Mathematics and Statistics 13(1): 28-40, 2025 http://www.hrpub.org
DOI: 10.13189/ms.2025.130104

Total 2 - Out Degree Equitable Domination Number for Distinct Graphs

T. Sindhuja[1], V. Maheswari[1,*], V. Balaji[2]

[1]Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Pallavaram, Chennai, Tamil Nadu, India
[2]Department of Mathematics, Sacred Heart College, India

Received August 10, 2024; Revised December 5, 2024; Accepted January 22, 2025

Cite This Paper in the Following Citation Styles

(a): *[1] T. Sindhuja, V. Maheswari, V. Balaji , "Total 2 - Out Degree Equitable Domination Number for Distinct Graphs," Mathematics and Statistics, Vol. 13, No. 1, pp. 28 - 40, 2025. DOI: 10.13189/ms.2025.130104.*

(b): *T. Sindhuja, V. Maheswari, V. Balaji (2025). Total 2 - Out Degree Equitable Domination Number for Distinct Graphs. Mathematics and Statistics, 13(1), 28 - 40. DOI: 10.13189/ms.2025.130104.*

Abstract For any graph G, "The out degree of u with respect to a dominating set D is denoted and defined as $od_D(u) = |N(u) \cap (V - D)|$". The subset D of V is referred to be a dominating set and the induced sub graph $<D>$ doesn't contain any isolated vertices. The total domination number is determined by the number of vertices of the minimal dominating set, which is denoted by $\gamma_t(G)$. Based on the above concept of out degree and total dominating set, we introduce a new domination called total 2 - out degree equitable domination (2 - ODED) number. A set D of V is referred to be total 2 – ODED set if D is a dominating set and induced sub graph $<D>$ that contains no isolated vertices also has the property $|od_D(u) - od_D(w)| \leq 2$ where $od_D(u) = |N(u) \cap (V - D)|$ for any two vertices $u, w \in D$. The minimum number of vertices of such dominating set is known as total 2 - ODED number. This paper is to investigate the proposed domination number for some general graphs like complete graph, star graph, path graph, cycle graph, double fan graph, bi star, fan graph, helm graph, crown graph, and triangular snake graph which are explained with examples. Finally, we discuss the application of total 2 – out degree equitable domination (2 – ODED) number in real life.

Keywords 2 - Out Degree Domination Number, Isolated Vertices, Dominating Set, Equitable Domination Number, Total 2 - Out Degree Domination, Helm Graph

1. Introduction

In this paper, we used the undirected and simple graphs [1]. A graph G = (V, E), contains a set of vertices V and a set of edges E. Here, p and q represent the cardinality of vertices and edges of a graph. The degree of any vertex u indicates the number of edges that are incident on a vertex, and it is represented by the symbol deg(u). u is referred to as an isolated vertex if deg(u) = 0.

Ore [2] and C. Berge [3] were the first to propose the notion of domination number. A subset D of V is called the dominating set of G if every vertex of $V - D$ is dominated by at least one vertex of D. A domination number is the number of vertices minimal dominating set, represented by $\gamma(G)$. Ali Sahal and V. Mathad [4] introduced the 2 - out degree equitable domination number. For any two vertices $u, w \in D$ such that $|od_D(u) - od_D(w)| \leq 2$ where $od_D(u) = |N(u) \cap (V - D)|$, then a dominating set D is referred as a 2 - ODED set. The term "2 - out degree equitable domination number" refers to the cardinality minimum 2 - ODED set, denoted by $\gamma_{2oe}(G)$. Based on the aforesaid domination number, Mahesh M.S, et al. [5,6,7,8] propose some new domination numbers. The notion of total domination number is proposed by Cokayne et al. [9].

The subset D of V is referred to be a total dominating set if D is a dominating set and the induced subgraph <D> doesn't contain any isolated vertices. The total domination number is determined by the number of vertices of the minimal total dominating set, which is denoted by $\gamma_t(G)$. Here we introduce a new domination parameter called total 2 - out degree equitable domination (ODED) number.

2. Total 2 - Out Degree Equitable Domination Number

2.1. Definition

A subset D of V is referred to be total 2 – ODED set if D is a dominating set and the induced sub graph $<D>$ containing no isolated vertices also has the property $|od_D(u) - od_D(w)| \leq 2$ where $od_D(u) = |N(u) \cap (V-D)|$ for any two vertices $u, w \in D$. The minimum number of vertices of such dominating set is known as total 2 - ODED number is represented as $\gamma_{t2oe}(G)$. The minimum total 2 - ODED set is called $\gamma_{t2oe} - set$.

2.2. Example

Consider the graph in Figure 1.

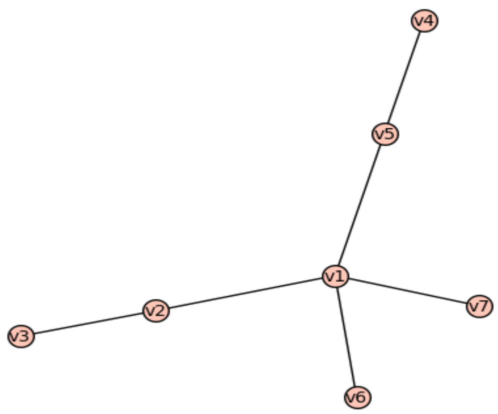

Figure 1. Example

“Let us consider

D = $\{v_1, v_2, v_5\}$ $and\ V - D = \{v_3, v_4, v_6, v_7\}$

$$od_D(v_1) = |N(v_1) \cap (V-D)|$$
$$= |\{v_2, v_5\} \cap \{v_3, v_4, v_6, v_7\}| = |\emptyset| = 0$$
$$od_D(v_2) = |N(v_2) \cap (V-D)|$$
$$= |\{v_1, v_3, v_7\} \cap \{v_3, v_4, v_6, v_7\}| = |\{v_3, v_7\}| = 2$$
$$od_D(v_5) = |N(v_5) \cap (V-D)|$$
$$= |\{v_1, v_4, v_6\} \cap \{v_3, v_4, v_6, v_7\}| = |\{v_4, v_6\}| = 2$$

Then any u, w $\in D, |od_D(u) - od_D(w)| \leq 2$

Now D = $\{v_1, v_2, v_5\}$ form 2 – ODED set of G and the induced subgraph <D> has no vertices of degree zero. So D is the minimum total 2 – ODED.

Thus $\gamma_{t2oe}(G) = 3$”.

3. Total 2 - ODED Number in Some General Graphs

For various general graphs, we calculate the above defined domination number.

3.1. Theorem

For any complete graph K_p, $\gamma_{t2oe}(K_p)$ = 2.

Proof.

Consider D ={u, w} G is complete, for all vertices u $\in$ D, N (u) = $V - \{u\}$.

Hence $od_D(u)$= |N(u) $\cap (V-D)|$ = $p - 2$.

For any two vertices u, w $\in$ D, $od_D(u) = p - 2$, $od_D(w) = p - 2$.

Hence $|od_D(u) - od_D(w)| = 0 \leq 2$ and D forms a 2 - ODED set.

Since G is complete, D = {u, w} has no isolated vertices. Then D is a $\gamma_{t2oe} - set$ and 2 $\leq \gamma_{t2oe}(K_p)$, $\gamma_{t2oe}(K_p) \leq$ 2.

Hence $\gamma_{t2oe}(K_p)$ = 2.

3.2. Example

For a complete Graph K_6, as shown in Figure 2.

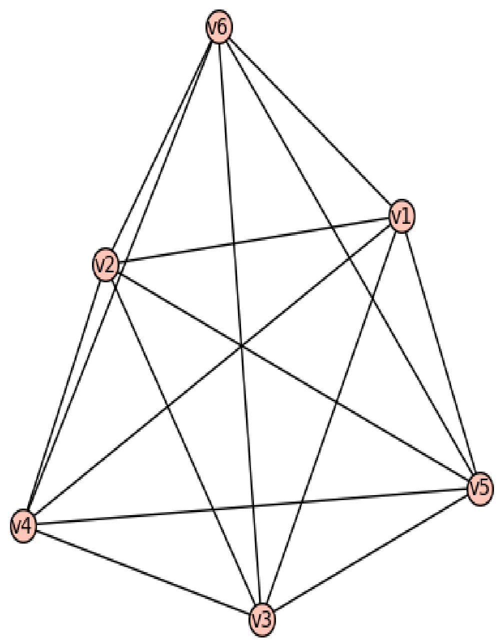

Figure 2. Complete Graph K_6

“Let us consider

D = $\{v_1, v_2\}$ $and\ V - D = \{v_3, v_4, v_5, v_6\}$

Now $od_D(v_1) = |N(v_1) \cap (V-D)|$

$$=|\{v_2,v_3,v_4,v_5,v_6\}\cap\{v_3,v_4,v_6,v_7\}|$$
$$=|v_3,v_4,v_6,v_7|=4$$
$$od_D(v_2)=|N(v_2)\cap(V-D)|$$
$$=|\{v_1,v_3,v_4,v_5,v_6\}\cap\ \{v_3,v_4,v_6,v_7\}|$$
$$=|v_3,v_4,v_6,v_7|=4$$

Now

$$|od_D(v_1)-od_D(v_2)|=0\le 2.$$

Then D = $\{v_1,v_2\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph.

Hence D is total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(G)=2$"

3.3. Theorem

For a star graph $K_{1,p}$, $\gamma_{t2oe}(K_{1,p})=p-2$.

Proof.

Let the set of vertices in $K_{1,p}$ be $\{u,w_1,w_2,w_3,\ldots,w_p\}$.

Consider the set D $=\{u,w_1,w_2,w_3,\ldots\ldots,w_{p-2}\}$ and $V-D=\{w_{p-1},w_p\}$.

By the definition of star graph $K_{1,p}$, for any $w_i\in$ D,

N(w_i) $=u$ for $i=1,2,\ldots\ldots,$ p and

N(w_i) $\cap(V-D)=\emptyset$.

Then $od_D(w_i)=|N(w_i)\cap(V-D)|=0$.

Now $N\ (u)=\{w_1,w_2,w_3,\ldots\ldots,w_p\}$ and $V-D\subseteq N(u)$.

Then N(u) $\cap(V-D)=V-D$.

Therefore $od_D(u)=|N(u)\cap(V-D)|=|V-D|=2$ and for any u, w $\in$ D,$|od_D(w_i)-od_D(u)|=2\le 2$.

Hence D is a $\gamma_{t2oe}-\ set$ and its induced subgraph has no isolated vertices.

Thus $\gamma_{t2oe}(K_{1,p})=p-2$.

3.4. Example

For a star graph $K_{1,8}$, as shown in Figure 3.

"Let us consider

D = $\{u,v_1,v_2,v_3,v_4,v_5,v_6\}\ and\ V-D=\{v_7,v_8\}$

Now

$$od_D(\text{u})=|N(\text{u})\cap(V-D)|$$
$$=|\{v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,\}\cap\{v_7,v_8\}|$$
$$=|v_7,v_8|=\ 2$$
$$od_D(v_1)=|N(v_1)\cap(V-D)|$$
$$=|\{\text{u}\}\cap\{v_7,v_8\}|=|\emptyset|=0$$
$$od_D(v_2)=|N(v_2)\cap(V-D)|$$
$$=|\{\text{u}\}\cap\{v_7,v_8\}|\quad=|\emptyset|=0$$
$$od_D(v_3)=|N(v_2)\cap(V-D)|$$
$$=|\{\text{u}\}\cap\{v_7,v_8\}|=|\emptyset|=0$$
$$od_D(v_4)=|N(v_2)\cap(V-D)|$$
$$=|\{\text{u}\}\cap\{v_7,v_8\}|=|\emptyset|=\ 0$$

Now $|od_D(\text{u})-od_D(v_\text{i})|=2\le 2$ and $|od_D(v_\text{i})-od_D(v_\text{j})|=2\le 2$.

Then D = $\{u,v_1,v_2,v_3,v_4,v_5,v_6\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph. Hence D is total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(G)=7$"

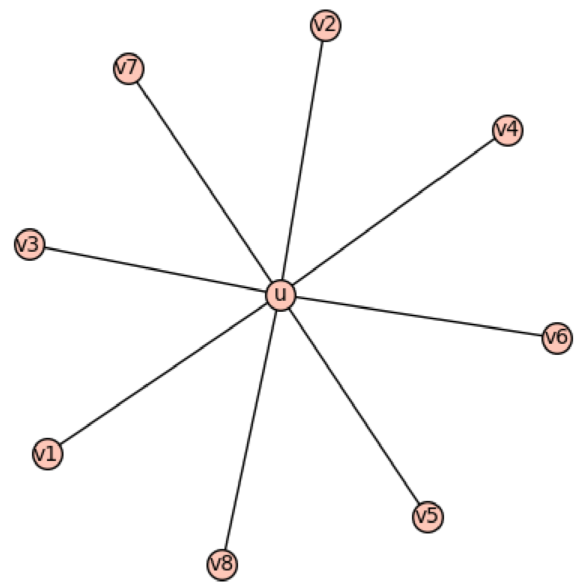

Figure 3. Star graph $K_{1,8}$

3.5. Theorem

For each Path P_p,

$$\gamma_{t2oe}(P_p) = \begin{cases} \frac{p}{2} & if \;\; p \equiv 0(mod4) \\ \left\lfloor \frac{p}{2} \right\rfloor + 1 & otherwise \end{cases} .$$

Proof.

Case 1

If $p \equiv 0(mod4)$

D = $\{w_{4n+2}, w_{4n+3} / \; n = 0,1,2\ldots\}$ and $V - D$ all the remaining vertices in D

Clearly, D is a minimum dominating set and induced subgraph <D> has no isolated vertices and any vertex in D has an out degree of 1.

Hence each total dominating set is a $\gamma_{t2oe} - \; set.$

Therefore $\gamma_{t2oe}(P_p) = \; \gamma_t(P_p) = \frac{p}{2}$.

Case 2

Otherwise, all the total dominating set is a $\gamma_{t2oe} - \; set$

Hence $\gamma_{t2oe}(P_p) = \; \left\lfloor \frac{p}{2} \right\rfloor + 1$

3.6. Example

For a path P_8, as shown in Figure 4

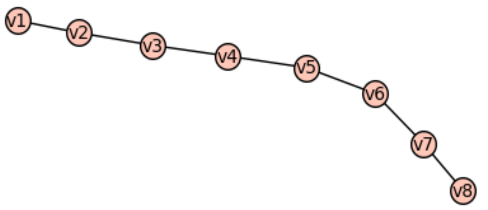

Figure 4. Path P_8

“Let us consider

D = $\{v_2, v_3, v_6, v_7\}$ $and \;\; V - D = \;\; \{v_1, v_4, v_5, v_8\}$

Now

$$od_D(v_2) = |N(v_2) \cap (V - D)|$$
$$= |\{v_1, v_3\} \cap \;\; \{v_1, v_4, v_5, v_8\}| = |\{v_1\}| = 1$$
$$od_D(v_3) = |N(v_3) \cap (V - D)|$$
$$= |\{v_2, v_4\} \cap \{v_1, v_4, v_5, v_8\}| = |\{v_4\}| = 1$$
$$od_D(v_6) = |N(v_6) \cap (V - D)|$$
$$= |\{v_5, v_7\} \cap \{v_1, v_4, v_5, v_8\}| = |\{v_5\}| = 1$$
$$od_D(v_7) = |N(v_7) \cap (V - D)|$$
$$= |\{v_6, v_8\} \cap \;\; \{v_1, v_4, v_5, v_8\}| = |\{v_8\}| = 1$$

Now

$$|od_D(v_i) - od_D(v_j)| = 0 \leq 2.$$

Then D = $\{v_2, v_3, v_6, v_7\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph. So D is total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(P_8) = 4$”

3.7. Theorem

For any Cycle C_p,

$$\gamma_{t2oe}(P_p) = \begin{cases} \left\lfloor \frac{p}{2} \right\rfloor & if \;\; p \not\equiv 2(mod4) \\ \frac{p+2}{2} & otherwise \end{cases}$$

Proof.

Let D be a minimum dominating set such that the induced subgraph <D> contains no isolated vertices, and every cycle within this subgraph is a 2-regular graph. Then the out degree of any vertices in D is 0 or 1.

For any u, w $\in D$, $|od_D(u) - od_D(w)| \leq 2$.

Hence D is $\gamma_{t2oe} - \;\; set$

$$\gamma_{t2oe}(C_p) = \;\; \gamma_t(C_p)$$
$$= \begin{cases} \left\lfloor \frac{p}{2} \right\rfloor & if \;\; p \not\equiv 2(mod4) \\ \frac{p+2}{2} & otherwise \end{cases}$$

3.8. Example

For a cycle C_6, as shown in Figure 5

“Let us consider

D = $\{v_2, v_3, v_5, v_6\}$ $and \;\; V - D = \;\; \{v_1, v_4\}$

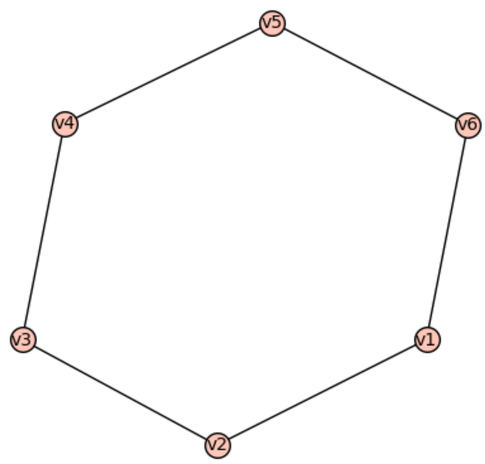

Figure 5. Cycle C_6

Now

$$od_D(v_2) = |N(v_2) \cap (V - D)|$$
$$= |\{v_1, v_3\} \cap \;\; \{v_1, v_4\}| = |\{v_1\}| = 1$$
$$od_D(v_3) = |N(v_3) \cap (V - D)|$$
$$= |\{v_2, v_4\} \cap \;\; \{v_1, v_4\}| = |\{v_4\}| = 1$$
$$od_D(v_5) = |N(v_5) \cap (V - D)|$$
$$= |\{v_4, v_6\} \cap \;\; \{v_1, v_4\}| = |\{v_4\}| = 1$$
$$od_D(v_6) = |N(v_6) \cap (V - D)|$$
$$= |\{v_5, v_1\} \cap \{v_1, v_4\}| = |\{v_1\}| = 1$$

Now

$$|od_D(v_i) - od_D(v_j)| = 0 \leq 2.$$

Then D = $\{v_2, v_3, v_6, v_7\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph. Hence D is total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(C_6) = 4$"

3.9. Theorem

For any double fan graph

$$\gamma_{t2oe}\big(D_2(F_{1,n})\big) = 2n - 1$$

Proof.

Consider

D= $\{u, u_1, u_2, u_3 \dots, u_{n-1}, u_{n+1}, u_{n+2}, u_{n+3} \dots, u_{2n-1}\}$ and $V - D = \{u_n, u_{2n}\}$.

Clearly the induced subgraph $< D >$ has no isolated vertices.

Here $od_D(u) = 2$, $od_D(u_{n-1}) = 1$, $od_D(v_{n-1}) = 1$, $od_D(u_i) = 0$ for $i = 1,2,3 \dots n-2$ and $od_D(v_i) = 0$ for $i = 1,2,3 \dots n-2$.

So, for any $u, w \in D$, $|od_D(u) - od_D(w)| \leq 2$.

Hence D is a $\gamma_{t2oe} -$ set and induced subgraph $< D >$ has no isolated vertices.

Therefore $\gamma_{t2oe}\big(D_2(F_{1,n})\big) = 2n - 1$.

3.10. Example

For a Double Fan $D_2(F_{1,4})$, as shown in Figure 6

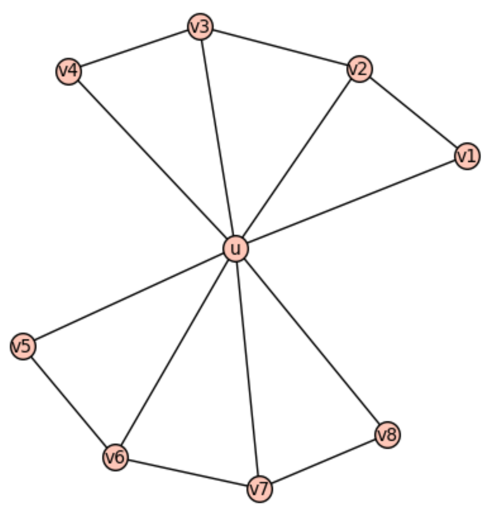

Figure 6. Double Fan $D_2(F_{1,4})$

"Let us consider

D = $\{u, v_1, v_2, v_3, v_5, v_6, v_7\}$ and $V - D = \{v_4, v_8\}$

Now

$$od_D(u) = |N(u) \cap (V - D)|$$
$$= |\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\} \cap \{v_4, v_8\}|$$
$$= |\{v_4, v_8\}| = 2$$
$$od_D(v_1) = |N(v_1) \cap (V - D)|$$
$$= |\{u, v_2\} \cap \{v_4, v_8\}| = |\emptyset| = 0$$
$$od_D(v_2) = |N(v_2) \cap (V - D)|$$
$$= |\{u, v_1, v_3\} \cap \{v_4, v_8\}| = |\emptyset| = 0$$
$$od_D(v_3) = |N(v_3) \cap (V - D)|$$
$$= |\{u, v_2, v_4\} \cap \{v_4, v_8\}| = |\{v_4\}| = 1$$
$$od_D(v_5) = |N(v_5) \cap (V - D)|$$
$$= |\{u, v_6\} \cap \{v_4, v_8\}| = |\emptyset| = 0$$
$$od_D(v_6) = |N(v_6) \cap (V - D)|$$
$$= |\{u, v_5, v_7\} \cap \{v_4, v_8\}| = |\emptyset| = 0$$
$$od_D(v_7) = |N(v_7) \cap (V - D)|$$
$$= |\{u, v_6, v_8\} \cap \{v_4, v_8\}| = |\{v_8\}| = 1$$

Now

$$|od_D(v_i) - od_D(v_j)| = 0 \leq 2.$$

Then D = $\{u, v_1, v_2, v_3, v_5, v_6, v_7\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph. Hence D is total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}\big(D_2(F_{1,4})\big) = 7$"

3.11. Theorem

For any bistar $\gamma_{t2oe}\big(B^2_{n,n}\big) = 2$

Proof.

Let V $= \{u, v, u_i, v_i\}, 1 \leq i \leq n$ where u_i, v_i are pendant vertices, u and v are support vertices.

Let us take D $= \{u, v\}$ and $V - D = \{u_1, u_2, u_3 \dots, u_n, v_1, v_2, v_3 \dots, v_n\}$.

Clearly the induced subgraph $< D >$ has no isolated vertices.

Here $od_D(u) =$ 2n and $|od_D(u) - od_D(v)| \leq 2$.

Hence D is a $\gamma_{t2oe} -$ set and induced subgraph $< D >$ has no vertices of degree zero.

Therefore $\gamma_{t2oe}\big(B^2_{n,n}\big) = 2$

3.12. Example

For a bistar $B^2_{4,4}$, as shown in Figure 7.

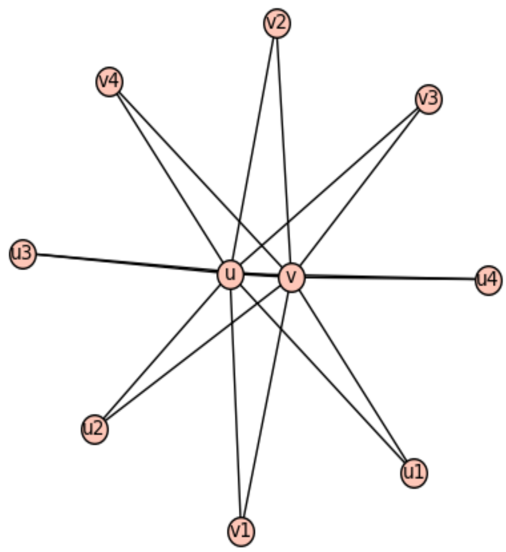

Figure 7. Bistar $B_{4,4}^2$

"Let us consider D = $\{u, \text{v}\}$ and

$$V - D = \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$$

Now

$$od_D(\text{u}) = |N(\text{u}) \cap (V - D)|$$

$$=|\{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\} \cap \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}|$$

$$= |\{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}| = 8$$

$$od_D(\text{v}) = |N(\text{v}) \cap (V - D)|$$

$$=|\{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\} \cap \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}|$$

$$= |\{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}| = 8$$

Now

$$|od_D(\text{u}) - od_D(\text{ v})| = 0 \leq 2.$$

Then D = $\{u, \text{v}\}$ form 2 – ODED set of G and there is no vertices of degree zero in the induced subgraph.

Thus D is a total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(B_{4,4}^2)) = 2$"

3.13. Theorem

For a double star $S_{r,t}$,

$$\gamma_{t2oe}(S_{r,t}) = \begin{cases} 2 & if \ |r - t| \leq 2 \\ not \ exist \ otherwise \end{cases}$$

Proof.

By the definition of double star,

$$\text{V}(S_{r,t}) = \{u, u_1, u_2, u_3 \ldots\ldots\ldots, u_r, w, w_1, w_2, w_3, \ldots\ldots, w_t\}$$

Let D ={u, w}be a dominating set and induced subgraph <D> doesn't have isolated vertices then by definition

$$N(u) \cap (V - D) = \{u_1, u_2, u_3 \ldots\ldots\ldots, u_t\}.$$

Then $od_D(u)$= $|N(u) \cap (V - D)|$ = t and

$N(w) \cap (V - D) = \{w_1, w_2, w_3, \ldots\ldots\ldots, w_r\}$ and $od_D(w)$= $|N(w) \cap (V - D)|$ = r.

Hence $|od_D(u) - od_D(w)| = |r - t| \leq 2$

Therefore D ={u, w} is $\gamma_{t2oe} - \ set$

Hence $\gamma_{t2oe}(S_{r,t}) = 2 \quad if \ |r - t| \leq 2$.

Clearly total 2 - out degree equitable dominating set not exists if $|r - t| \geq 2$.

3.14. Example

For a double star $B_{3,5}$, as shown in Figure 8

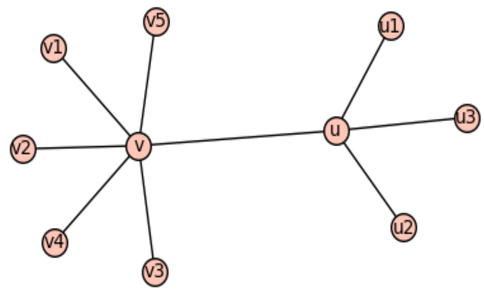

Figure 8. Double star $B_{3,5}$

"Let us consider D = $\{u, \text{v}\}$

$and \ V - D = \{u_1, u_2, u_3, v_1, v_2, v_3, v_4, v_5\}$

Now

$$od_D(\text{u}) = |N(\text{u}) \cap (V - D)|$$

$$= |\{u_1, u_2, u_3\} \cap \{u_1, u_2, u_3, v_1, v_2, v_3, v_4, v_5\}|$$

$$= |\{u_1, u_2, u_3\}| = 3$$

$$od_D(\text{v}) = |N(\text{v}) \cap (V - D)|$$

$$= |\{v_1, v_2, v_3, v_4, v_5\} \cap \{u_1, u_2, u_3, v_1, v_2, v_3, v_4, v_5\}|$$

$$= |\{v_1, v_2, v_3, v_4, v_5\}| = 5$$

Now

$$|od_D(\text{u}) - od_D(\text{v})| = 2 \leq 2.$$

Then D = $\{u, \text{v}\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph.

Thus D is total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(B_{3,5}) = 2$"

3.15. Theorem

For any helm graph H_n of order p, $\gamma_{t2oe}(H_n) = n$, where $p = 2n + 1$.

Proof.

Let $\{u_1, u_2, u_3 \ldots\ldots\ldots u_{n+1}, u_{n+2} \ldots\ldots\ldots\ldots u_{2n+1}\}$ be the vertices of helm graph. Here u_1 is the centre vertex $\{u_2, u_3 \ldots\ldots\ldots\ldots u_{n+1}\}$ be the vertices of cycle. $\{u_{n+2}, u_{n+3} \ldots\ldots\ldots u_{2n+1}\}$ be the pendant vertices and these vertices are adjacent to $u_2, u_3 \ldots\ldots, u_n$ respectively.

Take D = $\{u_2, u_3 \ldots\ldots\ldots\ldots\ldots u_{n+1}\}$ be a minimal dominating set and $V - D = \{u_1, u_{n+2}, u_{n+3} \ldots\ldots\ldots, u_{2n+1}\}$.

Now for any $u_i \in D$ then we get

$$od_D(u_i) = |N(u_i) \cap (V - D)|$$
$$= \{u_1, u_{i-1}, u_{i+1}, u_{n+i}\} \cap \{u_1, u_{n+2}, u_{n+3} \dots \dots, u_{2n+1}\}$$
$$= |\{u_1, u_{n-1}\}| = 2.$$

Then $|od_D(u_i) - od_D(u_j)| = 0 < 2$ and D is a $\gamma_{t2oe} - \ set$ and there are no vertices of degree zero in the induced subgraph <D>. Thus D is a $\gamma_{t2oe} - \ set$ and $\gamma_{t2oe}(H_n) = |D| = n.$

3.16. Example

For a helm graph H_5, as shown in Figure 9

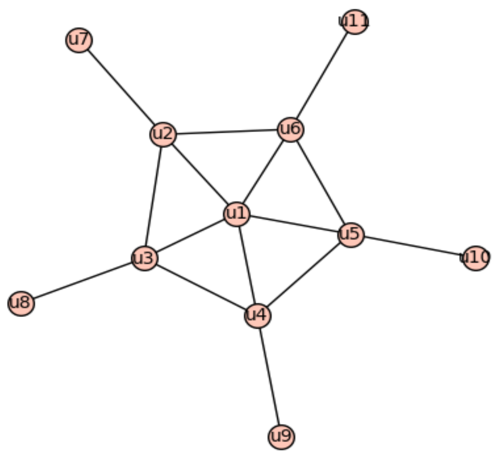

Figure 9. Helm graph H_5

"Let us consider D = $\{u_2, u_3, u_4, u_5, u_6\}$ and $V - D = \{u_1, u_7, u_8, u_9, u_{10}, u_{11}\}$

Now

$$od_D(u_2) = |N(u_2) \cap (V - D)|$$
$$= |\{u_1, u_7\} \cap \ \{u_1, u_7, u_8, u_9, u_{10}, u_{11}\}| = |\{u_1, u_7\}| = 2$$
$$od_D(u_3) = |N(u_3) \cap (V - D)|$$
$$= |\{u_1, u_8\} \cap \ \{u_1, u_7, u_8, u_9, u_{10}, u_{11}\}| = |\{u_1, u_8\}| = 2$$
$$od_D(u_4) = |N(u_4) \cap (V - D)|$$
$$= |\{u_1, u_9\} \cap \ \{u_1, u_7, u_8, u_9, u_{10}, u_{11}\}| = |\{u_1, u_9\}| = 2$$
$$od_D(u_5) = |N(u_5) \cap (V - D)|$$
$$= |\{u_1, u_{10}\} \cap \ \{u_1, u_7, u_8, u_9, u_{10}, u_{11}\}| = |\{u_1, u_{10}\}| = 2$$
$$od_D(u_6) = |N(u_6) \cap (V - D)|$$
$$= |\{u_1, u_{11}\} \cap \{u_1, u_7, u_8, u_9, u_{10}, u_{11}\}| = |\{u_1, u_{11}\}| = 2$$

Now

$$|od_D(u_i) - od_D(u_j)| = 0 \leq 2.$$

Then D = $\{u_2, u_3, u_4, u_5, u_6\}$ form 2 – ODED set of G and there are no isolated vertices of degree zero in the induced subgraph. Thus D is total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(H_5) = 2$"

3.17. Theorem

For any crown graph C_p^+ of order p, $\gamma_{t2oe}(C_p^+) = p.$

Proof.

Let

$$V(C_p^+) = \{w_1, w_2, \dots \dots \dots . w_p, w_{p+1}, w_{p+2} \dots \dots \dots, w_{2p}\}.$$

Here $\{w_1, w_2, w_3, \dots \dots, w_p\}$ be the vertices of cycle C_p and $\{w_{n+1}, w_{n+2}, \dots \dots, w_{2p}\}$ be the pendant vertices which are adjacent to $w_1, w_2, w_3, \dots\dots\dots w_p$ respectively.

Let D = $\{w_1, w_2, \dots \dots . . w_p\}$.

Clearly D is a minimal dominating set and $V - D = \{w_{p+1}, w_{p+2}, \dots, w_{2p}\}$.

Now $od_D(w_i) = |N(w_i) \cap (V - D)| = |\{w_{i-1}, w_{i+1}, \dots, w_{pi}\} \cap \{w_{p+1}, w_{p+2}, \dots, w_{2p}\}| = |\{w_{pi}\}|$=1.

Then $|od_D(w_i) - od_D(w_j)| = 0 < 2$ and D is a minimum 2 - ODED set and there are no vertices of degree zero in the induced subgraph <D>.

So D is a $\gamma_{t2oe} - \ set.$

Thus $\gamma_{t2oe}(C_p^+) = |D| = p.$

3.18. Example

For a crown graph C_5^+, as in Figure 10

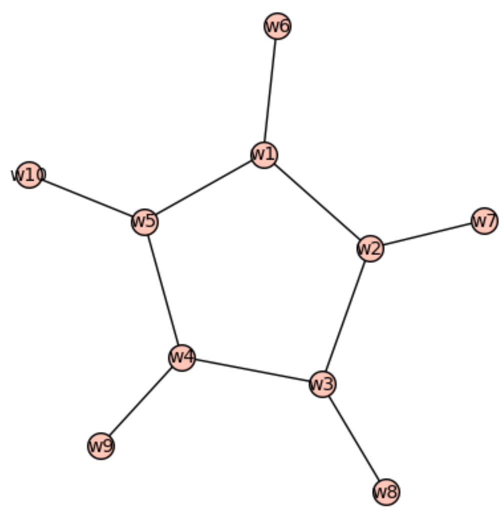

Figure 10. Crown graph C_5^+

"Let us consider D = $\{w_1, w_2, w_3, w_4, w_5\}$ and

$$V - D = \{w_6, w_7, w_8, w_9, w_{10}\}$$

Now

$$od_D(w_1) = |N(w_1) \cap (V - D)|$$
$$= |\{w_6\} \cap \{w_6, w_7, w_8, w_9, w_{10}\}| = |\{w_6\}| = 1$$
$$od_D(w_2) = |N(w_2) \cap (V - D)|$$

$= |\{w_7\} \cap \{w_6, w_7, w_8, w_9, w_{10}\}| = |\{w_7\}| = 1$

$$od_D(w_3) = |N(w_3) \cap (V - D)|$$
$$= |\{w_8\} \cap \{w_6, w_7, w_8, w_9, w_{10}\}| = |\{w_8\}| = 1$$
$$od_D(w_4) = |N(w_4) \cap (V - D)|$$
$$= |\{w_9\} \cap \{w_6, w_7, w_8, w_9, w_{10}\}| = |\{w_9\}| = 1$$
$$od_D(w_5) = |N(w_5) \cap (V - D)|$$
$$= |\{w_{10}\} \cap \{w_6, w_7, w_8, w_9, w_{10}\}| = |\{w_{10}\}| = 1$$

Now $|od_D(w_i) - od_D(w_j)| = 0 \leq 2.$

Then D = $\{w_1, w_2, w_3, w_4, w_5\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph. So D is total 2 - ODED set with minimum cardinality. Thus $\gamma_{t2oe}(C_5^+) = 5$”

3.19. Theorem

For any Fan graph,

$$\gamma_{t2oe}(F_{1,p-1}) = \begin{cases} 2 & if \;\; p = 2,3 \\ p - 2 & if \;\; p \geq 4 \end{cases}$$

Proof.

Let $V(F_{1,p-1})$ $=\{u, w_1, w_2, \dots \dots \dots \dots, w_{p-1}\}$

Case 1

For p = 2,3

Clearly $\gamma_{t2oe}(F_{1,1}) = 2$ for p = 2.

If p = 3, $V(F_{1,2})$ $=\{u, w_1, w_2\}$. Consider D $=\{u, w_1\}$ and $V - D$ $=\{w_2\}$

Then clearly $od_D(u) = 1$ and $od_D(w_1) = 1$.

Then $|od_D(u) - od_D(w_1)| = 0 < 2$ and <D> has no isolated vertices.

Hence D is a $\gamma_{t2oe} -$ set.

Then $\gamma_{\substack{t2oe\\t2oe}}(F_{1,2}) \leq 2$ and $2 \leq$ $\gamma_{t2oe}(F_{1,2})$

Hence $\gamma_{t2oe}(F_{1,2}) = 2$.

Case 2

For $p \geq 4$

Let $V(F_{1,p-1})$ $=\{u, w_1, w_2, \dots \dots \dots \dots, w_{p-1}\}$.

Let D = $\{u, w_1, w_2, \dots \dots ., w_{p-3}\}$ and $V - D$ $=\{w_{p-2}, w_{p-1}\}$.

Then

$$od_D(u) = |N(u) \cap (V - D)|$$
$$=|\{w_1, w_2, w_3, \dots \dots ., w_{p-2}, w_{p-1}\} \cap \{w_{p-2}, w_{p-1}\}|$$
$$= |\{w_{p-2}, w_{p-1}\}| = 2.$$

Now

$$od_D(w_1) = |N(w_1) \cap (V - D)|$$
$$= |\{u, w_2,\} \cap \{w_{p-2}, w_{p-1}\}| = \emptyset = 0.$$

Also

$$od_D(w_{p-3}) = |N(w_{p-3}) \cap (V - D)|$$
$$= |\{u, w_{p-4}, w_{p-2}\} \cap \{w_{p-2}, w_{p-1}\}| = 1.$$

For $2 \leq i \leq p - 4$, $od_D(w_i) = |N(w_i) \cap (V - D)| =$ $|\{u, w_{i-1}, w_{i+1}\} \cap \{w_{p-2}, w_{p-1}\}| = 0$

Then $|od_D(u) - od_D(w_i)| < 2$, for any u,v $\in D$.

Hence D is a $\gamma_{t2oe} -$ set and <D> has no isolated vertices.

Thus $\gamma_{t2oe}(F_{1,p-1}) = p - 2$.

3.20. Example

For a Fan graph $F_{1,5}$, as in Figure 11

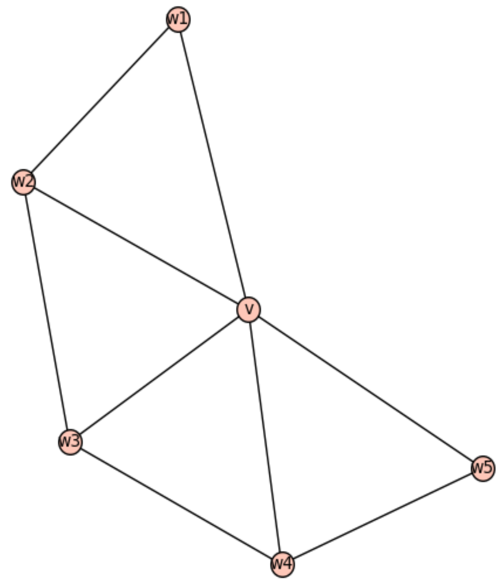

Figure 11. Fan graph $F_{1,5}$

“Let us consider D = $\{v, w_1, w_2, w_3\}$ *and*

$$V - D = \{w_4, w_5\}$$

Now

$$od_D(v) = |N(v) \cap (V - D)|$$
$$= |\{w_1, w_2, w_3, w_4, w_5\} \cap \{w_4, w_5\}| = |\{w_4, w_5\}| = 2$$
$$od_D(w_1) = |N(w_1) \cap (V - D)|$$
$$= |\{v, w_2\} \cap \{w_4, w_5\}| = |\emptyset| = 0$$
$$od_D(w_2) = |N(w_2) \cap (V - D)|$$
$$= |\{v, w_1, w_3\} \cap \{w_4, w_5\}| = |\emptyset| = 0$$
$$od_D(w_3) = |N(w_3) \cap (V - D)|$$
$$= |\{v, w_2, w_4\} \cap \{w_4, w_5\}| = |\{w_4\}| = 1$$

Now

$$|od_D(v) - od_D(w_3)| = 1 \leq 2$$
$$|od_D(w_2) - od_D(w_3)| = 1 \leq 2 \text{ and}$$
$$|od_D(v) - od_D(w_1)| = 2 \leq 2$$

Then D = $\{v, w_1, w_2, w_3\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph. So D is total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(F_{1,5}) = 5$”

3.21. Theorem

For any triangular snake graph $\gamma_{t2oe}(T_P^+) = p - 2$

Proof.

The graph G contains $2p-1$ vertices and $p-1$ triangles.

The upper vertices are labeled from w_1 to w_{p-1} and the lower vertices are labeled from w_p to w_{2p-1}.

Let $D = \{w_{p+1}, w_{p+2}, w_{p+3} \dots \dots .. w_{2p-3}, w_{2p-2}\}$ and $V - D = \{w_1, w_2, \dots \dots .. w_p, w_{2p-1}\}$.

Now w_{p+1} is adjacent to w_1, w_2, w_p and w_{p+2}.

Also w_{p+3} is adjacent to w_3, w_4, w_{p+2} and w_{p+4}.

The vertex w_{2p-2} is adjacent to $w_{p-2}, w_{p-1}, w_{2p+3}$ and w_{2p-1}.

Hence the set $\{w_{p+1}, w_{p+3}, \dots \dots w_{2p-2}\}$ is a minimum dominating set.

Therefore, consider all w_i is lying in the dominating set such that w_i is adjacent to w_j.

The set $D = \{w_{p+1}, w_{p+2}, w_{p+3} \dots \dots \dots w_{2p-3}, w_{2p-2}\}$ is a total dominating set.

Now

$$od_D(w_{p+1}) = |N(w_{p+1}) \cap (V - D)|$$

$$= |\{w_1, w_2, w_p\}| = 3$$

$$od_D(w_{2p-2}) = |N(w_{2p-2}) \cap (V - D)| =$$

$$|\{w_{p-2}, w_{p-1}, w_{2p-1}\}| = 3.$$

Now $od_D(w_i) = |N(w_i) \cap (V - D)|$

$$= |\{w_{j-1}, w_{j+1}\}| = 2$$

for $i = p+2, p+3, \dots, 2p-3$

Hence $|od_D(u) - od_D(w)| \leq 2 \; for \; all \; u, w \; \in D$ and therefore, D is a $\gamma_{t2oe} - \; set.$

Thus $\gamma_{t2oe}(T_P^+) = p - 2.$

3.22. Example

For a triangular snake graph T_6^+, as shown in Figure 12

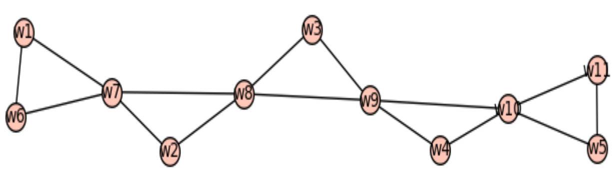

Figure 12. Triangular snake graph T_6^+

"Let us consider D = $\{w_7, w_8, w_9, w_{10}\}$ and $V - D = \{w_1, w_2, w_3, w_4, w_5, w_6, w_{11}\}$

Now

$$od_D(w_7) = |N(w_7) \cap (V - D)|$$

$$= |\{w_1, w_2, w_6, w_8\} \cap \{w_1, w_2, w_3, w_4, w_5, w_6, w_{11}\}|$$

$$= |\{w_1, w_2, w_6\}| = 3$$

$$od_D(w_8) = |N(w_8) \cap (V - D)|$$

$$= |\{w_2, w_3, w_7, w_9\} \cap \{w_1, w_2, w_3, w_4, w_5, w_6, w_{11}\}|$$

$$= \; |\{w_2, w_3\}| = 2$$

$$od_D(w_9) = |N(w_9) \cap (V - D)|$$

$$= \; |\{w_3, w_4, w_8, w_{10}\} \cap \{w_1, w_2, w_3, w_4, w_5, w_6, w_{11}\}|$$

$$= |\{w_3, w_4\}| = 2$$

$$od_D(w_{10}) = |N(w_{10}) \cap (V - D)|$$

$$= \; |\{w_4, w_5, w_9, w_{11}\} \cap \{w_1, w_2, w_3, w_4, w_5, w_6, w_{11}\}|$$

$$= |\{w_4, w_5, w_{11}\}| = 3$$

Now

$$|od_D(w_7) - od_D(w_8)| = 1 \leq 2$$

$$(w_7) - od_D(w_9)| = 1 \leq 2$$

$$|od_D(w_7) - od_D(w_{10})| = 0 \leq 2$$

$$|od_D(w_8) - od_D(w_9)| = \; 0 \leq 2$$

$|od_D(w_8) - od_D(w_{10})| = 1 \leq 2$ and

$$|od_D(w_9) - od_D(w_{10})| = 1 \leq 2$$

Then D = $\{w_7, w_8, w_9, w_{10}\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph. So D is a total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(T_6^+) = 4$".

3.23. Theorem

For any double triangular snake graph,

$$\gamma_{t2oe}(D(T_P^+)) = p - 2$$

Proof.

The graph G contains $3p-2$ vertices and $p-1$ triangles.

The upper vertices are labeled $\{w_1, w_2, \dots \dots .., w_{p-1}\}$ and the middle vertices are labeled $\{w_p, w_{p+1}, \dots ,$ $w_{2p-1}\}$ and the lower vertices are labeled $\{w_{2p},$ $w_{2p+1}, \dots \dots .. w_{3p-2}\}$.

Take the set

$D = \{w_{p+1}, w_{p+2}, w_{p+3} \dots \dots .. w_{2p-3}, w_{2p-2}\}$ and $V - D = \{w_1, w_2, \dots \dots .. w_p, w_{2p-1}\}$.

The vertex w_{p+1} is adjacent to $w_1, w_2, w_p, w_{p+2}, w_{2p}$, and w_{2p+1}.

The vertex w_{p+3} is adjacent to $w_3, w_4, w_{p+2}, w_{p+4}$ and w_{2p+3}.

In similar manner, the vertex w_{2p-2} is adjacent to $w_{p-2}, w_{p-1}, w_{2p+3}, w_{3p-3}$ and w_{3p-2}.

Hence the set $\{w_{p+1}, w_{p+3}, \dots \dots ., w_{2p-2}\}$ is a minimum dominating set but it is not a total dominating set.

Now consider all w_i is lying in the dominating set such that w_i is adjacent to w_j.

The set $D = \{w_{p+1}, w_{p+2}, w_{p+3}, \dots \dots .., w_{2p-3}, w_{2p-2}\}$ is a total dominating set.

Now

$$od_D(w_{p+1}) = |N(w_{p+1}) \cap (V-D)|$$
$$= |\{w_1, w_2, w_p, w_{2p}, w_{2p+1}\}| = 5.$$
$$od_D(w_{2p-2}) = |N(w_{2p-2}) \cap (V-D)|$$
$$= |\{w_{p-2}, w_{p-1}, w_{2p-1}, w_{3p-3}, w_{3p-2}\}| = 5.$$
$$od_D(w_i) = |N(w_i) \cap (V-D)|$$
$$= |\{w_{i-p}, w_{i-p+1}, w_{i+p}, w_{i+p-1}\}| = 4 \quad \text{for}$$
$$i = p+2, p+3, \ldots, 2p-3.$$

Hence $|od_D(u) - od_D(w)| \leq 2 \ for\ all\ u, w \ \in D.$
Hence D is a $\gamma_{t2oe} -\ set.$
Then $\gamma_{t2oe}(G) = p - 2.$

3.24. Example

For a double triangular snake graph $D(T_5^+)$, as shown in Figure 13

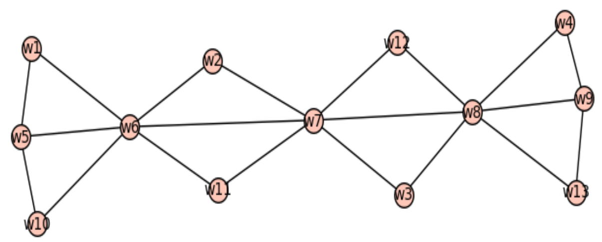

Figure 13. Double triangular snake graph $D(T_5^+)$

"Let us consider D = $\{w_6, w_7, w_8\}\ and$

$V - D = \{w_1, w_2, w_3, w_4, w_5, w_9, w_{10}, w_{11}, w_{12}, w_{13}\}$

Now

$$od_D(w_6) = |N(w_6) \cap (V-D)| =$$
$$|\{w_1, w_2, w_5, w_7, w_{10}, w_{11}\}$$
$$\cap \ \{w_1, w_2, w_3, w_4, w_5, w_9, w_{10}, w_{11}, w_{12}, w_{13}\}|$$
$$= \ |\{\{w_1, w_2, w_5, w_{10}, w_{11}\}\}| = 5$$
$$od_D(w_7) = |N(w_7) \cap (V-D)| =$$
$$|\{w_2, w_3, w_6, w_8, w_{11}, w_{12},\}$$
$$\cap \ \{w_1, w_2, w_3, w_4, w_5, w_9, w_{10}, w_{11}, w_{12}, w_{13}\}|$$
$$= \ |\{w_2, w_3, w_{11}, w_{12}\}| = 4$$
$$od_D(w_8) = |N(w_8) \cap (V-D)| =$$
$$|\{w_3, w_4, w_7, w_9, w_{12}, w_{13}\}$$
$$\cap \ \{w_1, w_2, w_3, w_4, w_5, w_9, w_{10}, w_{11}, w_{12}, w_{13}\}|$$
$$= |\{w_3, w_4, w_7, w_9, w_{12}, w_{13}\}| = 5$$

Now

$$|od_D(w_6) - od_D(w_7)| = 1 \leq 2.$$
$$(w_6) - od_D(w_8)| = 0 \ \ \leq 2$$
$$|od_D(w_7) - od_D(w_8)| = 1 \leq 2$$

Then D = $\{w_6, w_7, w_8\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph.

Hence D is a total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(D(T_5^+)) = 4$".

3.25. Theorem

For any path $\gamma_{t2oe}(P_2 \times P_p) = p$ for p $\geq$ 2.

Proof.

V($P_2 \times P_p$) = $\left\{\begin{matrix} (w_1, w_1), (w_1, w_2), (w_1, w_3), .., (w_1, w_p) \\ (w_2, w_1), (w_2, w_2), .., (w_2, w_p) \end{matrix}\right\}$

containing 2p vertices.

Consider a minimal dominating set

$$D = \{(w_1, w_1), (w_1, w_2), (w_1, w_3), \ldots \ldots . . (w_1, w_p)\}$$
$$V - D \ = \ \{(w_2, w_1), (w_2, w_2), (w_2, w_3), \ldots \ldots . . (w_2, w_p)\}.$$

Since <D> is a path, <D> has no isolated vertices.
Let u = $(w_1, w_j) \in$ D, j = 1,2,3, ……, p
If u =(w_1, w_1), then

$$od_D(w_1, w_1) = |N(w_1, w_1) \cap (V-D)|$$
$$= |(w_2, w_1), (w_1, w_2) \cap (V-D)| = |(w_2, w_1)| = 1.$$

If u =(w_1, w_p),then

$$od_D(w_1, w_p) = |N(w_1, w_p) \cap (V-D)|$$
$$= |(w_1, w_{p-1}), (w_2, w_p) \cap (V-D)|$$
$$= |(w_2, w_p)| = 1$$

If u =(w_1, w_j), j = 2,3…$p-1$
Then $od_D(w_1, w_j)$= $|(w_2, w_j)|$=1.
Then for any $u, w \in D$, $|od_D(u) - od_D(w)| \leq 2.$
Hence D is a $\gamma_{t2oe} -\ set.$
Thus $\gamma_{t2oe}(P_2 \times P_p) = p$ for $p \ \geq \ 2.$

3.26. Example

For any path $(P_2 \times P_6)$, as shown in Figure 14.

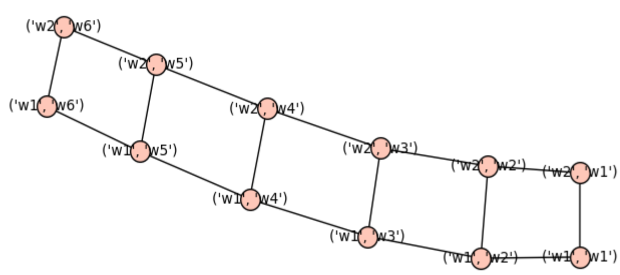

Figure 14. Path $(P_2 \times P_6)$

"Let us consider

D=$\left\{\begin{matrix} (w_1, w_1), (w_1, w_2), (w_1, w_3), \\ (w_1, w_4), (w_1, w_5), (w_1, w_6) \end{matrix}\right\} and$

$$V - D = \begin{Bmatrix} (w_2, w_1), (w_2, w_2), (w_2, w_3), \\ (w_2, w_4), (w_2, w_5), (w_2, w_6) \end{Bmatrix}$$

Now

$$od_D\big((w_1, w_1)\big) = \big|N\big((w_1, w_1)\big) \cap (V - D)\big|$$

$$= \left|\{(w_1, w_2)(w_2, w_1)\} \cap \begin{Bmatrix} (w_2, w_1), (w_2, w_2), (w_2, w_3), \\ (w_2, w_4), (w_2, w_5), (w_2, w_6) \end{Bmatrix}\right|$$

$$= |\{(w_2, w_1)\}| = 1$$

Now

$$od_D\big((w_1, w_2)\big) = |N((w_1, w_2)) \cap (V - D)|$$

$$= \left|\{(w_1, w_1), (w_1, w_2)(w_2, w_2)\} \cap \begin{Bmatrix} ((w_2, w_1), (w_2, w_2), (w_2, w_3), \\ (w_2, w_4), (w_2, w_5), (w_2, w_6) \end{Bmatrix}\right|$$

$$= |\{(w_2, w_2)\}| = 1$$

Now

$$od_D\big((w_1, w_3)\big) = |N((w_1, w_3)) \cap (V - D)| = \left|\{(w_1, w_2), (w_1, w_4)(w_2, w_3)\} \cap \begin{Bmatrix} (w_2, w_1), (w_2, w_2), (w_2, w_3), \\ (w_2, w_4), (w_2, w_5), (w_2, w_6) \end{Bmatrix}\right|$$

$$= |\{(w_2, w_3)\}| = 1$$

Now

$$od_D\big((w_1, w_4)\big) = |N((w_1, w_4)) \cap (V - D)| = \left|\{(w_1, w_3), (w_1, w_5)(w_2, w_4)\} \cap \begin{Bmatrix} (w_2, w_1), (w_2, w_2), (w_2, w_3), \\ (w_2, w_4), (w_2, w_5), (w_2, w_6) \end{Bmatrix}\right|$$

$$= |\{(w_2, w_4)\}| = 1$$

Now

$$od_D\big((w_1, w_5)\big) = |N((w_1, w_5)) \cap (V - D)| = \left|\{(w_1, w_4), (w_1, w_6)(w_2, w_5)\} \cap \begin{Bmatrix} (w_2, w_1), (w_2, w_2), (w_2, w_3), \\ (w_2, w_4), (w_2, w_5), (w_2, w_6) \end{Bmatrix}\right|$$

$$= |\{(w_2, w_5)\}| = 1$$

Now

$$od_D\big((w_1, w_6)\big) = |N((w_1, w_6)) \cap (V - D)| = \left|\{(w_1, w_5), (w_2, w_6)\} \cap \begin{Bmatrix} (w_2, w_1), (w_2, w_2), (w_2, w_3), \\ (w_2, w_4), (w_2, w_5), (w_2, w_6) \end{Bmatrix}\right|$$

$$= |\{(w_2, w_6)\}| = 1$$

Now

$$|od_D((w_1, w_i) - od_D((w_1, w_j))| = 0 \leq 2.$$

Then

D = $\{(w_1, w_1), (w_1, w_2), (w_1, w_3), (w_1, w_4), (w_1, w_5), (w_1, w_6)\}$ form 2 – ODED set of G and there are no isolated vertices in the induced subgraph. Hence D is a total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(P_2 \times P_6) = 6$".

3.27. Corollary

For any path $\gamma_{t2oe}(P_3 \times P_p)$ = p for p $\geq$ 3.

3.28. Theorem

For any combo graph P_n^+ of order p, $\gamma_{t2oe}(P_n^+) = n.$

Proof.

Let $\{w_1, w_2, \dots\dots, w_n, w_{n+1}, w_{n+2}, \dots\dots, w_{2n}\}$ be the vertices of combo graph of P_n^+.

Here $\{w_1, w_2, w_3, \dots\dots, w_n\}$ be the vertices of path P_n *and* $\{w_{n+1}, w_{n+2}, \dots\dots, w_n\}$ be the pendant vertices which are adjacent to $w_1, w_2, w_3,$ ……, w_n respectively.

Consider a minimal dominating set D= $\{w_1, w_2, \dots\dots, w_n\}$ and $V - D = \{w_{n+1}, w_{n+2}, \dots, w_{2n}\}$.

Now we have

$$od_D(w_i) = |N(w_i) \cap (V - D)|$$

$$= |\{w_{i-1}, w_{i+1}, w_{ni}\} \cap \{w_{n+1}, w_{n+2}, \dots, w_{2n}\}| = |\{w_{ni}\}| = 1.$$

Then $|od_D(w_i) - od_D(w_j)| = 0 < 2$ and D is a minimum 2 - ODED set. Clearly the induced subgraph $< D >$ has no isolated vertices.

Hence D is a $\gamma_{t2oe} - set$

Hence $\gamma_{t2oe}(P_n^+) = |D| = n.$

3.29. Example

For combo graph P_6^+, as shown in Figure 15.

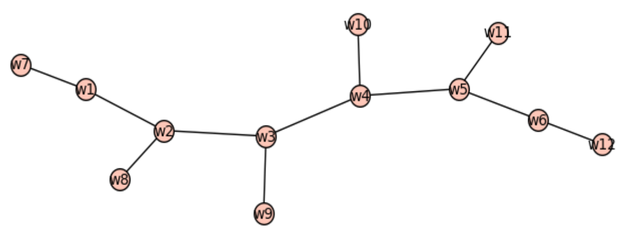

Figure 15. Combo graph P_6^+

"Let us consider D = $\{w_1, w_2, w_3, w_4, w_5, w_6\}$ *and* $V - D = \{w_7, w_8, w_9, w_{10}, w_{11}, w_{12}\}$

Now

$$od_D(w_1) = |N(w_1) \cap (V - D)|$$

$$= |\{w_2, w_7\} \cap \{w_7, w_8, w_9, w_{10}, w_{11}, w_{12}\}|$$

$$= |\{w_7\}| = 1$$

$$od_D(w_2) = |N(w_2) \cap (V - D)|$$

$$= |\{w_1, w_3, w_8\} \cap \{w_7, w_8, w_9, w_{10}, w_{11}, w_{12}\}|$$

$$= |\{w_8\}| = 1$$

$$od_D(w_3) = |N(w_3) \cap (V - D)|$$
$$=|\{w_2, w_4, w_9\} \cap \{w_7, w_8, w_9, w_{10}, w_{11}, w_{12}\}|$$
$$= |\{w_9\}| = 1$$
$$od_D(w_4) = |N(w_4) \cap (V - D)|$$
$$=|\{w_3, w_5, w_{10}\} \cap \{w_7, w_8, w_9, w_{10}, w_{11}, w_{12}\}|$$
$$= |\{w_{10}\}| = 1$$
$$od_D(w_5) = |N(w_5) \cap (V - D)|$$
$$= |\{w_4, w_6, w_{11}\} \cap \{w_7, w_8, w_9, w_{10}, w_{11}, w_{12}\}|$$
$$= |\{w_{11}\}| = 1$$
$$od_D(w_6) = |N(w_6) \cap (V - D)|$$
$$=|\{w_5, w_{12}\} \cap \{w_7, w_8, w_9, w_{10}, w_{11}, w_{12}\}|$$
$$= |\{w_{12}\}|=1$$

Now

$$|od_D(w_i) - od_D(w_j)| = 0 \leq 2 \quad \text{for all D}$$

Then D = $\{w_6, w_7, w_8\}$ form 2 – ODED set of G and there are no vertices of degree zero in the induced subgraph. So D is a total 2 - ODED set with minimum cardinality.

Thus $\gamma_{t2oe}(D(P_6^+)) = 6$".

3.30. Theorem

For all combo graph P_n^+, $\gamma_{t2oe}(T_2(P_n^+) = n$.

Proof.

$V\big((P_n^+)\big) = \{v_1, v_2 \ldots.., v_n, v_1', v_2', \ldots v_n'\}$ and $E\big((P_n^+)\big) = \{e_1, e_2 \ldots.., e_{2n-1}\}$.

Then $V\big(T_2(P_n^+)\big) = \{v_1, v_2 \ldots.., v_n, v_1', v_2', \ldots v_n', e_1, e_2 \ldots.., e_{2n-1}\}$ is vertices of $T_2(P_3^+)$.

Let us take $D = \{v_1, v_2 \ldots.., v_n\}$ and $V - D = \{v_1', v_2', \ldots v_n', e_1, e_2 \ldots.., e_{2n-1}\}$

Clearly D is a dominating set.

Also $od_D(v_1) = 3 = od_D(v_n)$ and $od_D(v_i) = 4$

Where i=2,3,4….., $n - 1$.

For any $u, v \in D, such\ that\ |od_D(u) - od_D(v)| \leq 2$.

Hence D is a 2- out degree equitable dominating set and <D> has no isolated vertices.

Thus $\gamma_{t2oe}(T_2(P_n^+) = n$.

3.31. Theorem

For all triangular snake graph,

$$\gamma_{t2oe}(T_2(T_n^+) = 2n - 1.$$

Proof.

Let $V\big((T_n^+)\big) = \{v_1, v_2 \ldots, v_n, v_{n+1}, v_{n+2} \ldots., v_{2n-1}\}$ and $E\big((T_n^+)\big) = \{e_1, e_2 \ldots.., e_{2n+1}\}$

Then $V\big(T_2(T_n^+)\big) = \{v_1, v_2 \ldots, v_n, v_{n+1}, v_{n+2} \ldots, v_{2n-1}, e_1, e_2 \ldots, e_{2n+1}\}$ is vertices of $T_2(T_n^+)$.

Let us take $D = \{v_1, v_2 \ldots, v_n, v_{n+1}, v_{n+2} \ldots, v_{2n-1}\}$ and $V - D = \{e_1, e_2 \ldots, e_{2n+1}\}$

Clearly D is a dominating set.

Also $od_D(v_i) = 2$ for $i = 1, n, n + 1 \ldots..2n - 1$ and $od_D(v_j) = 4$ for $j = 2,3, \ldots..., n - 1$.

For any $\in D, such\ that\ |od_D(u) - od_D(v)| \leq 2$.

Hence D is a 2 - out degree equitable dominating set and <D> has no isolated vertices.

Thus $\gamma_{t2oe}(T_2(T_n^+)) = 2n - 1$.

4. Application of Total 2 – ODED Number

The concept of a total 2 – ODED set is useful for the formation of any committee. It is desirable that each committee member might feel comfortable knowing at least one member of the committee. In this situation, a total 2 – out degree equitable domination is useful while there is no difference of opinion between any two members or they differ on at most one issue. In this situation, the concept of equitable domination is applicable.

5. Conclusions

In this paper, we introduce a new domination number called the total 2 – ODED number. Also, we investigate the proposed domination number for some general graphs. Finally, we discuss the real life application of the proposed domination number. We would like to extend our research work to include an additional set of graphs, as well as investigate the limitations of the total 2 - out degree equitable domination number.

REFERENCES

[1] Harary Frank, "Graph Theory", Addison-Wesley Publishing Company, pp. 1-274, 1969.

[2] Oystein Ore, "Theory of graphs", Colloquium Publications, American Mathematical Society, vol. 38, pp. 1–270, 1962.

[3] Claude Berge, "The theory of graphs", Courier Corporation, pp. 1 – 247, 2001.

[4] Ali Sahal, Veena Mathad. "Two-out degree equitable Domination in graphs", Transactions on Combinatorics, vol. 2, no. 3, pp. 13-19, 2013.

[5] Mahesh M.S., P. Namasivayam, "Connected two out degree equitable domination number in different graphs", International Journal of Mathematical Archive, vol. 5, no. 1, pp. 67-74, 2014.

[6] Mahesh, M. S., P. Namasivayam, "Triple connected two out degree equitable domination number", Journal of Discrete Mathematical Sciences and Cryptography, vol. 20, no. 3, pp. 653-661, 2017.

[7] K. Mathevan Pillai, M. S. Mahesh, G. Selvam, "Non spit two out dcgree equitable domination number in graphs",

Journal of Chemical and Pharmaceutical Sciences, vol. 9, no. 4, pp. 2266-2270, 2016.

[8] K. Mathevan Pillai, M.S. Mahesh, A. Santiago Stephan, G. Selvam, "Some more results on Non split two out degree equitable domination number", International Journal of Mechanical and Production Engineering Research and Development, vol. 8, no. 2, pp. 361-369, 2018.

[9] Cockayne, Ernest J., R. M. Dawes, Stephen T. Hedetniemi., "Total domination in graphs", Networks, vol. 10, no. 3, pp. 211-219, 1980. DOI: 10.1002/net.3230100304.