

The Collection of Various β ws – Closed Sets in Topological Spaces

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Abstract:

This research presents an innovative class of Beta weakly semi-closed sets namely, The collection of various β ws – closed sets in Topological spaces. Throughout this paper, β ws-Semi closure sets, β ws-Interior and β ws – Neighbourhood sets were examined to get the fundamental facts in the Beta weakly semi-closed sets. In this paper, the notion of countable β ws-semi-closed sets and β ws-Interior in TS were explored and β ws – Neighbourhood (β ws-nbd) in TS were also studied to get results. Here, many characterizations were obtained along with some of their feature results.

Keywords: Beta weakly semi interior sets (β ws-int), Beta weakly semi closure sets (β ws-cl), Beta weakly semi neighbourhood sets (β ws-nbd), Topological spaces(TS).

1. Introduction

The fundamental concepts of the collection of various β ws – closed sets in topological spaces play crucial roles in general topology. Numerous researchers have investigated into exploring their fundamental properties, leading to inspirations for generalizing these concepts to innovative extents. R.S. Suriyaand, T. Shyla Issac Mary [1] introduced Alpha weakly semi closed-sets in topological spaces. Levine N [2] introduced and he was studied on Generalized CS in topology in 1970. V. Kavitha, V.E. Sasikala, [3] introduced and studied on the closed sets in Beta generalized CS in topological spaces. D.Sivaraj and V.E. Sasikala,[4] studied on A Study on soft α -open sets in 2016. V.E. Sasikala, D. Sivaraj et al., [5, 6] studied on soft semi weakly generalized closed set in soft topological spaces in the year 2019. Many researchers introduced the soft topological spaces, V.E. Sasikala et al.,[7] introduced and investigated Soft swg separation axioms in soft topological spaces in 2019. In 1986, D. Andrijevic [8] discussed the Semi-preopen sets. Balachandran [9] studied on Generalized - α - closed maps and Maki, H. and α - generalized closed maps” in 1998. Basavaraj M.Ittanagi & Veerasha A Sajjanar, [10] introduced and investigated On weakly semi CS in topological Spaces, in the year 2017. Caldas, M, Jafari, S, Noiri, T & Simoes, M, [11] have defined and studied on A new generalization of contra continuity via Levine’s g-CS in 2007. G.B.Navalagi, [12] discussed the Semi pre neighbourhoods and properties of gspr -closed sets in topology in the year 2001. Indirani, K, Sathesh mohan, P & Rajendran, [14], explained Almost contra gr^* -continuous function in topological Space, in the year 2014. Mariappa. K & Sekar.S, [15] discussed the details of regular generalized b-CS, in the year 2013. V.E. Sasikala and D. Sivaraj [16]

introduced and studied on soft β -open sets, in 2017. Noiri, T., Popa, V. [17] presented a new class of between closed-sets and g-closed-sets in the year 2006. V.E. Sasikala, D. Sivaraj, R. Thirumalaisamy and J. Venkatesan[18-20] have defined and studied on soft regular star generalized star closed sets in soft topological spaces. This research aims to introduce the ideas extending β ws – closure, β ws-interior and β ws – neighbourhood within TS, along with providing characterizations for these concepts

2. PRELIMINARIES

Assume we recall that the following definitions the before said are used by our sequel.

Definition: 2.1 [1] $A \subseteq B$ of a TS (X, τ) it's called a semi – OS if $B \subseteq \text{cl}(\text{int}(B))$ and also a semi- CS and $\text{int}(\text{cl}(B)) \subseteq B$.

Definition: 2.2 [6] $A \subseteq P$ of a TS (X, τ) it's called a weakly CS (briefly, wg-C) if $\text{cl}(\text{int}(P)) \subseteq G$ whenever $P \subseteq G$ and G open in X .

Definition: 2.3 [5] $A \subseteq P$ of a TS (X, τ) it's called a regular generalized α - CS (briefly $\text{rg}\alpha$ -C) if $\alpha\text{cl}(P) \subseteq G$ whenever $P \subseteq G$ and G is regular α -OS in X .

3. BETA WEAKLY SEMI – INTERIOR:

Definition: 3.1 Let P be a subset of a TS in X , β ws-interior of P be symbolized by $\beta\text{wsint}(P)$ it was described as $\beta\text{wsint}(P) = \cup \{Q \subseteq P \text{ and } Q \text{ be } \beta\text{ws-OS in } X\}$ or $\cup \{Q: Q \subseteq P \text{ and } Q \text{ is } \beta\text{wsO}(X) \text{ or } \beta\text{wsint}(P)\}$ be a combination of all β ws-OS with includes P .

Theorem: 3.2 If P and E are subsets of a TS in X , then

- i) $\beta\text{wsint}(X) = X$, $\beta\text{wsint}(\phi) = \phi$, $\beta\text{wsint}(P) \subseteq P$
- ii) When E is a β ws-OS included in P , that the $E \subseteq \beta\text{wsint}(P)$.
- iii) Assume that $P \subseteq E$ then $\beta\text{wsint}(P) \subseteq \beta\text{wsint}(E)$, $\beta\text{wsint}(P) = \beta\text{wsint}(\beta\text{wsint}(P))$.

Proof:

i) For the aim of defining on β ws- interior, $\beta\text{wsint}(X) = \cup \{Q: Q \subseteq X \text{ and } Q \text{ are } \beta\text{ws – OS in } X\}$. Since X be β ws – open, $\beta\text{wsint}(X) = X$. By using Definition 3.1, $\beta\text{wsint}(\phi) = \cup \{Q: Q \subseteq \phi \text{ and } B \text{ is } \beta\text{ws – OS in } X\}$. But ϕ are the only β ws – OS included in ϕ . Therefore $\beta\text{wsint}(\phi) = \phi$.

ii) If $X \in \beta\text{wsint} - \text{int}(P)$. Then there exists an β ws – OS $Q \ni X \in P$. Thus $\beta\text{wsint}(P) \subseteq P$.

iii) Assume that E be any β ws – OS such that $E \subseteq P$. Let $X \in E$, E an β ws – OS included in P then $X \in \beta\text{wsint}(P)$. Hence $E \subseteq \beta\text{wsint}(P)$.

Let us assume $X \in \beta\text{wsint}(P)$. There is a β ws – OS $Q \ni X \in Q \subseteq P$. Considering $P \subseteq E$, $X \in Q \subseteq E$. Hence, $X \in \beta\text{wsint}(E)$. Therefore, $\beta\text{wsint}(P) \subseteq \beta\text{wsint}(E)$.

Let P be any subset of X . By using Definition 3.1, if $Q \subseteq P$ and $Q \in \beta\text{wsO}(X)$ then $Q \subseteq \beta\text{ws-int}(P)$. Since Q exists a β ws-OS contained within $\beta\text{wsint}(P)$, by (iii), $Q \subseteq \beta\text{wsint}(P)$. Hence $\beta\text{wsint}(\beta\text{wsint}(P)) \subseteq \cup \{Q: Q \subseteq P \text{ and } Q \in \beta\text{ws O}(X)\} = \beta\text{wsint}(P)$. Therefore $\beta\text{wsint}(\beta\text{wsint}(P)) = \beta\text{wsint}(P)$.

Theorem: 3.3 Let P is a subset TS in X . If P be β ws – OS then β wsint (P) = P .

Proof:

Assume P is the β ws-OS of a TS in X . It is known that β wsint (P) \subseteq (P) \Rightarrow (1). Similarly, P denotes a β ws-OS that contains P in Theorem 3.2 (iii) $P \subseteq \beta$ wsint (P) \Rightarrow (2). Therefore, β wsint (P) = P based on (1) and (2).

Theorem: 3.4 Assume that \wp and \mathfrak{Q} subsets of TS in X , and then β ws int (\wp) \cup (β ws int \mathfrak{Q}) \subseteq β ws int ($\wp \cup \mathfrak{Q}$).

Proof:

If \wp and \mathfrak{Q} be any two subsets in X . We know that $\wp \subseteq \wp \cup \mathfrak{Q}$ and $\mathfrak{Q} \subseteq \wp \cup \mathfrak{Q}$. Since $\wp \subseteq \wp \cup \mathfrak{Q}$, β ws – int (\wp) \subseteq β ws – int ($\wp \cup \mathfrak{Q}$). Since $\mathfrak{Q} \subseteq \wp \cup \mathfrak{Q}$, β ws – int (\mathfrak{Q}) \subseteq β ws – int ($\wp \cup \mathfrak{Q}$). This implies that $[\beta$ ws - int (\wp) \cup (β ws – int (\mathfrak{Q}))] \subseteq β ws - int ($\wp \cup \mathfrak{Q}$ s).

Remark: 3.5

For any two subsets P and Q of X . β ws – int (P) \cup β ws – int (Q) \neq β ws – int ($P \cup Q$) given in that following explanation.

Example: 3.6

Let $X = \{\mathfrak{m}, \mathfrak{b}, \mathfrak{p}\}$, $\tau = \{X, \wp, \{\mathfrak{m}\}, \{\mathfrak{p}\}, \{\mathfrak{m}, \mathfrak{p}\}\}$, β ws $O(X) = \{X, \wp, \{\mathfrak{b}, \mathfrak{p}\}, \{\mathfrak{m}, \mathfrak{p}\}, \{\mathfrak{p}\}\}$. Let $P = \{\mathfrak{b}, \mathfrak{p}\}$, $Q = \{\mathfrak{m}, \mathfrak{p}\}$, $P \cup Q = \{\mathfrak{m}, \mathfrak{b}, \mathfrak{p}\}$. Now β ws – int (P) = $\{\mathfrak{p}\}$, β ws – int (Q) = $\{\mathfrak{m}\}$ and $[\beta$ ws – int (P) \cup β ws – int (Q)] = $\{\mathfrak{p}, \mathfrak{m}\}$ and β ws – int \mathcal{J} , $\{\mathfrak{p}, \mathcal{J}\}$, $\{\mathfrak{m}, \mathfrak{p}\}$, $\{\mathfrak{m}, \mathcal{J}\}$, $\{\mathfrak{m}, \mathfrak{p}, \mathcal{J}\}$ β ws $O(X) = \{X, \wp, \{\mathfrak{b}, \mathfrak{p}, \mathcal{J}\}, \{\mathfrak{m}, \mathfrak{p}, \mathcal{J}\}, \{\mathfrak{m}, \mathfrak{b}, \mathcal{J}\}, \{\mathfrak{p}, \mathcal{J}\}, \{\mathfrak{b}, \mathcal{J}\}, \{\mathfrak{m}, \mathcal{J}\}\}$. Let $P = \{\mathfrak{b}, \mathcal{J}\}$, $Q = \{\mathfrak{b}, \mathfrak{p}, \mathcal{J}\}$, int (P) = $\{\mathfrak{b}\}$, β int (P) = $\{\mathfrak{b}\}$ this implies β wsint (P) = $\{\mathfrak{b}, \mathfrak{p}, \mathcal{J}\}$, $\therefore, \{\mathfrak{b}, \mathcal{J}\} \subseteq \{\mathfrak{b}, \mathfrak{p}, \mathcal{J}\}$ Hence int (P) \subseteq β ws- int (P) and β int (P) \subseteq β ws –int (P).

4. Methods and Discussion

BETA WEAKLY SEMI – CLOSURE:

Definition: 4.1 Assume P is a subset of a TS in X , Beta weakly semi –closure on P be denoted by β wscl (P) also it is defined as β ws cl (P) = $\cap \{Q: P \subseteq Q, Q \text{ is } \beta$ ws-CS in $X\}$ or intersection on all β ws –CS in X .

Theorem: 4.2 In that instance, P and E are elements of a TS in X and then

- i) β ws cl(X) = X , β ws cl(\wp) = \wp ,
- ii) $P \subseteq \beta$ ws cl(P).
- iii) If E has any β ws – CS in X including P , as the result β ws cl(P) \subseteq E .
- iv) In that case $P \subseteq E$ then β ws cl(P) \subseteq β ws cl(E),
- v) β ws cl(\wp) = β ws cl(β ws cl(\wp)).

Proof:

i) According to the definition β ws-closure, β ws cl(X) = \cap among them β ws – CS in X including X = $X \cap \{\beta$ ws – CS including $X\} = X \cap X = X$. \therefore , β ws cl(X) = X . According to the definition about β ws – closure, β ws cl(ϕ) = \cap among them β ws – CS in X containing $\phi = \phi \cap$ any β ws – CS in X which includes $\phi = \phi \cap = \phi$. \therefore , β ws cl(ϕ) = ϕ .

ii) In accordance with explanation on β ws – closure on P , It's clear which $P \subseteq \beta$ ws cl(P).

iii) Let E is part of β ws – CS in X including A . Considering that β ws cl(P) is the intersection all β ws – CS in X which includes P . β ws cl(P) exists in each and every β ws – CS through X which includes P . Therefore, particularly β ws cl(P) $\subseteq E$.

iv) Suppose P and E are subsets of $X \ni P \subseteq E$. By the definition of β ws – closure, β ws cl(E) = $\cap \{Q: Q \text{ is } \beta$ ws – CS in $X\}$. If $E \subseteq Q, Q \text{ is } \beta$ ws – CS with X then β ws cl(E) $\subseteq Q$. Since $Q \subseteq E \subseteq Q \text{ is } \beta$ ws – CS in X , we have β ws cl(P) $\subseteq Q$, β ws cl(P) $\subseteq \cap \{Q: \text{is } \beta$ ws – CS in $X\} = \beta$ ws cl(E). \therefore , β ws cl(P) $\subseteq \beta$ ws cl(E).

v) Assume that ϕ is a brief subset of X . According to a definition of β ws – closure, β ws cl(ϕ) = $\cap \{Q : \phi \subseteq Q, Q \text{ is } \beta$ ws – CS in $X\}$. If $\phi \subseteq Q$, then β ws cl(ϕ) $\subseteq Q$. Since Q be a β ws – CS which includes β ws cl(ϕ), from (iii) β ws cl(β ws cl(ϕ)) $\subseteq Q$. \therefore , β ws cl(ϕ) = $\cap \{Q : \phi \subseteq Q, Q \text{ is } \beta$ ws – CS in $X\} = \beta$ ws cl(ϕ). Hence β ws cl(ϕ) = β ws cl(β ws cl(ϕ)).

THEOREM 4.3: Let us consider $x \in X$. Then, $x \in \beta$ ws- cl A iff $\forall V \cap A \neq \emptyset \forall \beta$ ws – OS V including the points x .

Proof: Let assume that $x \in \beta$ ws – cl A . To prove $\forall V \cap A \neq \emptyset \forall \beta$ ws – OS V including the x . Suppose that $\exists \beta$ ws – OS V which includes $x \ni V \cap A = \emptyset$, and then $A \subseteq X - V$, because $V \cap A = \emptyset$. By theorem 4.2 (iv) β ws- cl $A \subseteq \beta$ ws- cl ($X - V$). Given V is an β ws – OS then $X - V$ is β ws – C. Since $X - V$ is β ws – C. Then β ws- cl ($X - V$) = $X - V$. Thus β ws – cl $A \subseteq X - V$. \therefore , $x \notin \beta$ ws – cl A which is a contradiction to our assumption. \therefore , $\forall V \cap A \neq \emptyset \forall \beta$ ws – OS V containing the point x . Conversely, Assume that $\forall V \cap A \neq \emptyset \forall \beta$ ws – OS V containing the point x . To prove $x \in \beta$ ws – cl A . Suppose $x \notin \beta$ ws – cl A , \exists an β ws – C $\subseteq F$ which includes $A \ni x \notin F$. \therefore , $x \in X - F$. \therefore , $(X - F) \cap A = \emptyset$, because $A \subseteq F$. That's contradiction, because $x \in X - F$. \therefore , $x \in \beta$ ws – cl A .

THEOREM 4.4: If A and B are subsets of the space X , then β ws – cl($A \cap B$) $\subseteq (\beta$ ws – cl A) $\cap (\beta$ ws – cl B).

Proof: Assume that A and B are any two subsets of X . Since $A \cap B \subseteq A$, from using theorem 4.2(iv), β ws – cl ($A \cap B$) $\subseteq \beta$ ws – cl A . Also $A \cap B \subseteq B \Rightarrow \beta$ ws – cl($A \cap B$) $\subseteq \beta$ ws – cl B . Thus β ws – cl ($A \cap B$) $\subseteq (\beta$ ws – cl A) $\cap (\beta$ ws – cl B).

Remark 4.5 :

In generally, $(\beta$ ws – cl A) $\cap (\beta$ ws – cl B) $\not\subseteq \beta$ ws – cl ($A \cap B$) as the example below explains.

Example: 4.6

Let $X = \{m, b, p, d\}$, $\tau = \{X, \phi, \{m\}, \{p\}, \{d\}, \{p, d\}, \{m, d\}, \{m, p, d\}$ following that using β ws $C(X, \tau) = \{X, \phi, \{m\}, \{b\}, \{p\}, \{m, b\}, \{m, p\}, \{b, p\}\}$. Let $A = \{m\}$, $B = \{p\}$, then $A \cap B = \{\phi\} \Rightarrow \beta$ ws $-cl(A \cap B) = \phi \rightarrow (1)$. Also β ws $-cl A = \{m, b\}$, β ws $-cl B = \{m, p\}$ then $(\beta$ ws $-cl A) \cap (\beta$ ws $-cl B) = \{m\} \rightarrow 2$, From 1 and 2, $(\beta$ ws $-cl A) \cap (\beta$ ws $-cl B) \not\subseteq \beta$ ws $-cl (A \cap B)$.

5. BETA WEAKLY SEMI CLOSED NEIGHBORHOOD AND LIMIT POINTS:

β ws – Neighborhood through is TS introduced by using Notion of β ws – OS.

Definition: 5.1 If X is a TS with the $x \in X$. If subset N of X it is said β ws – Neighborhood (briefly - β ws – nbd) of x if \exists an β ws – OSG that the $x \in G' \subseteq N$.

Definition: 5.2 Let subset N of a space X is known as β ws – nbd of $P \subseteq N$ if \exists an β ws – OSG which $P \subseteq G \subseteq N$. A group of all β ws – Neighborhood of $x \in X$ is known as β ws – nbd of X and that will be represented by β ws – $N(X)$.

Theorem: 5.3 Each of open Neighborhood on $x \in X$ is an β ws – Neighborhood of $x \in X$.

Proof:

Assume that N be an open– Neighborhood N on $x \in X$. To prove N is an β ws – Neighborhood on x . From definition 5.1, \exists an OS $G' \ni x \in G' \subseteq N$. Now G' is an β ws – O, since every OS is β ws – O in X . \therefore , N be an β ws – Neighborhood of x .

Theorem: 5.4 Let a subset N of a space X is β ws – O, and N be a β ws– Neighborhood everyone it's points.

Proof: Assume N be β ws – O. If $x \in N$. To prove N is β ws – Neighborhood of x . Considering N is a β ws – OS, $x \in N \subseteq N$. From, definition 5.1, N is β ws – Neighborhood of x . Considering X represents an arbitrary N point, N is a β ws – Neighborhood everyone it's points.

Theorem: 5.5 Let us consider X be a TS. Let F be a β ws – C subset on X and $x \in X - F$, Also, there is a β ws – Neighborhood N of $x \ni N \cap F = \phi$.

Proof: Assume F is a β ws – C $\subseteq X$ also $x \in X - F$. Since $X - F$ be β ws – O, by using Theorem 3.3.6, $X - F$ contains a β ws – Neighborhood each one of its points. Then \exists an β ws – Neighborhood N on $x \ni N \subseteq X - F$. Hence $N \cap F = \phi$.

Theorem: 5.6 Let us consider N be an β ws – Neighborhood of a TS in X and of every $x \in X$ and let the β ws – $N(x)$ is a collection of every β ws – Neighborhood of x .

i) For each $x \in X$, β ws – $N(x) \neq \phi$

ii) If $N \in \beta$ ws – $N(x)$, then $x \in N$.

iii) If $N \in \beta$ ws – $N(x)$ and $N \subseteq M$, then $M \in \beta$ ws – $N(x)$.

iv) If $N \in \beta$ ws – $N(x)$ or $M \in \beta$ ws – $N(x)$ then $N \cup M \in \beta$ ws – $N(x)$.

v) If $N \in \beta_{ws} - N(x)$, then $\exists M \in \beta_{ws} - N(x) \ni M \subseteq N$ and $M \in \beta_{ws} - N(y)$ for all $y \in M$.

vi) If $N \in \beta_{ws} - N(x)$ and $M \in \beta_{ws} - N(x)$ then $N \cap M \in \beta_{ws} - N(x)$.

Proof:

i) Since X be an $\beta_{ws} - OS$, by using Theorem 5.7, it be an $\beta_{ws} - nbd$ on every $x \in X$. Then \exists the least of any $\beta_{ws} - Neighborhood$ (known as X) for every $x \in X$. Therefore, $\beta_{ws} - N(x) \neq \emptyset$ for each $x \in X$.

ii) If $N \in \beta_{ws} - N(x)$, N be an $\beta_{ws} - Neighborhood$ of x , from definition 5.1, $x \in N$.

iii) Let us assume $N \in \beta_{ws} - N(x)$. Let $N \subseteq M$. \therefore , there be a $\beta_{ws} - OS$ $G \ni x \in G \subseteq N$. Now $x \in G \subseteq M$, Since $N \subseteq M$. \therefore , M is an $\beta_{ws} - Neighborhood$ of x . Hence $M \in \beta_{ws} - N(x)$.

iv) Given $N \in \beta_{ws} - N(x)$ or $M \in \beta_{ws} - N(x)$. From Definition 5.1, \exists an $\beta_{ws} - OS$ G_1 and $G_2 \ni x \in G_1 \subseteq N$ or $x \in G_2 \subseteq M$. Thus $x \in G_1 \cup G_2 \subseteq N \cup M$. Assume G_1 and G_2 are $\beta_{ws} - OS$, $G_1 \cup G_2$ is an $\beta_{ws} - OS$. Thus $N \cup M \in \beta_{ws} - N(x)$.

v) Let $N \in \beta_{ws} - N(x)$. \therefore , \exists an $\beta_{ws} - OS$ $M \ni x \in M \subseteq N$. Since M is a $\beta_{ws} - OS$, by using theorem 5.7, it is an $\beta_{ws} - Neighborhood$ each one it's points. Hence $M \in \beta_{ws} - N(y)$ for each $y \in M$.

vi) Given $N \in \beta_{ws} - N(x)$ and $M \in \beta_{ws} - N(x)$. From using definition 5.1, \exists an $\beta_{ws} - OS$ G_1 and $G_2 \ni x \in G_1 \subseteq N$ and $x \in G_2 \subseteq M$. Thus $x \in G_1 \cap G_2 \subseteq N \cap M$. \therefore , G_1 and G_2 are $\beta_{ws} - OS$, by using 3.10, $G_1 \cap G_2$ is an $\beta_{ws} - OS$. Thus $N \cap M$ is an $\beta_{ws} - Neighborhood$ of x . \therefore , $N \cap M \in \beta_{ws} - N(x)$.

Definition: 5.7 Let X be a TS with P as a subset of X , point $x \in X$ is known as Beta weakly semi – limit point of P if each $\beta_{ws} - Neighborhood$ on X has a P point that is different through X it has $((N - \{x\}) \cap P) \neq \emptyset \forall \beta_{ws} - Neighborhood N$ of x .

Remark: 5.8

In generally, β_{ws} -neighborhood N of $x \in X$ can't be a neighborhood of x in X as demonstrated by the following explanation.

Example: 5.9

Let $X = \{a, b, p, q\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, p\}\}$ be a topology on X . Therefore, $\beta_{ws} O(X) = \{X, \emptyset, \{a\}, \{b\}, \{p\}, \{q\}, \{a, b\}, \{b, p\}, \{p, q\}, \{a, b, p\}\}$. The set $\{b, p, q\}$ is a $\beta_{ws} - nbd$ of the point q , \therefore , $\beta_{ws} - OS \{p, q\} \ni \{q\} \subseteq \{p, q\} \subseteq \{b, p, q\}$. The set $\{b, p, q\}$ is not an neighborhood of the point q , \therefore , $OS G_1$ exists $\ni p \in G_1 \subseteq G_1 \{p, q\}$.

Definition: 5.10 If $P \subset X$. Let the points $x \in X$ is known as $\beta_{ws} - limit$ points of P if each $\beta_{ws} - limit$ points on P if every $\beta_{wg} - CS$ which includes x contains a points on P apart from x . The collection of all $\beta_{wg} - limit$ points of P is known as $\beta_{ws} - derived$ set of P with be devoted by $D\beta_{ws} - (P)$. Some properties of the above concepts are as follows:

Theorem: 5.11 $\beta_{wg} - clP = P \cup D\beta_{wg} (P)$.

Proof:

Let $x \in \beta\text{wg} - \text{cl} P$. Then either $x \in P$ or $x \notin P$. Let $x \notin P$ as well as if x is not a $\beta\text{wg} - \text{limit point}$ on P . Then $\exists \beta\text{wg} - \text{OS } Q$ containing $x \ni Q \cap P = \emptyset$. This means that is a $\beta\text{wg} - \text{CS}$ containing P . So $\beta\text{wg} - \text{cl } P \subset Q$ and $x \notin Q^c$. Thus we get a contradiction.

This proves that either $x \in P$ or x is $\beta\text{wg} - \text{limit point}$ of P , In other words $\beta\text{wg} - \text{cl } P \subset P \cup D\beta\text{wg}(P)$.

Again in order to prove that $P \cup D\beta\text{wg}(P) \subset \beta\text{wg} - \text{cl } P$, we Need to prove that if $x \notin \beta\text{wg} - \text{cl } (P)$ then $x \notin P \cup D\beta\text{wg}(P)$. Now $x \notin \beta\text{wg} - \text{cl } (P) \Rightarrow x \in (\beta\text{wg} - \text{cl}(P))^c$ which is $\beta\text{wg} - \text{O}$. $\Rightarrow x$ is Neither a point on P Nor a $\beta\text{wg} - \text{limit point}$ on $P \Rightarrow x \notin P \cup D\beta\text{wg}(P)$. Hence the result.

COROLLARY: 5.12 A subset P of X is $\beta\text{wg} - \text{C}$ iff $D\beta\text{wg}(P) \subset P$.

Proof:

Let P be $\beta\text{wg} - \text{closed}$. Then $\beta\text{wg} - \text{cl } P = P \cup D\beta\text{wg}(P)$. This proves that $D\beta\text{wg}(P) \subset P$. Conversely, if $D\beta\text{wg}(P) \subset P$, then $\beta\text{wg} - \text{cl } P = P \cup D\beta\text{wg}(P) = P$. Hence P is $\beta\text{wg} - \text{closed}$.

5. Conclusion

We discovered fresh cases as well as the weaker and stronger βws -limits and βws -CS neighborhoods. The concepts of βws -neighborhood, βws -closure, βws -interior, and βws -limit points are introduced and examined in order to characterize the features of βws -CS.

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