-C of Ģ by βws-O ⊆ X. After that, $\mathcal{S}^*$ = $\mathcal{S}$ ∪ $Ģ^c$ is a βws-O-C of X. That is X = [∪{$Ģ_i$:i∊I}]∪ $Ģ^c$. By the assumption that, X will be considered as βws-compact, ∴, $\mathcal{S}^*$ reduction to a finite subcover of X for example, X = $Ģ_{i1}$∪ $Ģ_{i2}$∪…∪ $Ģ_{in}$ ∪ $Ģ^c$, $Ģ_{ik}$∈ $\mathcal{S}^*$. Since Ģ and $Ģ^c$ are disjoint. ∴, Ģ ⊆ $Ģ_{i1}$∪ $Ģ_{i2}$∪…..∪ $Ģ_{in}$∈$\mathcal{S}$. Thus a βws-O-C $\mathcal{S}$ of Ģ ⊂ finite subcover, proving that Ģ is the βws - Compact related to X.

Theorem 3.6:

Each βws-compact space are considered as compact.

Proof.

Assume that X are considered as βws-Compact space.{$Ģ_i$:i∊I} is an O-C of X. Thenceforth {$Ģ_i$:i∊I} will be a βws-O-C of X Since every O-S is also a βws – O-S, ∴, X is βws-compact, the βws-O-C {$Ģ_i$:i∊I} of X includes a finite subcover talks {$Ģ_i$, i ∊ n} within X. ∴, X is compact.

Theorem 3.7:

Each and all βGO-Compact space are considered as βws-Compact.

Proof.

Assuming X relate as a βGO-Compact space. Assuming, {$Ģ_i$:i∊I} relate as a βws-O-C of X by βws-O-S in X. Afterward, every βws-O-S is βg-O,∴, {$Ģ_i$:i∊I} is βg-O-C regarding X. ∴, X is βGO-Compact, the βg-O-C {$Ģ_i$:i∊I} X has a finite sub cover say {$Ģ_i$ i ∊ n} of X . ∴, X is βws - Compact.

4. Countable βws-Compactness in TS:

In this section, we explored that the notion of countable βws-compact in TS besides that investigate most of its characteristics.

Definition 4.1.

A TS X is named as countable βws-Compact if each and all countable βws-O-C of` X includes a finite sub cover.

Theorem 4.2.

Suppose X considered as countable βws-Compact space, thenceforth m would be considered as countable compact.

Proof.

Consider, {$Ģ_i$:i∊I} be present countable O-C of X along with O-S in X. Thenceforth {$Ģ_i$:i∊I} is countable βws-O-C regarding X. Given that X would be countable βws-Compact, that countable βws-O-C of X includes a finite subcover, for example {Ģi .i ϵn}. ∴, X are considered as countable Compact.

Theorem 4.3.

Each and every βws-compact space is considered to be Countable βws-compact.

Proof.

Suppose that Ҳ will be considered as βws-Compact space, {Ģi :iϵI} will be considered as countable βws-O-C of Ҳ including βws-O-S. Following that {Ģi :iϵI} is a βws-O-C {Ģi :iϵI} of X whose finite sub cover say {Ģi, iϵn}. ∴, X is countable βws-Compact.

Theorem 4.4.

The countable βws-Compact space under βws-irresolute function are considered as countable βws-Compact.

Proof.

Suppose ϖ : X → Y have a βws-irresolute function after a countable βws-Compact space Ҳ continuously TS Y, {Ģi : iϵI} will be considered as countable βws-O-C of Y. Subsequently, {ϖ^{-1}(Ģi): iϵI} are considered as countable βws-O-S of X such as ϖ remains βws-irresolute. when X is countable βws-Compact, the countable βws-O-C {ϖ^{-1}(Ģi) : iϵI} of X include a finite subcover say {ϖ^{-1}(Ģi) : i∈n}. ∴, X = $\cup_{i\in I}$ ϖ^{-1} (Ģi) ϖ **(**X**) =** $\cup_{i\in I}$ Ģi. Then Y = $\cup_{i\in I}$ Ģi {Ģ1, Ģ2, Ģ3 ,…, Ģn} is a finite ⊆ { Ģi : iϵI} used for Y. ∴, Y is countable βws - Compact.

Theorem 4.5:

A space X is countable βws - Compact if all countable unit of βws-CS that is X having finite intersection property has a non-empty intersection.

Proof:

The X is considered as countable βws-Compact, {F_i:iϵI} is included to countable unit of βws-CS along through finite ∩ property. To show that $\cap_{i\in I}^{n}$ (F_i) ≠ ϕ. To take $\cap_{i\in I}^{n}$ (F_i) = ϕ, Then X $\cup_{i\in I}$ F_i = X ⇒ $\cup_{i\in I}$ (X - F_i) = X. The {X - F_i : iϵI} is a countable βws-O-C { X - F_i:iϵI} has finite sub cover say { X - F_i:i = 1,…,n}. Where, X = $\cup_{i=1}^{n}$ (X - F_i) ⇔ X = X =$\cap_{i=1}^{n}$ F_i ⟹ X - X = X -$\cap_{i=1}^{n}$ F_i. Hence, ϕ =$\cap_{i=1}^{n}$ F_i. ∃ finite subcollection {F_i.i∈n} of {F_i : iϵI}∋ X = $\cup_{i=1}^{n}$(X - F_i). Then, ϕ =$\cap_{i=1}^{n}$ F_i. which contradicts the assumption, $\cap_{i\in I}^{n}$ (F_i) ≠ ϕ.

Theorem 4.6.

In case a map ϖ : X → Y is βws-irresolute besides B subset of X is βws–Compact related to X, thereafter, the illustration ϖ (B) is βws-Compact related towards Y.

Proof:

Suppose that {Ģi : i∈Δ_0} be multiples of βws-O ⊆ Y ∋ ϖ (B)⊂∪ {Ģi : i∈Δ }holds. By hypothesis ∃ a finite ⊆ ∃ B⊂∪ {ϖ^{-1}(Ģi) : i∈Δ_0}. ∴, Here, we have ϖ (B) ⊂ ∪(Ģi : i∈Δ_0} , showing that ϖ (B) are considered as βws-Compact related towards Y.

Theorem 4.7.

The X × Y of two non-empty space is βws-Compact.

Proof.

Considering X →Y represent the non-empty spaces' product space X and Y also considering X × Y is a βws - Compact. Subsequently, the projection ∏ : X × Y are considered as βws -irresolute map. ∴, (X × Y) = X is βws - Compact. Likewise, we illustrate aimed at the space Y.

5. βws – Connectedness ($C_{\beta ws}$) in TS:

Definition 5.1.

A TS X is named as βws-$C_{\beta ws}$ if X cannot be indicated as a disjoint union of two non - empty βws-O-S.

Example 5.2.

Ҳ = {L, G, H, K} τ = {Ҳ, φ, {L}, { K }, {L, K}}, βws – CS are { Ҳ, φ, {G }, { H }, {K }, {G , H}, {G, K }, {H, K }}. βws – O-S are {Ҳ, φ, {L, H, K }, { L, G, K }, { L, G, H}, {L, K}, { L, H }, { L, G}}. Let U = {L, G} and V = {L, K}. Here X cannot be expressed as the union of two non-empty disjoint βws – O-S. ∴, X is βws – $C_{\beta ws}$.

Theorem 5.3.

Every βws-$C_{\beta ws}$ space is $C_{\beta ws}$.

Proof.

Considering (Ҳ, τ) is known as βws-$C_{\beta ws}$ space. Supposing that (Ҳ, τ) is not that $C_{\beta ws}$. Later, Ҳ = Ģ∪Q, where Ģ and Q be there disjoint nonempty O-S in (Ҳ, τ). As it is already known, arbitrary ∪ βws-O-S is βws-O, Ģ and Q are βws - O, Ҳ = Ģ∪Q, where Ģ and Q are disjoint nonempty and βws-O-S in (Ҳ, τ). This opposes the point that (Ҳ, τ) is βws-$C_{\beta ws}$ and so (Ҳ, τ) is $C_{\beta ws}$.

Definition 5.4.

A function m : X →Y is named as βws-irresolute if m^{-1}(V) is βws-C in X ∀ βws-CS V of Y.

Theorem: 5.5

If m X → Y is a βws - irresolute surjection and X is βws-$C_{\beta ws}$, hence Y is βws-$C_{\beta ws}$.

Proof.

Considering Y is not that βws-$C_{\beta ws}$, Y = Ģ∪Q where Ģ and Q are disjoint non-empty βws-O-S throughout Y. Subsequently, m is βws-irresolute besides on, X = m^{-1}(Ģ) ∪ m^{-1} (Q) where m^{-1}(Ģ) and m^{-1}(Q) are disjoint non-empty βws-O-S in X. The fact X is contradicted by this βws-$C_{\beta ws}$. ∴, Y is $C_{\beta ws}$.

Remark 5.6.

Any indiscrete space with two points is βws – $C_{\beta ws}$, but any two – point set with discrete topology is not βws- $C_{\beta ws}$.

Theorem 5.7.

If the βws-O-S Q and ℘ leads to separation of X besides if Y is βws-$C_{\beta ws}$ subspace of X, then Y exists mainly in Q and ℘.

Proof.

When Q and ℘ are both βws-O in X the sets Q ∩℘ and ℘∩Y are βws-O in Y these two sets are disjoint ∪ in Y. If both were non-empty, they would represent a division y. ∴, There are only one empty one. ∴, Y has to completely lie in Q or in ℘.

6. Conclusion:

The definition of two new concepts of connectedness and compactness βws - (compactness) and βws - (connectedness). An X represents βws – compact if every βws – O – C of X have a finite subcover. The βws - $C_{\beta ws}$ and βws – Compactness fulfilled most of the connectedness and compactness properties in TS.

The primary goal of this work was to get a deeper understanding of the consequences of Compactness and Connectedness in Beta weakly semi-CS in TS and suggest future lines of research. We have obtained several noteworthy results from our analysis of beta weakly closed sets. The study has strengthened the theoretical foundations of these concepts by developing and verifying several theorems that show these ideas may hold true in various topological domains. There are a number of interesting avenues for further research in the future. The application of connectedness and compactness in beta weakly semi-CS in TS. The study of compactness and connectedness in beta weakly semi-CS in TS has led to significant discoveries in the TS.

References

[1] D. Andrijevic, On b-open sets, Math. Vesnik, 48 (1996), No. 1-2, 59-64. DOI.10.12691/tjant-6-6-4

[2] K. Rekha, "**b-Compactness and **b-Connectedness in Topological Spaces," International Journal of Mathematics Trends and Technology (IJMTT), vol. 60, no. 5, pp. 286-293, 2018. Crossref, https://doi.org/10.14445/22315373/IJMTT-V60P542.

[3] J. H. Park, θ-b-continuous functions, Acta Math. Hungar,110(2006), No. 4, 347-359. https://doi.org/10.1016/j.mcm.2008.05.007

[4] D.Sivaraj and V.E. Sasikala, 'A Study on Soft α−open sets', IOSR Journal of Mathematics, vol.12,pp.70-74.Vol.12,issue.5,Version-6/M1205067074. 2016 http://www.iosrjournals.org/iosr-jm/papers

[5] V.E. Sasikala, D. Sivaraj and A.P. Ponraj, On soft semi weakly generalized closed set in soft topological spaces, International Journal of Innovative Technology and Exploring Engineering, (Impact Factor.0.546), ISSN No. 2278-3075, Vol.8, Issue No.12, Year.2019, pp.1252-1256. DOI.10.35940/ijitee.L3907.1081219 .

[6] V.E. Sasikala, D. Sivaraj and A.P. Ponraj, Soft swg Separation Axioms in Soft Topological Spaces, Journal of Applied Science and Computations, (Impact Factor.5.8), ISSN No. 1076-5161, Vol.6, Issue No.5, Year.2019, pp. 2236-2245. (UGC)

[7] V. Kavitha, V.E. Sasikala (Corresponding author), Beta Generalized CS in Topological Spaces, Journal of Algebraic Statistics, (Impact factor 0.21), ISSN.1309-3452, Volume 13, No. 3, July, 2022 pp. 891-898, Web of Science. Volume 13, No. 3, 2022, p. 891-898 https://publishoa.com ISSN.1309-3452.

[8] R.S.Wali and Vivekananda Dembre, "Minimal weakly open sets and maximal weakly closed sets in topological spaces"; International Journal of Mathematical Archieve; Vol-4(9)-Sept-2014. DOI .https://doi.org/10.14445/22315373/IJMTT-V66I4P506

[9] A. A. Nasef, Some properties of contra-γ-continuous functions, Chaos Solitons Fractals, 24 (2005), No. 2, 471-477. DOI 10.1016/j.chaos.2003.10.033

[10] K. Balachandran, P. Sundaram, H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Kochi Univ. Ser. A, Math. 12 (1991) 5–13. DOI.https://doi.org/10.14445/22315373/IJMTT-V6P506

[11] R. Devi, Studies on Generalizations of Closed Maps and Homeomorphisms in Topological Spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 1994. https://doi.org/10.26713/jims.v9i3.755

[12] Y. Gnanambal, K. Balachandran, On gpr-continuous functions in topological spaces, Indian J. Pure Appl. Math. 30 (6) (1999) 581–593. DOI .https://doi.org/10.14445/22315373/IJMTT-V64P513

[13] A. Pushpalatha, Studies on Generalizations of Mappings in Topological Spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 2000. DOI:10.29322/IJSRP.11.11.2021.p11959

[14] M. Sheik John, A Study on Generalizations of Closed Sets and Continuous Maps in Topological and Bitopological Spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 2002. 10.26637/MJM0S20/0108.

[15] αg*s-Compactness and αg*s- Connectedness in Topological Spaces Sarika M. Patil1 and T. D. Rayanagoudar 2* Department of Mathematics Government First Grade College, Rajnagar, Hubli -580 032, Karnataka State, India.

[16] "gso-connectedness and gso-compactness in topological spaces", International Journal of Emerging Technologies and Innovative Research (www.jetir.org), ISSN:2349-5162, Vol.9, Issue 11,pageno.c119-c122,November-2022,Available :http://www.jetir.org/papers/JETIR2211209.pdf