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The Study on Schizophrenia Using Brownian Motion and Network Analysis

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Abstract. This paper investigates the impact of schizophrenia on the human brain through a multi- approach. Schizophrenia, a complex neuropsychiatric disorder, poses significant challenges to understanding its underlying mechanisms and developing effective treatments. Brownian motion, the study aims to quantify abnormality regions within the brain, the network analysis providing valuable insights into the spatial distribution and connectivity patterns associated with the disorder. Additionally, the research analyzes grey matter levels in the brains of patients before and after treatment interventions, employing statistical techniques to assess treatment efficacy.

Keywords: schizophrenia, Brownian motion, Diffusion, Network, Connectivity, Statistics

INTRODUCTION

Schizophrenia is a complex and multifaceted disorder that involves both structural and functional abnormalities in the brain [1]. The exact causes of schizophrenia are still not fully understood, and research also believes that an irregularity in the intricate chemical processes within the brain, which involve neurotransmitters like dopamine and glutamate, and potentially others, contributes to schizophrenia [2]. One of the most widely studied aspects of schizophrenia is the dysregulation of dopamine, a neurotransmitter involved in various brain functions, including motivation, reward, and movement. It's believed that excessive dopamine activity, particularly in certain regions of the brain, may contribute to symptoms like hallucinations and delusions [1-2]. Glutamate is another neurotransmitter that plays a crucial role in brain function, particularly learning and memory. Research has indicated that abnormalities in the glutamatergic system, including deficits in glutamate receptors and signaling pathways, may be implicated in schizophrenia. This imbalance could affect information processing and cognitive function. Furthermore, certain individuals with schizophrenia exhibit slight variations in brain structures compared to those without the disorder. For example- In certain individuals with schizophrenia, the fluid-filled spaces known as ventricles in the center of the brain appear to be enlarged compared to those without the condition.

LITERATURE REVIEW

Graph theory is a mathematical framework extensively used in network analysis across various domains, including social networks, transportation systems, communication networks, and, notably, the analysis of brain networks in neuroscience, including in the study of schizophrenia [3], it explores the organization and connectivity patterns within the brain's networks, by representing brain regions as nodes and the connections between them as edges. Brownian motion refers to the random movement of microscopic particles suspended in a fluid, first observed by the botanist Robert Brown in 1827while looking through a microscope at pollen grains suspended in water, Brownian motion is a consequence of the constant collision of fluid molecules with the suspended particles [4-5]. In a fluid, molecules are in constant motion due to thermal energy. When these molecules collide with larger particles (like pollen grains or microscopic particles), they exert random forces on them, causing them to move in a zigzag or erratic manner. Brownian motion is observable at the microscopic level and plays a crucial role in various natural processes, including the dispersion of pollutants in the atmosphere, the diffusion of molecules in biological systems, and the movement of particles in colloidal suspensions [6-7]. Regional Abnormality of Grey Matter in Schizophrenia: Effect from the Illness or Treatment? [8]. This study found that antipsychotic-naïve first-episode schizophrenia patients had reduced grey matter in the right superior temporal gyrus. After 8 weeks of treatment, increases in grey matter were observed in several brain regions, linked to symptom improvement. This suggests early schizophrenia affects the right superior temporal gyrus, and antipsychotics can positively impact brain structure [8]. The data was also collected and analyzed in this study.

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METHODOLOGY

Connectivity abnormalities in schizophrenia represent a complex interplay of structural and functional disruptions within the brain's networks. It's a debilitating mental disorder characterized by disturbances in thought, perception, and behavior. Structural connectivity typically refers to the anatomical connections between different regions or components within a system, such as the brain [2]. It's a key concept in understanding how information is transmitted and processed within complex networks like the brain [2]**.** Network analysis in this paper focuses on the analysis and interpretation of complex systems represented as networks or graphs. These systems can be found in various domains, their analysis involves studying the structure, dynamics, and properties of these networks to understand their behavior, interactions, and emergent properties. [3] In this section network analysis on analyzing regional grey matter abnormalities in schizophrenia patients treated with antipsychotic medication, they compare their differences. It then explored the impact of antipsychotic treatment on brain structures in schizophrenia patients after 8 weeks [8].

RESULTS AND DISCUSSION

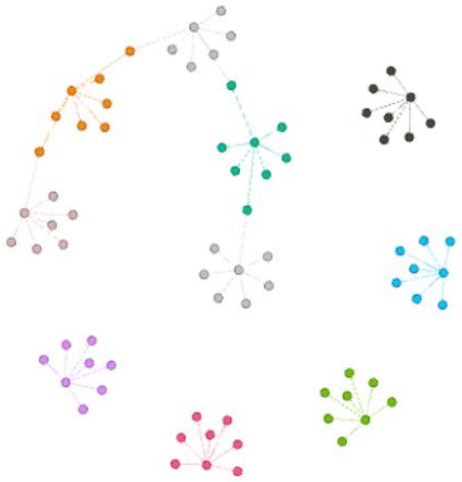

FIGURE 1. Baseline grey matter

Fig 1 shows Baseline grey matter volume refers to the initial quantity of grey matter found in particular brain regions before the beginning of treatment [8]. Fig. 2 shows Follow-up grey matter volume denotes the extent of grey matter augmentation observed in particular brain regions following an 8-week treatment period [8]. Baseline grey matter volume is the amount of grey matter in specific brain regions before treatment. Follow-up grey matter volume measures it after an 8-week treatment period, allowing researchers to assess changes. Gephi, with its powerful tools and intuitive interface, is excellent for data analysis, visualization, and exporting results."

TABLE 1. Grey matter volume before and after treatment

Id	L-insula	L-mid_frontal	L-sup_frontal	L-sup_occipital	R-Cerebellum	R-insula	R-mid_frontal	R-Thalamus
1	0.203	0.248	0.272	0.362	0.349	0.182	0.192	0.213
2	0.243	0.251	0.2	0.292	0.237	0.239	0.287	0.247
3	0.283	0.377	0.364	0.356	0.404	0.294	0.305	0.21
4	0.393	0.375	0.324	0.351	0.396	0.374	0.418	0.244
5	0.26	0.288	0.274	0.249	0.299	0.306	0.338	0.188
6	0.194	0.356	0.358	0.271	0.27	0.191	0.23	0.172
7	0.246	0.341	0.353	0.286	0.445	0.25	0.348	0.255

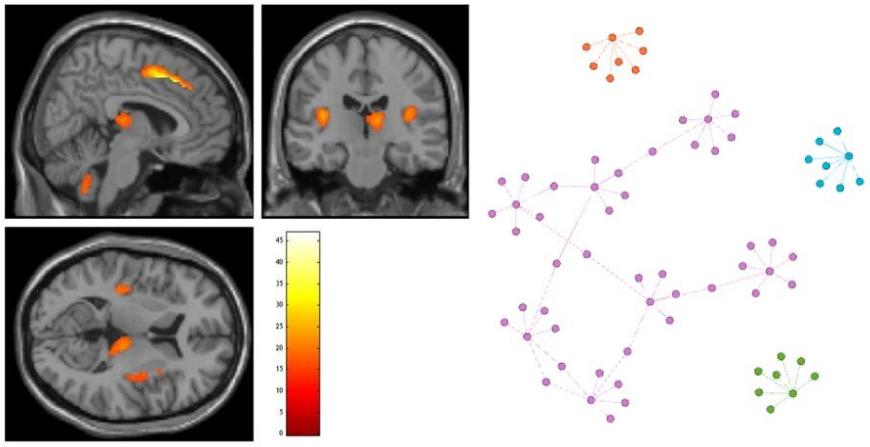

FIGURE 2. Grey matter augmentation

RESULTS AND DISCUSSION

A Brownian motion is a random process that starts at 0, has independent and normally distributed increments, and its paths exhibit certain characteristics of randomness and unpredictability [5, 10]. Brownian motion $(B_t)_t$ 0 is the real-valued probabilistic process such that [10].

$$B_0 = 0 \tag{1}$$

Independent increments: the random variables $B_y - B_x, B_p - B_q$ are independent whenever $x \le y \le p \le q$ (so the intervals (x, y) (p, q) are disjoint.). Normal increments is $B_{p+q} - B_q \sim M(0, p)$ for all $p, q \ge 0$ and with probability 1, the function $p \to B_p$ is continuous. Brownian motion is a stochastic process describing the random motion of particles suspended in a fluid, such as gas or liquid. In this context, the particles are assumed to be undergoing Brownian motion in one dimension along the real line.[4]

Theorem

Let assume $0 < \mu < 1/2$ along with probability 1 Brownian sample function Y: [0,1] $\to \mathbb{R}$ satisfies [2].

$$|Y(s+k) - Y(s)| \le b|k|^{\mu} \tag{2}$$

For some (random) $K_0 > 0$ where b depends only on μ Equivalently, there is a random constant B such that

$$|Y(s+k) - Y(s)| \le B|k|^{\mu} \ (s, s+k \in [0,1]) \tag{3}$$

Proof

Let $k > 0$, It has $\forall$ $s \ge 0$ and $k > 0$, the increment $Y(s+k) - Y(s)$, if it is a normal distribution the value of the mean is 0 and the value of the variance is k, Therefore,

$$P(Y(s+k)-Y(S)\le y=\frac{1}{\sqrt{2\pi k}}\int_{-\infty}^{y}\exp\left(\frac{v^2}{2k}\right)dv$$

$$P(\left|Y(s+k)-Y(s)\right|\le k^{\mu})=\frac{2}{\sqrt{2\pi h}}\int_{k}^{\infty}\exp\left(\frac{-v}{2k}\right)dv \tag{4}$$

$$=C_1\int_{k^{\mu-1/2}}^{\infty}\exp\left(\frac{-Z^2}{2}\right)dz$$

$$\le C_2\int_{k_{-}^{\mu-1/2}}^{\infty}\exp(-Z)dz$$

$$=C_2\exp(-k^{\mu-1/2)}$$

$$\le C_3k^2 \tag{5}$$

After a substitution $Z=\mu k^{-1/2}$ and some estimates, including $\exp(-y)\le const\ y^{-\gamma}$ for each $\gamma>0$, where C_1, C_2 and C_3 do not depend on h or t (though C_3 depends on μ). Taking $[s,s+k]$ as the binary intervals $(n-1)2^{-i},n^{-i}$, for each positive integer l,

$$P\left(\left|Y(n-1)2^{-i}-Y(n_2^{-i})\right|\right)>2^{-iu} \tag{6}$$

For some $i\ge l$ and $1\le n\le 2^i$, $\le C_3\sum_{i=1}^{\infty}2^i2^{-2i}=C_3-2^{-I+1}$. Thus, with probability 1, there is an integer K such that $\left|Y(n-1)2^{-i})-Y(n_2^{-i})\right|\le 2^{-i\mu}$ for all $i>l$ and $1\le n\le 2^i$. If $h<H_0=2^{-i}$, the interval $[s,s+k]$"can be represented, except potentially at the endpoints, as a countable union of continuous binary intervals of the form" $[(n-1)2^{-i}]$ with $2^{-i}\le h$ and with two intervals of one length at most. Following that, let's use the continuity of X, if k is the smallest integer with $2^{-i}\le k$, $\left|Y(s)-Y(s+k)\right|\le 2\sum_{i=1}^{\infty}2^{-i\mu}$

$$=\frac{2^{-l\mu 2}}{(1-2^{-\mu})} \tag{7}$$

$$\le\frac{2^{-k\mu 2}}{(1-2^{-\mu})} \tag{8}$$

Since, Y(s) is almost surely continuous and therefore bound on [0,1], the inequality in (1) holds for some (different) value of b for $K_0\le|k|\le 1$, so (2) follows for all $0\le s,s+k\le 1$ for some random B [4]. This theorem provides a fundamental understanding of the local behavior of Brownian motion, highlighting its smoothness properties over short time intervals and its almost sure continuity. Einstein's insight into the random movement of molecules paved the way for the mathematical investigation of Brownian motion, revealing its role in the macroscopic process of diffusion.[6] Unsurprisingly, there exist significant ties between the theory of Brownian motion. In 1827, botanist Robert Brown noticed the irregular movement of pollen particles suspended in water when observing them through his microscope [6]. Diffusion in the white and grey matter of the brain refers to the movement of molecules within the extracellular space of the cerebral cortex and certain deeper brain structures that

primarily consist of neuronal cell bodies, dendrites, and synapses [6, 8]. This diffusion process is governed by Brownian motion, driven by the thermal energy of the surrounding fluid (cerebrospinal fluid) and other factors [6, 7]. Diffusion Tensor Imaging (DTI) is a specialized technique that is used to study the microstructural organization of white and grey matter in the brain [7, 11]. It relies on the observation that water molecules within biological tissues preferentially diffuse along the alignment of fiber tracts rather than across them.[6,7] By assessing the diffusion of water molecules in various directions, also it deduces the orientation and arrangement of these underlying fiber pathways.

Tracking Measures

The overall amount of water is reflected in the diffusion tensor trace. Trace is a widely used clinical metric that represents the total diffusivity inside a given voxel. It is calculated as three eigenvalues δ_1, δ_2 and δ_3 or diagonal elements T T_{uu}, T_{vv}, T_{ww}).[6] Tractography measures maintain consistency irrespective of rotational changes, ensuring a reliable depiction of white matter pathways [6, 7].

$$M_r(T) = \delta_1 + \delta_2 + \delta_3 = T_{uu} + T_{vv} + T_{ww} \tag{9}$$

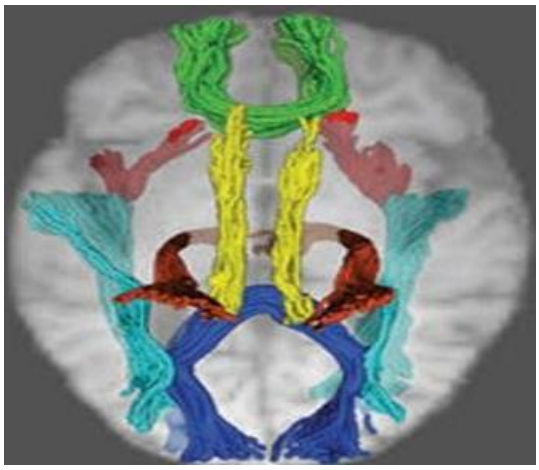

FIGURE 3. Brain connectivity through fiber tractography

Fig. 3 DTI data enables the creation of virtual representations of white matter tracts, aiding in the exploration of brain connectivity through fiber tractography [6, 7]. Diffusion Tensor Imaging (DTI) is a magnetic resonance imaging (MRI) technique used to investigate the microstructural organization of white matter in the brain. While DTI primarily focuses on white matter tracts, it can indirectly assist in identifying and analyzing grey matter structures as well [6, 7]. Mean diffusivity offers insight into the general diffusion characteristics by averaging the diffusion values across all three axes of the tensor within a specified volume $M_r(T)$.

Functional connectivity refers to the statistical relationship between the activities of different brain regions, It is commonly extrapolated from neuroimaging modalities like functional magnetic resonance imaging. It measures how synchronous or correlated the activities of different brain regions are over time, without necessarily implying direct anatomical connections between those regions [9]. Here, Two-way analysis of variance (ANOVA) is used to analyze the data. to assess the impact of two sample groups on gender, considering three categories: positive score of grey matter range, negative score of grey matter range, and general psychopathology of grey matter range.[8,9]The scale comprises 20 items that evaluate positive symptoms (e.g., hallucinations, delusions), negative symptoms (e.g., blunted affect, social withdrawal), and general psychopathology (e.g., anxiety, depression, disorientation). Each of these three categories is associated with separate scores for males and females [8]. The data is presented in tabular format, organized into rows and columns.

TABLE 2. Positive score, negative score, and general psychopathology of grey matter range

Gender	Positive Score	Negative Score	General Psychopathology
Male	39	39	78
	35	35	70
	32	32	64
	13	11	48
	16	15	41
	36	36	72
	15	12	38
	40	40	80
	22	29	44
	23	26	72
	34	34	68
	21	21	42
	41	41	82
	39	39	78
Female	18	20	55
	25	25	50
	23	8	43
	27	24	52
	20	12	51
	19	16	47

The PANSS (Positive and Negative Syndrome Scale) includes the general psychopathology subscale as one of its components, the general psychopathology subscale specifically focuses on symptoms that do not fall under the positive or negative symptom categories. It provides a more comprehensive assessment of a broad range of psychopathological symptoms beyond those directly related to schizophrenia.

TABLE 3. Analysis

Source of Variation	Sum of squares	Degree of freedom	Mean square	F	P-value	F critical value
Sample	101.4	1	101.4	0.682131	0.412488	4.019541
Columns	14031.63	2	7015.817	47.1963	1.401205	3.168246
Interaction	36.7	2	18.35	0.123443	0.884121	3.168246
Within	8027.2	54	148.6519			
Total	22196.93	59				

The Decision indicates a significant difference between the categories Positive score, negative score, and general psychopathology there's no significant difference between the two samples or interaction effect between gender and categories. therefore the variation in scores is primarily due to differences between the categories (Positive score, negative score, and general psychopathology), while differences between the two samples and interaction effects are not significant. Network analysis assists to, visualizing, and analyzing interactions and distinctions among different entities. and helps to understand changes in grey matter volume before and after a treatment intervention is crucial for assessing the effectiveness of the treatment and understanding its impact on brain structure and function. These figures and network graphs likely provide valuable insights into how the treatment affects grey matter volume and

brain network organization. Brownian motion and diffusion are key in medical imaging like diffusion tensor imaging (DTI), revealing tissue microstructure. DTI offers metrics like trace, mean, radial, and axial diffusivity to assess diffusion in the brain and tissues. These metrics aid in understanding disorders, tracking disease, and evaluating treatments for tissue integrity and function. Overall, DTI is a valuable tool for non-invasively studying the structural connectivity of the brain and investigating how alterations in white matter organization may contribute to neurological and psychiatric disorders. The two-factor ANOVA with replication revealed significant variations in the measured outcomes across different categories and factors. The analysis included data from three categories labeled Positive score, Negative score, and general psychopathology. The male category statistics for each factor and the total across all factors. It shows the count, sum, average, and variance. For factor Positive score, there are 10 observations with a total sum of 271, an average of 27.1, and a variance of 109.43. Factor Negative score, there are also 10 observations with a total sum of 275, an average of 27.5, and a variance of 123.39. Factor general psychopathology, there are 10 observations with a total sum of 607, an average of 60.7, and a variance of 263.12. Therefore total count across all factors is 30, with a total sum of 1153, an average of approximately 38.43, and a variance of 410.39. The female category statistics for each factor and the total but for a different set of observations. For factor Positive score, there are 10 observations with a total sum of 267, an average of 26.7, and a variance of 70.9. For factor Negative score, there are 10 observations with a total sum of 240, an average of 24, and a variance of 122.67. For factor General psychopathology, there are 10 observations with a total sum of 568, an average of 56.8, and a variance of 202.4. Following that the total count across all factors is again 30, with a total sum of 1075, an average of approximately 35.83, and a variance of 351.52. The overall data, including the total count, sum, average, and variance for each factor. The total count across all factors is 20 for each factor, with a total sum of 538 for factor Positive score, 515 for the factor Negative score, and 1175 for the factor general psychopathology. The average across all factors is approximately 26.9 for the factor Positive score, 25.75 for the factor Negative score, and 58.75 for the factor general psychopathology. The variance for the factor Positive score is approximately 85.46, for the factor Negative score is 119.78, and for the factor general psychopathology is 224.51. The ANOVA results indicate no significant effects from Sample, Columns, or their interaction on the outcome. This suggests that observed differences are likely due to random variation.

CONCLUSION

This paper inspects the various aspects of schizophrenia such as its causes, symptoms, progression, and treatments. Brownian motion theory plays a vital role in this study. it analysis to quantify abnormal regions within the brain. Furthermore, the study examines changes in grey matter levels in patients' brains before and after treatment interventions. By applying network analysis techniques, as well as statistical analysis examine schizophrenia patients, on a gender basis. Overall, this multi-approach offers a comprehensive understanding of the impact of schizophrenia on the human brain, providing valuable insights for both clinical practice and future research endeavors.

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