

Neutrosophic soft set matrix and their applications based on multi criteria decision-making problems

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Abstract The significance of addressing decision-making challenges in uncertain environments cannot be overstated. Recently developed advanced mathematical techniques, such as soft set theory and neutrosophic soft set theory, aim to enhance the handling of data sets that are uncertain, incomplete, and inconsistent. This paper introduces the concepts of neutrosophic soft set, neutrosophic soft matrix, and their operations. Additionally, comparison measures, including score, certainty, and accuracy functions, are provided for evaluating neutrosophic soft sets. Subsequently, novel operators, namely the neutrosophic arithmetic operator (NA), neutrosophic weighted arithmetic operator (NA_w), neutrosophic geometric operator (NG), neutrosophic weighted geometric operator (NG_w), neutrosophic harmonic operator (NH), and neutrosophic weighted harmonic operator (NH_w), are introduced to aggregate neutrosophic information. Building on these operators, the paper develops a neutrosophic multi-criteria decision-making framework using the Laplace criterion, optimism criterion, and savage criterion. Finally, the effectiveness of the proposed technique is demonstrated through a numerical example, showcasing its practical application

Keywords: soft set, neutrosophic soft set matrix, Laplace criterion, optimism criterion, Savage criterion, multi-criteria decision-making

1. Introduction

In various real-world domains such as economics, engineering, agriculture, the environment, social sciences, medical sciences, and business management, uncertainty is a prevalent issue. The complexity and challenges associated with traditional mathematical modelling contribute to uncertain data in different disciplines. Managing real-life problems requires addressing uncertain and partial information in ambiguous and uncertain environments. Despite this, fuzzy sets and intuitionistic fuzzy sets fall short in handling contradictory and uncertain data. To address uncertainties, imprecision, incompleteness, and determinacy, researchers have identified a new mathematical tool. Smarandache (2005) introduced the concept of a neutrosophic set as a mathematical tool to tackle problems involving imprecise and indeterminate data. Molodtsov (1997) proposed the concept of soft sets to handle decision-making problems in an indistinct environment. Maji (2013) introduced various operators for soft set theory, but Ali et al. (2009) highlighted gaps in some of these definitions and their features. Çağman and Enginoğlu (2010) made modifications to the operations of soft sets to address these gaps. Çağman (2014) redefined soft sets using a single parameter set. Maji combined the concepts of soft set and neutrosophic set to create the innovative notion of a neutrosophic soft set, which he applied to a decision-making issue. Çağman (2010) introduced fuzzy soft matrices, a crucial idea in decision-making problems. Broumi et al. (2014) established numerous relations on interval-valued neutrosophic soft sets, applicable in various decision-making contexts. In 2014, Irfan and Broumi described neutrosophic soft matrices and used them in decision-making. Subsequently, Loganathan and Pushpalatha (2018) proposed an application using fuzzy matrices. Mary et al. (2021) applied neutrosophic soft sets in agriculture for decision-making. This paper focuses on solving multi-criteria decision-making problems using neutrosophic soft set matrices. We utilise three criteria: the Laplace criterion, the optimism criterion, and the Savage criterion for decision-making problems.

2. Materials and methods

2.1. Definition (Karaaslan, 2015) (Soft set)

Let U be a starting universe, and E is a set of conditions. Let $P(U)$ stand for the power set U . Then if $A \subseteq E$, a pair (F, A) is called a soft set over U , where $F: A \rightarrow P(U)$.

2.1.1. Example

Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the universe which are six carpets and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be the set of parameters. Here, e_i ($i = 1, 2, 3, 4, 5, 6$) stands for the Parameters "modern", "woollen", "expensive", "cheap", "large" and "beautiful". Then, the consecutive soft sets are described in sequence.

$$f_A = \{(e_1, \{x_1, x_4, x_5\}), (e_2, \{x_1, x_3, x_5, x_6\}), (e_4, \{x_2, x_4, x_6\})\}$$

$$f_B = \{(e_2, \{x_1, x_3, x_5\}), (e_3, X), (e_6, \{x_2, x_4, x_5, x_6\})\}$$

2.2. Definition (Neutrosophic set)

Let X be a space of points (objects), with a generic element in X denoted by x . A neutrosophic set A on the universe of discourse X is defined as

$$A = \{x, T_A(x), I_A(X), F_A(X) / x \in X\}$$

Where $T_A(x), I_A(X), F_A(X): X \rightarrow (0,1)$ and $-0 \leq T_A(X) + I_A(X) + F_A(X) \leq 3$. From the neutrosophic set takes real standard or non-standard subset value of $-]0,1[+ .T_A$ (Truth), I_A (Indeterminacy), F_A (falsity) referred to as neutrosophic components.

2.2.1. Example

Let us assume that $X = \{m_1, m_2, m_3\}$ be the attributes of the merchandise whereby m_1 the product's quality, m_2 is the product's price and m_3 is the product's dependability. Furthermore m_1, m_2, m_3 are in $[0,1]$ and they originate from surveys used by certain scholars. To characterize the characteristics of the product, researchers have proposed three components: the degree of goodness, the degree of indeterminacy, and the degree of poorness. Assume A is a neutrosophic set of X such that, $A = \{(m_1, 0.8, 0.6, 0.4), (m_2, 0.6, 0.3, 0.2), (m_3, 0.7, 0.6, 0.3)\}$, The quality score is 0.8 for goodness of quality, 0.6 for indeterminacy of quality, 0.4 for falsity of quality, etc.

2.3. Definition (Tuhin and Nirmal 2017) (Neutrosophic soft set)

Let U be an initial universe set and E be a set of parameters. Let $N(U)$ denote the set of all neutrosophic soft set of U . Then for $A \subseteq E$, Then the collection of (F, A) is called a **Neutrosophic soft set** (N_{ss}) over U , where $F: A \rightarrow N(U)$.

2.3.1. Example

Let U be an universal set $U = \{m_1, m_2, m_3, m_4, m_5\}$ be five different oil makers and $E = \{h_1, h_2, h_3, h_4\}$ represent the many types of oils, such as coconut oil, sesame oil, sunflower oil, and groundnut oil, using a set of metrics. The expert E_1 is currently assessing which product yields a higher profit,

$$f_N(h_1) = \left\{ \begin{array}{l} \langle m_1, (0.6, 0.4, 0.3) \rangle, \langle m_2, (0.9, 0.5, 0.3) \rangle, \langle m_3, (0.5, 0.6, 0.2) \rangle, \\ \langle m_4, (0.7, 0.5, 0.4) \rangle, \langle m_5, (0.5, 0.4, 0.1) \rangle \end{array} \right\}$$

$$f_N(h_2) = \left\{ \begin{array}{l} \langle m_1, (0.8, 0.6, 0.7) \rangle, \langle m_2, (0.6, 0.4, 0.4) \rangle, \langle m_3, (0.2, 0.1, 0.1) \rangle, \\ \langle m_4, (0.6, 0.5, 0.2) \rangle, \langle m_5, (0.3, 0.2, 0.2) \rangle \end{array} \right\}$$

$$f_N(h_3) = \left\{ \begin{array}{l} \langle m_1, (0.3, 0.5, 0.2) \rangle, \langle m_2, (0.6, 0.4, 0.8) \rangle, \langle m_3, (0.8, 0.5, 0.6) \rangle, \\ \langle m_4, (0.6, 0.5, 0.1) \rangle, \langle m_5, (0.5, 0.5, 0.1) \rangle \end{array} \right\}$$

$$f_N(h_4) = \left\{ \begin{array}{l} \langle m_1, (0.5, 0.5, 0.2) \rangle, \langle m_2, (0.6, 0.3, 0.7) \rangle, \langle m_3, (0.9, 0.5, 0.3) \rangle, \\ \langle m_4, (0.7, 0.5, 0.4) \rangle, \langle m_5, (0.8, 0.5, 0.1) \rangle \end{array} \right\}$$

Then $N = \{[h_1, f_N(h_1)], [h_2, f_N(h_2)], [h_3, f_N(h_3)], [h_4, f_N(h_4)], \}$ is an N_{ss} over (U, E) .

The tabular illustrate of the N_{ss} of N is shown

Table 1 Tabular form of N_{ss} .

	$f_N(h_1)f_N(h_2)f_N(h_3)f_N(h_4)$
M_1	(0.6,0.4,0.3)(0.8,0.6,0.7)(0.3,0.5,0.2)(0.5,0.5,0.2)(0.9,0.5,0.3)(0.6,0.4,0.4)(0.6,0.4,0.8)(0.6,0.3,0.7)
M_2	(0.5,0.6,0.2)(0.2,0.1,0.1)(0.8,0.5,0.6)(0.9,0.5,0.3)
M_3	(0.7,0.5,0.4)(0.6,0.5,0.2)(0.6,0.5,0.1)(0.7,0.5,0.4)
M_4	(0.5,0.4,0.1)(0.3,0.2,0.2)(0.5,0.5,0.1)(0.8,0.5,0.1)
M_5	

2.4. Definition (Tanushree and Shyamal 2015) (Neutrosophic soft set matrix(N_{ssm}))

If $x_{ij} = (T_A(u_i, e_j), I_A(u_i, e_j), F_A(u_i, e_j))$, then Neutrosophic Soft Set Matrix

(N_{ssm}) of order $m \times n$, is $(x_{ij})_{m \times n} = (x_{ij})_m$

2.4.1. Example

Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$. N be a neutrosophic soft sets over U neutrosophic as

$$N = \left\{ \begin{array}{l} (x_1, \{ \langle u_1, (0.7, 0.5, 0.3) \rangle, \langle u_2, (0.6, 0.5, 0.2) \rangle, \langle u_3, (0.5, 0.4, 0.2) \rangle \}), \\ (x_2, \{ \langle u_1, (0.8, 0.6, 0.2) \rangle, \langle u_2, (0.9, 0.5, 0.3) \rangle, \langle u_3, (0.7, 0.5, 0.4) \rangle \}), \\ (x_3, \{ \langle u_1, (0.2, 0.2, 0.1) \rangle, \langle u_2, (0.6, 0.6, 0.4) \rangle, \langle u_3, (0.5, 0.6, 0.8) \rangle \}) \end{array} \right\}$$

Then, the N –matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} (0.7, 0.5, 0.3) & (0.6, 0.5, 0.2) & (0.5, 0.4, 0.2) \\ (0.8, 0.6, 0.2) & (0.9, 0.5, 0.3) & (0.7, 0.5, 0.4) \\ (0.2, 0.2, 0.1) & (0.6, 0.6, 0.4) & (0.5, 0.6, 0.8) \end{bmatrix}$$

2.5. Definition (Linguistic variable and values)

What distinguishes a linguistic variable is $(x, T(x), U, M)$ where x is the variable's name. $T(x)$ is the term set of x , The fuzzy variables defined in are the set of names or linguistic values assigned to each x value. U, M s a semantic rule that applies to every variate component. U is a universe of discourse.

For example. $X = "Age"$ is characterized as a language variable. $T(Age) = \{Young, Not Young, Very Young, More or less old, old\}$. $U = \{0, 100\}$, M = defined each fuzzy variable's membership function, for instance, $M(Young)$ = the fuzzy set for those participating in Young and under 26 years old.

Linguistic values and variables

Variables	Values
Low	0.1 – 0.3
Medium	0.4 – 0.6
High	0.7 – 1.0

2.6. Definition (Value Matrix)

Suppose $C = [T_{ij}^c, I_{ij}^c, F_{ij}^c]$ Nssm of order $m \times n$ then, C is called the value matrix of N denoted by $V(C)$ and $V(C) = [T_{ij}^c + 4I_{ij}^c + F_{ij}^c] / 6$ for all i and j , respectively where $i = 1, 2, 3 \dots m$ and $j = 1, 2, 3 \dots n$

3. Operator's on neutrosophic soft matrices

If $A = [(T_{ij}^A, I_{ij}^A, F_{ij}^A)](N_{ssm})$ of order $m \times n$.

$B = [(T_{ij}^B, I_{ij}^B, F_{ij}^B)](N_{ssm})$ of order $m \times n$.

Where, $W_1, W_2 = 0.6, 0.4$ respectively,

Then the defuzzification value of:

a) Neutrosophic Arithmetic Operator (NA): $(NA) = [T_{ij}^c, I_{ij}^c, F_{ij}^c]$ where, $T_{ij}^c = \frac{(T_{ij}^A + T_{ij}^B)}{2}$, $I_{ij}^c = \frac{(I_{ij}^A + I_{ij}^B)}{2}$ and $F_{ij}^c = \frac{(F_{ij}^A + F_{ij}^B)}{2}$

b) Neutrosophic Weighted Arithmetic Operator (NA_W): $(NA_W) = [T_{ij}^c, I_{ij}^c, F_{ij}^c]$ where, $T_{ij}^c = \frac{(w_1 T_{ij}^A + w_2 T_{ij}^B)}{(w_1 + w_2)}$, $I_{ij}^c = \frac{(w_1 I_{ij}^A + w_2 I_{ij}^B)}{(w_1 + w_2)}$ and $F_{ij}^c = \frac{(w_1 F_{ij}^A + w_2 F_{ij}^B)}{(w_1 + w_2)}$.

c). Neutrosophic Geometric Operator (NG): $(NG) = [T_{ij}^c, I_{ij}^c, F_{ij}^c]$ Where, $T_{ij}^c = \sqrt{T_{ij}^A * T_{ij}^B}$, $I_{ij}^c = \sqrt{I_{ij}^A * I_{ij}^B}$, and $F_{ij}^c = \sqrt{F_{ij}^A * F_{ij}^B}$.

d) Neutrosophic Weighed Geometric Operator (NG_W)

$$(NG_W) = [T_{ij}^c, I_{ij}^c, F_{ij}^c], \text{ Where } T_{ij}^c = ((w_1 + w_2)) * \left\{ \sqrt{(w_1 T_{ij}^A) * (w_2 T_{ij}^B)} \right\}, I_{ij}^c = ((w_1 + w_2)) * \left\{ \sqrt{(w_1 I_{ij}^A) * (w_2 I_{ij}^B)} \right\}, \text{ and } F_{ij}^c = ((w_1 + w_2)) * \left\{ \sqrt{(w_1 F_{ij}^A) * (w_2 F_{ij}^B)} \right\}.$$

e) Neutrosophic Harmonic Operator (NH) : $(NH) = [T_{ij}^c, I_{ij}^c, F_{ij}^c]$ where,

$$T_{ij}^c = \frac{(2 * T_{ij}^A * T_{ij}^B)}{(T_{ij}^A + T_{ij}^B)}, I_{ij}^c = \frac{(2 * I_{ij}^A * I_{ij}^B)}{(I_{ij}^A + I_{ij}^B)} \text{ and } F_{ij}^c = \frac{(2 * F_{ij}^A * F_{ij}^B)}{(F_{ij}^A + F_{ij}^B)}$$

f). Neutrosophic Weighed Harmonic Operator (NH_W): $(NH_W) = [T_{ij}^c, I_{ij}^c, F_{ij}^c]$ where $T_{ij}^c = \frac{(w_1 + w_2)}{\left(\frac{w_1}{T_{ij}^A} \right) + \left(\frac{w_2}{T_{ij}^B} \right)}, I_{ij}^c =$

$$\frac{(w_1 + w_2)}{\left(\frac{w_1}{I_{ij}^A} \right) + \left(\frac{w_2}{I_{ij}^B} \right)} \text{ and } F_{ij}^c = \frac{(w_1 + w_2)}{\left(\frac{w_1}{F_{ij}^A} \right) + \left(\frac{w_2}{F_{ij}^B} \right)}$$

4. Methodology

- Step 1: Construct neutrosophic soft set matrices (A&B)
- Step 2: Compute C by using the operators of neutrosophic soft set matrix
- Step 3: Calculate the value matrices $V(C)$
- Step 4: Obtain the decision by using Laplace criterion, optimism criterion and savage criterion.

5. Numerical example

Let F be a set of soft neutrosophic poultry that produce a variety of domestic birds to meet people's year-round needs for meat and eggs. When the goods are prepared, the vendor will either benefit or lose money depending on whether they sell them on the market or straight to customers. Assume for the moment that a group of vendors is offering the goods in two distinct markets. M_1 and M_2 . E is a set of parameters. let $A \subseteq E$ and $B \subseteq E$.

The first $N_{ss}(X, A)$ over F is the seller production of meat and egg sold in market M_1 , where $X: A \rightarrow P(F)$. The second $N_{ss}(Y, B)$ over F is the seller production of meat and egg sold in market M_2 , where $Y: A \rightarrow P(F)$, $P(F)$ is a set of all neutrosophic soft subset of F . Let a universal set $U = \{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8\}$ be an eight different poultry forms and $E = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8\}$ depict the various domestic poultry, such as chickens, turkeys, geese, ducks, guinea fowl, quile, pigeons, and ostriches, using parameters. Two outbound P_1 and P_2 evaluated the production of the seller in the market M_1 and M_2 , respectively.

Report of expert P_1 in the market place M_1 is representing in the form of (N_{ss}) as

$$(X, A) = (x(C_1), x(C_2), x(C_3), x(C_4), x(C_5), x(C_6), x(C_7), x(C_8))$$

$$X(C_1) = \left\{ \begin{array}{l} f_1(0.6, 0.3, 0.2), f_2(0.2, 0.2, 0.1), f_3(0.6, 0.4, 0.3), f_4(0.4, 0.3, 0.2), f_5(0.5, 0.5, 0.1), \\ f_6(0.6, 0.2, 0.2), f_7(0.7, 0.3, 0.1), f_8(0.8, 0.8, 0.6) \end{array} \right\}$$

$$X(C_2) = \left\{ \begin{array}{l} f_1(0.4, 0.2, 0.2), f_2(0.6, 0.5, 0.5), f_3(0.9, 0.6, 0.3), f_4(0.4, 0.1, 0.1), f_5(0.7, 0.3, 0.2), \\ f_6(0.5, 0.3, 0.3), f_7(0.2, 0.1, 0.1), f_8(0.7, 0.6, 0.6) \end{array} \right\}$$

$$X(C_3) = \left\{ \begin{array}{l} f_1(0.3, 0.3, 0.2), f_2(0.9, 0.6, 0.3), f_3(0.8, 0.7, 0.6), f_4(0.6, 0.4, 0.4), f_5(0.7, 0.7, 0.3), \\ f_6(0.3, 0.3, 0.1), f_7(0.6, 0.5, 0.3), f_8(0.7, 0.5, 0.2) \end{array} \right\}$$

$$X(C_4) = \left\{ \begin{array}{l} f_1(0.6, 0.5, 0.2), f_2(0.7, 0.3, 0.2), f_3(0.7, 0.6, 0.6), f_4(0.4, 0.3, 0.2), f_5(0.4, 0.4, 0.1), \\ f_6(0.6, 0.5, 0.3), f_7(0.7, 0.7, 0.6), f_8(0.2, 0.2, 0.1) \end{array} \right\}$$

$$X(C_5) = \left\{ \begin{array}{l} f_1(0.4, 0.4, 0.3), f_2(0.6, 0.2, 0.1), f_3(0.8, 0.8, 0.6), f_4(0.6, 0.3, 0.1), f_5(0.7, 0.5, 0.5), \\ f_6(0.3, 0.2, 0.1), f_7(0.6, 0.4, 0.4), f_8(0.3, 0.2, 0.2) \end{array} \right\}$$

$$X(C_6) = \left\{ \begin{array}{l} f_1(0.7, 0.3, 0.2), f_2(0.4, 0.3, 0.3), f_3(0.6, 0.5, 0.5), f_4(0.4, 0.3, 0.2), f_5(0.7, 0.6, 0.5), \\ f_6(0.6, 0.6, 0.3), f_7(0.6, 0.5, 0.5), f_8(0.8, 0.7, 0.4) \end{array} \right\}$$

$$X(C_7) = \left\{ \begin{array}{l} f_1(0.9, 0.3, 0.2), f_2(0.6, 0.3, 0.2), f_3(0.8, 0.8, 0.5), f_4(0.6, 0.5, 0.3), f_5(0.4, 0.3, 0.2), \\ f_6(0.7, 0.6, 0.6), f_7(0.9, 0.3, 0.2), f_8(0.6, 0.1, 0.1) \end{array} \right\}$$

$$X(C_8) = \left\{ \begin{array}{l} f_1(0.3, 0.1, 0.1), f_2(0.8, 0.6, 0.6), f_3(0.9, 0.3, 0.2), f_4(0.7, 0.7, 0.1), f_5(0.3, 0.2, 0.2), \\ f_6(0.5, 0.4, 0.3), f_7(0.8, 0.3, 0.3), f_8(0.6, 0.5, 0.4) \end{array} \right\}$$

Report of expert P_2 in the market place M_2 is representing in the form of (N_{ss}) as

$$(Y, B) = \{y(\mathbb{C}_1), y(\mathbb{C}_2), y(\mathbb{C}_3), y(\mathbb{C}_4), y(\mathbb{C}_5), y(\mathbb{C}_6), y(\mathbb{C}_7), y(\mathbb{C}_8)\}$$

$$Y(\mathbb{C}_1) = \left\{ \begin{array}{l} f_1(0.4, 0.3, 0.2), f_2(0.8, 0.6, 0.3), f_3(0.8, 0.4, 0.1), f_4(0.6, 0.7, 0.4), f_5(0.7, 0.7, 0.3), \\ f_6(0.8, 0.4, 0.2), f_7(0.3, 0.3, 0.1), f_8(0.6, 0.4, 0.2) \end{array} \right\}$$

$$Y(\mathbb{C}_2) = \left\{ \begin{array}{l} f_1(0.6, 0.4, 0.2), f_2(0.8, 0.7, 0.3), f_3(0.7, 0.4, 0.1), f_4(0.6, 0.3, 0.34), f_5(0.7, 0.5, 0.4), \\ f_6(0.7, 0.5, 0.3), f_7(0.4, 0.3, 0.1), f_8(0.5, 0.2, 0.2) \end{array} \right\}$$

$$Y(\mathbb{C}_3) = \left\{ \begin{array}{l} f_1(0.7, 0.5, 0.4), f_2(0.7, 0.4, 0.3), f_3(0.8, 0.5, 0.4), f_4(0.6, 0.6, 0.2), f_5(0.7, 0.5, 0.3), \\ f_6(0.6, 0.3, 0.3), f_7(0.8, 0.7, 0.5), f_8(0.5, 0.3, 0.2) \end{array} \right\}$$

$$Y(\mathbb{C}_4) = \left\{ \begin{array}{l} f_1(0.8, 0.5, 0.4), f_2(0.9, 0.5, 0.4), f_3(0.7, 0.4, 0.2), f_4(0.6, 0.5, 0.1), f_5(0.8, 0.4, 0.3), \\ f_6(0.6, 0.3, 0.3), f_7(0.7, 0.5, 0.2), f_8(0.4, 0.4, 0.3) \end{array} \right\}$$

$$Y(\mathbb{C}_5) = \left\{ \begin{array}{l} f_1(0.6, 0.4, 0.1), f_2(0.8, 0.4, 0.3), f_3(0.8, 0.6, 0.4), f_4(0.4, 0.3, 0.1), f_5(0.7, 0.3, 0.3), \\ f_6(0.5, 0.4, 0.1), f_7(0.6, 0.2, 0.2), f_8(0.5, 0.4, 0.1) \end{array} \right\}$$

$$Y(\mathbb{C}_6) = \left\{ \begin{array}{l} f_1(0.6, 0.5, 0.4), f_2(0.6, 0.3, 0.3), f_3(0.8, 0.7, 0.5), f_4(0.5, 0.5, 0.4), f_5(0.9, 0.4, 0.3), \\ f_6(0.8, 0.6, 0.3), f_7(0.4, 0.3, 0.1), f_8(0.6, 0.5, 0.4) \end{array} \right\}$$

$$Y(\mathbb{C}_7) = \left\{ \begin{array}{l} f_1(0.7, 0.5, 0.4), f_2(0.4, 0.3, 0.2), f_3(0.6, 0.4, 0.3), f_4(0.4, 0.3, 0.1), f_5(0.5, 0.3, 0.2), \\ f_6(0.9, 0.6, 0.4), f_7(0.7, 0.5, 0.2), f_8(0.8, 0.3, 0.3) \end{array} \right\}$$

$$Y(\mathbb{C}_8) = \left\{ \begin{array}{l} f_1(0.5, 0.3, 0.1), f_2(0.8, 0.4, 0.4), f_3(0.7, 0.5, 0.4), f_4(0.5, 0.5, 0.3), f_5(0.4, 0.4, 0.2), \\ f_6(0.3, 0.2, 0.1), f_7(0.6, 0.5, 0.5), f_8(0.4, 0.3, 0.2) \end{array} \right\}$$

The payoff matrices **A** and **B** are,

	\mathbb{C}_1	\mathbb{C}_2	\mathbb{C}_3	\mathbb{C}_4	\mathbb{C}_5	\mathbb{C}_6	\mathbb{C}_7	\mathbb{C}_8
f_1	(0.6, 0.3, 0.2)	(0.4, 0.2, 0.2)	(0.3, 0.3, 0.2)	(0.6, 0.5, 0.2)	(0.4, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.9, 0.3, 0.2)	(0.3, 0.1, 0.1)
f_2	(0.2, 0.2, 0.1)	(0.6, 0.5, 0.5)	(0.9, 0.6, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.2, 0.1)	(0.4, 0.3, 0.3)	(0.6, 0.3, 0.2)	(0.8, 0.6, 0.6)
f_3	(0.6, 0.4, 0.3)	(0.9, 0.6, 0.3)	(0.8, 0.7, 0.6)	(0.7, 0.6, 0.6)	(0.8, 0.8, 0.6)	(0.6, 0.5, 0.5)	(0.8, 0.8, 0.5)	(0.9, 0.3, 0.2)
f_4	(0.4, 0.3, 0.2)	(0.4, 0.1, 0.1)	(0.6, 0.4, 0.4)	(0.4, 0.3, 0.2)	(0.6, 0.3, 0.1)	(0.4, 0.3, 0.2)	(0.6, 0.5, 0.3)	(0.7, 0.7, 0.1)
f_5	(0.5, 0.5, 0.1)	(0.7, 0.3, 0.2)	(0.7, 0.7, 0.3)	(0.4, 0.4, 0.1)	(0.7, 0.5, 0.5)	(0.7, 0.6, 0.5)	(0.4, 0.3, 0.2)	(0.3, 0.2, 0.2)
f_6	(0.6, 0.2, 0.2)	(0.5, 0.3, 0.3)	(0.3, 0.3, 0.1)	(0.6, 0.5, 0.3)	(0.3, 0.2, 0.1)	(0.6, 0.6, 0.3)	(0.7, 0.6, 0.6)	(0.5, 0.4, 0.3)
f_7	(0.7, 0.3, 0.1)	(0.2, 0.1, 0.1)	(0.6, 0.5, 0.3)	(0.7, 0.7, 0.6)	(0.6, 0.4, 0.4)	(0.6, 0.5, 0.5)	(0.9, 0.3, 0.2)	(0.8, 0.3, 0.3)
f_8	(0.8, 0.8, 0.6)	(0.7, 0.6, 0.6)	(0.7, 0.5, 0.2)	(0.2, 0.2, 0.1)	(0.3, 0.2, 0.2)	(0.8, 0.7, 0.4)	(0.6, 0.1, 0.1)	(0.6, 0.5, 0.4)

	\mathbb{C}_1	\mathbb{C}_2	\mathbb{C}_3	\mathbb{C}_4	\mathbb{C}_5	\mathbb{C}_6	\mathbb{C}_7	\mathbb{C}_8
f_1	(0.4, 0.3, 0.2)	(0.6, 0.4, 0.2)	(0.7, 0.5, 0.4)	(0.8, 0.5, 0.4)	(0.6, 0.4, 0.1)	(0.6, 0.5, 0.4)	(0.7, 0.5, 0.4)	(0.5, 0.3, 0.1)
f_2	(0.8, 0.6, 0.3)	(0.8, 0.7, 0.3)	(0.7, 0.4, 0.3)	(0.9, 0.5, 0.4)	(0.8, 0.4, 0.3)	(0.6, 0.3, 0.3)	(0.4, 0.3, 0.2)	(0.8, 0.4, 0.4)
f_3	(0.8, 0.4, 0.1)	(0.7, 0.4, 0.1)	(0.8, 0.5, 0.4)	(0.7, 0.4, 0.2)	(0.8, 0.6, 0.4)	(0.8, 0.7, 0.5)	(0.6, 0.4, 0.3)	(0.7, 0.5, 0.4)
f_4	(0.6, 0.7, 0.4)	(0.6, 0.3, 0.3)	(0.6, 0.6, 0.2)	(0.6, 0.5, 0.1)	(0.4, 0.3, 0.1)	(0.5, 0.5, 0.4)	(0.4, 0.3, 0.1)	(0.5, 0.5, 0.3)
f_5	(0.7, 0.7, 0.3)	(0.7, 0.5, 0.4)	(0.7, 0.5, 0.3)	(0.8, 0.4, 0.3)	(0.7, 0.3, 0.3)	(0.9, 0.4, 0.3)	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.2)
f_6	(0.8, 0.4, 0.2)	(0.7, 0.5, 0.3)	(0.6, 0.3, 0.3)	(0.6, 0.3, 0.3)	(0.5, 0.4, 0.1)	(0.8, 0.6, 0.3)	(0.9, 0.6, 0.4)	(0.3, 0.2, 0.1)
f_7	(0.3, 0.3, 0.1)	(0.4, 0.3, 0.1)	(0.8, 0.7, 0.5)	(0.7, 0.5, 0.2)	(0.6, 0.2, 0.2)	(0.4, 0.3, 0.1)	(0.7, 0.5, 0.2)	(0.6, 0.5, 0.5)
f_8	(0.6, 0.4, 0.2)	(0.5, 0.2, 0.2)	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.3)	(0.5, 0.4, 0.1)	(0.6, 0.5, 0.4)	(0.4, 0.3, 0.3)	(0.4, 0.3, 0.2)

To find crisp value by using Neutrosophic Arithmetic Operator (NA)

	\mathbb{C}_1	\mathbb{C}_2	\mathbb{C}_3	\mathbb{C}_4	\mathbb{C}_5	\mathbb{C}_6	\mathbb{C}_7	\mathbb{C}_8
f_1	(0.5, 0.3, 0.2)	(0.5, 0.3, 0.2)	(0.5, 0.4, 0.3)	(0.7, 0.5, 0.3)	(0.5, 0.4, 0.2)	(0.65, 0.4, 0.3)	(0.8, 0.4, 0.3)	(0.4, 0.2, 0.1)
f_2	(0.5, 0.4, 0.2)	(0.7, 0.6, 0.4)	(0.8, 0.5, 0.3)	(0.8, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.5, 0.3, 0.3)	(0.5, 0.3, 0.2)	(0.8, 0.5, 0.5)
f_3	(0.7, 0.4, 0.2)	(0.8, 0.5, 0.2)	(0.8, 0.6, 0.5)	(0.7, 0.5, 0.4)	(0.8, 0.7, 0.5)	(0.7, 0.6, 0.5)	(0.7, 0.6, 0.4)	(0.8, 0.4, 0.3)
f_4	(0.5, 0.5, 0.3)	(0.5, 0.2, 0.2)	(0.6, 0.5, 0.3)	(0.5, 0.4, 0.15)	(0.5, 0.3, 0.1)	(0.45, 0.4, 0.3)	(0.5, 0.4, 0.2)	(0.6, 0.6, 0.2)
f_5	(0.6, 0.6, 0.2)	(0.7, 0.4, 0.3)	(0.7, 0.6, 0.3)	(0.6, 0.4, 0.2)	(0.7, 0.4, 0.4)	(0.8, 0.5, 0.4)	(0.45, 0.3, 0.2)	(0.3, 0.3, 0.2)
f_6	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.45, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.4, 0.3, 0.1)	(0.7, 0.6, 0.3)	(0.8, 0.6, 0.5)	(0.4, 0.3, 0.2)
f_7	(0.5, 0.3, 0.1)	(0.3, 0.2, 0.1)	(0.7, 0.6, 0.4)	(0.7, 0.6, 0.4)	(0.6, 0.3, 0.3)	(0.5, 0.4, 0.3)	(0.8, 0.4, 0.2)	(0.7, 0.4, 0.4)
f_8	(0.7, 0.6, 0.4)	(0.6, 0.4, 0.4)	(0.6, 0.4, 0.2)	(0.3, 0.3, 0.2)	(0.4, 0.3, 0.15)	(0.7, 0.6, 0.4)	(0.5, 0.2, 0.2)	(0.5, 0.4, 0.3)

$$V(c) = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{matrix} & \begin{bmatrix} 0.32 & 0.32 & 0.4 & 0.5 & 0.38 & 0.425 & 0.45 & 0.22 \\ 0.38 & 0.58 & 0.52 & 0.45 & 0.35 & 0.33 & 0.32 & 0.55 \\ 0.42 & 0.5 & 0.62 & 0.52 & 0.68 & 0.6 & 0.58 & 0.45 \\ 0.47 & 0.25 & 0.48 & 0.375 & 0.3 & 0.39 & 0.38 & 0.53 \\ 0.53 & 0.43 & 0.57 & 0.4 & 0.45 & 0.53 & 0.31 & 0.29 \\ 0.35 & 0.42 & 0.31 & 0.42 & 0.28 & 0.57 & 0.61 & 0.3 \\ 0.3 & 0.2 & 0.58 & 0.58 & 0.35 & 0.4 & 0.43 & 0.45 \\ 0.58 & 0.43 & 0.4 & 0.28 & 0.29 & 0.58 & 0.25 & 0.4 \end{bmatrix} \end{matrix}$$

Decisions are made under the following circumstances:

1. Laplace Criterion method analysis:

$$\text{Laplace Criterion} = 1 \div n(C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8)$$

f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
0.38	0.43	0.54	0.39	0.43	0.4	0.41	0.4

Therefore, f_3 ($\max = 0.54$) attains maximum profit.

2. Optimism Criterion method analysis:

Minimum	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Maximum
0.22 f_1	0.32	0.32	0.4	0.5	0.38	0.425	0.45	0.22	0.5
0.32 f_2	0.38	0.58	0.52	0.45	0.35	0.33	0.32	0.55	0.58
0.42 f_3	0.42	0.5	0.62	0.52	0.68	0.6	0.58	0.45	0.68
0.25 f_4	0.47	0.25	0.48	0.375	0.3	0.39	0.38	0.53	0.53
0.29 f_5	0.53	0.43	0.57	0.4	0.45	0.53	0.31	0.29	0.57
0.28 f_6	0.35	0.42	0.31	0.42	0.28	0.57	0.61	0.3	0.61
0.2 f_7	0.3	0.2	0.58	0.58	0.35	0.4	0.43	0.45	0.58
0.25 f_8	0.58	0.43	0.4	0.28	0.29	0.58	0.25	0.4	0.58

Therefore, f_3 ($\minimax = 0.42$), ($\maximax = 0.68$) attains maximum profit.

3. Savage Criterion method analysis:

a) Maximum Regret

$$\text{Regret payoff} = \text{maximum payoff from } C_j - \text{payoff}, (j = 1, 2, 3, 4, \dots, 8)$$

Regret table for maximum									
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Min
f_1	0.26	0.26	0.22	0.08	0.3	0.17	0.16	0.33	0.33
f_2	0.2	0.0	0.1	0.13	0.33	0.27	0.29	0.0	0.33
f_3	0.16	0.08	0.0	0.06	0.0	0.0	0.03	0.1	0.16
f_4	0.11	0.33	0.14	0.21	0.38	0.21	0.23	0.02	0.38
f_5	0.05	0.15	0.05	0.18	0.23	0.07	0.3	0.26	0.3
f_6	0.23	0.16	0.31	0.16	0.04	0.03	0.0	0.25	0.4
f_7	0.28	0.38	0.04	0.0	0.33	0.2	0.18	0.1	0.38
f_8	0.0	0.15	0.22	0.3	0.39	0.02	0.36	0.15	0.39

b) Minimum Regret

$$\text{Regret payoff} = \text{payoff} - \text{maximum payoff from } C_j, (j = 1, 2, 3, \dots, 8)$$

Regret Table for Minimum									
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Max
f_1	0.02	0.12	0.09	0.22	0.1	0.095	0.2	0.0	0.22
f_2	0.08	0.38	0.21	0.17	0.07	0.0	0.07	0.33	0.38
f_3	0.12	0.3	0.31	0.24	0.4	0.27	0.33	0.23	0.4
f_4	0.17	0.05	0.17	0.095	0.02	0.06	0.13	0.31	0.31
f_5	0.23	0.23	0.26	0.12	0.17	0.2	0.06	0.07	0.26
f_6	0.05	0.22	0.0	0.14	0.0	0.24	0.36	0.08	0.36
f_7	0.0	0.0	0.27	0.3	0.07	0.07	0.18	0.23	0.3
f_8	0.28	0.23	0.09	0.0	0.01	0.25	0.0	0.18	0.28

Therefore, $f_3(max = 0.4)$ attains maximum profit.
II To find the Crisp Value by using Neutrosophic Weighted Arithmetic Operator (NA_w)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
f_1	(0.52, 0.3, 0.2)	(0.48, 0.28, 0.2)	(0.46, 0.38, 0.28)	(0.68, 0.5, 0.28)	(0.48, 0.4, 0.22)	(0.66, 0.38, 0.28)	(0.82, 0.38, 0.28)	(0.38, 0.18, 0.1)
f_2	(0.4, 0.36, 0.18)	(0.68, 0.58, 0.42)	(0.82, 0.52, 0.3)	(0.78, 0.38, 0.28)	(0.68, 0.28, 0.18)	(0.48, 0.3, 0.3)	(0.52, 0.3, 0.2)	(0.8, 0.52, 0.52)
f_3	(0.68, 0.4, 0.2)	(0.82, 0.52, 0.22)	(0.8, 0.62, 0.52)	(0.7, 0.52, 0.44)	(0.8, 0.72, 0.52)	(0.68, 0.58, 0.5)	(0.72, 0.64, 0.42)	(0.82, 0.38, 0.28)
f_4	(0.48, 0.46, 0.28)	(0.48, 0.18, 0.18)	(0.6, 0.48, 0.32)	(0.48, 0.38, 0.16)	(0.52, 0.3, 0.1)	(0.44, 0.38, 0.28)	(0.52, 0.42, 0.22)	(0.62, 0.62, 0.18)
f_5	(0.6, 0.6, 0.2)	(0.7, 0.38, 0.28)	(0.7, 0.62, 0.3)	(0.56, 0.4, 0.18)	(0.7, 0.42, 0.42)	(0.78, 0.52, 0.42)	(0.44, 0.3, 0.2)	(0.34, 0.28, 0.2)
f_6	(0.68, 0.28, 0.2)	(0.58, 0.38, 0.3)	(0.42, 0.3, 0.18)	(0.6, 0.42, 0.3)	(0.38, 0.28, 0.1)	(0.68, 0.6, 0.3)	(0.78, 0.6, 0.52)	(0.42, 0.32, 0.22)
f_7	(0.54, 0.3, 0.1)	(0.28, 0.18, 0.1)	(0.68, 0.58, 0.38)	(0.7, 0.62, 0.44)	(0.6, 0.32, 0.32)	(0.52, 0.42, 0.34)	(0.82, 0.38, 0.2)	(0.72, 0.38, 0.38)
f_8	(0.72, 0.64, 0.44)	(0.62, 0.44, 0.44)	(0.62, 0.42, 0.2)	(0.28, 0.28, 0.18)	(0.38, 0.28, 0.16)	(0.72, 0.62, 0.4)	(0.52, 0.18, 0.18)	(0.52, 0.42, 0.32)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
f_1	0.32	0.3	0.37	0.49	0.38	0.41	0.43	0.2
f_2	0.34	0.57	0.53	0.43	0.33	0.33	0.32	0.57
f_3	0.42	0.52	0.63	0.53	0.7	0.58	0.62	0.44
f_4	0.43	0.23	0.47	0.36	0.3	0.37	0.4	0.56
f_5	0.51	0.41	0.58	0.39	0.47	0.54	0.31	0.27
f_6	0.33	0.4	0.3	0.43	0.26	0.56	0.62	0.32
f_7	0.31	0.18	0.56	0.6	0.36	0.42	0.42	0.43
f_8	0.62	0.47	0.42	0.26	0.27	0.6	0.23	0.42

Decisions are made under the following circumstances:

1. Laplace Criterion method analysis:

Laplace Criterion= $1 \div n(C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7 + C_8)$

f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
0.36	0.43	0.55	0.39	0.43	0.41	0.41	0.41

Therefore, $f_3(max = 0.55)$ attains maximum profit.

2. Optimism Criterion method analysis:

Minimum		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Maximum
0.20	f_1	0.32	0.3	0.37	0.49	0.38	0.41	0.43	0.2	0.49
0.32	f_2	0.34	0.57	0.53	0.43	0.33	0.33	0.32	0.57	0.57
0.42	f_3	0.42	0.52	0.63	0.53	0.7	0.58	0.62	0.44	0.70
0.23	f_4	0.43	0.23	0.47	0.36	0.3	0.37	0.4	0.56	0.56
0.27	f_5	0.51	0.41	0.58	0.39	0.47	0.54	0.31	0.27	0.58
0.26	f_6	0.33	0.4	0.3	0.43	0.26	0.56	0.62	0.32	0.62
0.18	f_7	0.31	0.18	0.56	0.6	0.36	0.42	0.42	0.43	0.60
0.23	f_8	0.62	0.47	0.42	0.26	0.27	0.6	0.23	0.42	0.62

Therefore, $f_3\{(minimax = 0.42), (maximax = 0.70)\}$ attains maximum profit.

3. Savage Criterion method analysis:

a) Maximum Regret

Regret payoff = maximum payoff from C_j - payoff, ($j = 1,2,3,4...8$)

Regret Table for Maximum									
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Min
f_1	0.3	0.27	0.26	0.11	0.32	0.19	0.18	0.37	0.37
f_2	0.28	.0	0.1	0.17	0.37	0.27	0.29	0.0	0.37
f_3	0.2	0.05	0.0	0.07	.0	0.02	.0	0.13	0.2
f_4	0.17	0.34	0.16	0.24	0.4	0.23	0.21	0.01	0.4
f_5	0.11	0.16	0.05	0.21	0.23	0.06	0.3	0.3	0.31
f_6	0.29	0.17	0.33	0.17	0.44	0.04	0.0	0.25	0.44
f_7	0.31	0.39	0.07	0.0	0.34	0.18	0.19	0.14	0.38
f_8	0.0	0.1	0.21	0.34	0.43	0.0	0.38	0.15	0.43

b) Minimum Regret

$$\text{Regret payoff} = \text{payoff} - \text{minimum payoff from } C_j, (j = 1, 2, 3, 4, \dots, 8)$$

Regret Table for Minimum									
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	MAX
f_1	0.02	0.12	0.07	0.23	0.12	0.08	0.19	0.0	0.23
f_2	0.04	0.39	0.23	0.17	0.07	0.0	0.08	0.37	0.39
f_3	0.12	0.34	0.33	0.27	0.44	0.25	0.38	0.24	0.43
f_4	0.13	0.05	0.17	0.1	0.04	0.04	0.16	0.36	0.36
f_5	0.21	0.23	0.28	0.13	0.2	0.21	0.07	0.07	0.28
f_6	0.03	0.22	0.0	0.17	0.0	0.23	0.38	0.12	0.38
f_7	0.01	0.0	0.26	0.34	0.1	0.09	0.18	0.23	0.34
f_8	0.32	0.29	0.12	0.0	0.01	0.27	0.0	0.22	0.32

Therefore, $f_3(\text{max} = 0.43)$ attains maximum profit.

III. To find the Crisp Value by using Neutrosophic Geometric ator(NG)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
f_1	(0.49,0.30,0.20)	(0.49,0.28,0.20)	(0.46,0.39,0.28)	(0.69,0.50,0.28)	(0.49,0.40,0.17)	(0.65,0.39,0.28)	(0.79,0.39,0.28)	(0.39,0.17,0.10)
f_2	(0.40,0.35,0.17)	(0.69,0.59,0.39)	(0.79,0.49,0.30)	(0.79,0.39,0.28)	(0.69,0.28,0.17)	(0.49,0.30,0.30)	(0.49,0.30,0.20)	(0.80,0.49,0.49)
f_3	(0.69,0.40,0.17)	(0.79,0.49,0.17)	(0.80,0.59,0.49)	(0.70,0.49,0.35)	(0.80,0.69,0.49)	(0.69,0.59,0.50)	(0.69,0.57,0.39)	(0.79,0.39,0.28)
f_4	(0.49,0.46,0.28)	(0.49,0.17,0.17)	(0.60,0.49,0.28)	(0.49,0.39,0.14)	(0.49,0.30,0.10)	(0.45,0.39,0.28)	(0.49,0.39,0.17)	(0.59,0.59,0.17)
f_5	(0.59,0.59,0.17)	(0.70,0.39,0.28)	(0.70,0.59,0.30)	(0.57,0.40,0.17)	(0.70,0.39,0.39)	(0.79,0.49,0.39)	(0.45,0.30,0.20)	(0.35,0.28,0.20)
f_6	(0.69,0.28,0.20)	(0.59,0.39,0.30)	(0.42,0.30,0.17)	(0.60,0.39,0.30)	(0.39,0.28,0.10)	(0.69,0.60,0.30)	(0.79,0.60,0.49)	(0.39,0.28,0.17)
f_7	(0.46,0.30,0.10)	(0.28,0.17,0.10)	(0.69,0.59,0.39)	(0.70,0.59,0.35)	(0.60,0.28,0.28)	(0.49,0.39,0.22)	(0.79,0.39,0.20)	(0.69,0.39,0.39)
f_8	(0.69,0.57,0.35)	(0.59,0.35,0.35)	(0.59,0.39,0.20)	(0.28,0.28,0.17)	(0.39,0.28,0.14)	(0.69,0.59,0.40)	(0.49,0.17,0.17)	(0.49,0.39,0.28)

$$V(C) = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{matrix} & \begin{bmatrix} 0.31 & 0.30 & 0.38 & 0.50 & 0.38 & 0.41 & 0.44 & 0.20 \\ 0.33 & 0.57 & 0.51 & 0.44 & 0.33 & 0.33 & 0.31 & 0.54 \\ 0.41 & 0.49 & 0.61 & 0.50 & 0.68 & 0.59 & 0.56 & 0.44 \\ 0.43 & 0.23 & 0.47 & 0.36 & 0.30 & 0.38 & 0.37 & 0.52 \\ 0.52 & 0.42 & 0.56 & 0.39 & 0.44 & 0.52 & 0.31 & 0.28 \\ 0.34 & 0.41 & 0.30 & 0.41 & 0.27 & 0.57 & 0.61 & 0.28 \\ 0.29 & 0.18 & 0.57 & 0.57 & 0.34 & 0.38 & 0.42 & 0.44 \\ 0.55 & 0.39 & 0.39 & 0.26 & 0.28 & 0.58 & 0.23 & 0.39 \end{bmatrix} \end{matrix}$$

Decisions are made under the following circumstances:

1. Laplace Criterion method analysis:

$$\text{Laplace Criterion} = 1 \div n(C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7 + C_8)$$

Therefore, $f_3(\text{max} = 0.53)$ attains maximum profit.

2. Optimism Criterion method analysis:

Therefore, $f_3\{(\text{minimax} = 0.4), (\text{maximax} = 0.68)\}$ attains maximum profit

3. Savage Criterion method analysis:

a) Maximum Regret

$$\text{Regret payoff} = \text{maximum payoff from } C_j - \text{payoff}, (j = 1, 2, 3, 4, \dots, 8)$$

Therefore, $f_3(\text{min} = 0.14)$ attains maximum profit.

b) Minimum Regret

$$\text{Regret payoff} = \text{payoff} - \text{minimum payoff from } C_j, (j = 1, 2, 3, \dots, 8)$$

Therefore, $f_3(\text{max} = 0.41)$ attains maximum profit.

IV To find the Crisp Value by using Neutrosophic Weighted Geometric Operator (NG_w)

	\mathbb{C}_1	\mathbb{C}_2	\mathbb{C}_3	\mathbb{C}_4	\mathbb{C}_5	\mathbb{C}_6	\mathbb{C}_7	\mathbb{C}_8
f_1	(0.51,0.30,0.20)	(0.47,0.26,0.20)	(0.42,0.37,0.26)	(0.67,0.50,0.26)	(0.47,0.40,0.19)	(0.66,0.37,0.26)	(0.81,0.37,0.26)	(0.37,0.16,0.10)
f_2	(0.35,0.31,0.16)	(0.67,0.57,0.41)	(0.81,0.51,0.30)	(0.77,0.37,0.26)	(0.67,0.26,0.16)	(0.47,0.30,0.30)	(0.51,0.30,0.20)	(0.80,0.51,0.51)
f_3	(0.67,0.40,0.19)	(0.81,0.51,0.19)	(0.80,0.61,0.51)	(0.70,0.51,0.39)	(0.80,0.71,0.51)	(0.67,0.57,0.50)	(0.71,0.61,0.41)	(0.81,0.37,0.26)
f_4	(0.47,0.42,0.26)	(0.47,0.16,0.16)	(0.60,0.47,0.30)	(0.47,0.37,0.15)	(0.51,0.30,0.10)	(0.44,0.37,0.26)	(0.51,0.41,0.19)	(0.61,0.61,0.16)
f_5	(0.57,0.57,0.16)	(0.70,0.37,0.26)	(0.70,0.61,0.30)	(0.53,0.40,0.16)	(0.70,0.41,0.41)	(0.77,0.51,0.41)	(0.44,0.30,0.20)	(0.34,0.26,0.20)
f_6	(0.67,0.26,0.20)	(0.57,0.37,0.30)	(0.40,0.30,0.16)	(0.60,0.41,0.30)	(0.37,0.26,0.10)	(0.67,0.60,0.30)	(0.77,0.60,0.51)	(0.41,0.30,0.19)
f_7	(0.50,0.30,0.10)	(0.26,0.16,0.10)	(0.67,0.57,0.37)	(0.70,0.61,0.39)	(0.60,0.30,0.30)	(0.51,0.41,0.26)	(0.81,0.37,0.20)	(0.71,0.37,0.37)
f_8	(0.71,0.61,0.39)	(0.61,0.39,0.39)	(0.61,0.41,0.20)	(0.26,0.26,0.16)	(0.37,0.26,0.15)	(0.71,0.61,0.40)	(0.51,0.16,0.16)	(0.51,0.41,0.30)

	\mathbb{C}_1	\mathbb{C}_2	\mathbb{C}_3	\mathbb{C}_4	\mathbb{C}_5	\mathbb{C}_6	\mathbb{C}_7	\mathbb{C}_8
f_1	0.32	0.29	0.36	0.49	0.38	0.40	0.42	0.18
f_2	0.29	0.56	0.53	0.42	0.31	0.33	0.32	0.56
f_3	0.41	0.51	0.63	0.52	0.69	0.58	0.59	0.42
f_4	0.40	0.21	0.46	0.35	0.30	0.36	0.39	0.54
f_5	0.50	0.41	0.57	0.38	0.46	0.54	0.31	0.27
f_6	0.32	0.39	0.29	0.42	0.25	0.56	0.61	0.30
f_7	0.30	0.16	0.55	0.59	0.35	0.40	0.41	0.43
f_8	0.59	0.42	0.41	0.25	0.26	0.59	0.21	0.41

Decisions are made under the following circumstances:

1. Laplace Criterion method analysis:

$$\text{Laplace Criterion} = 1 \div n(\mathbb{C}_1 + \mathbb{C}_2 + \mathbb{C}_3 + \mathbb{C}_4 + \mathbb{C}_5 + \mathbb{C}_6 + \mathbb{C}_7 + \mathbb{C}_8)$$

Therefore, $f_3(\text{max} = 0.53)$ attains maximum profit.

2. Optimism Criterion method analysis:

Therefore, $f_3\{(\text{minimax} = 0.41), (\text{maximax} = 0.69)\}$ attains maximum profit.

3. Savage Criterion method analysis:

a) Maximum Regret

$$\text{Regret payoff} = \text{maximum payoff from } \mathbb{C}_j - \text{payoff}, (j = 1, 2, 3, 4, \dots, 8)$$

Therefore, $f_3(\text{min} = 0.18)$ attains maximum profit.

b) Minimum Regret

$$\text{Regret payoff} = \text{payoff} - \text{minimum payoff from } \mathbb{C}_j, (j = 1, 2, 3, \dots, 8)$$

Therefore, $f_3(\text{max} = 0.44)$ attains maximum profit.

V. To find the Crisp Value by using Neutrosophic Harmonic Operator(NH)

	\mathbb{C}_1	\mathbb{C}_2	\mathbb{C}_3	\mathbb{C}_4	\mathbb{C}_5	\mathbb{C}_6	\mathbb{C}_7	\mathbb{C}_8
f_1	(0.48,0.30,0.20)	(0.48,0.27,0.20)	(0.42,0.38,0.27)	(0.69,0.50,0.27)	(0.48,0.40,0.15)	(0.65,0.38,0.27)	(0.79,0.38,0.27)	(0.38,0.15,0.10)
f_2	(0.32,0.30,0.15)	(0.69,0.58,0.38)	(0.79,0.48,0.30)	(0.79,0.38,0.27)	(0.69,0.27,0.15)	(0.48,0.30,0.30)	(0.48,0.30,0.20)	(0.80,0.48,0.48)
f_3	(0.69,0.40,0.15)	(0.79,0.48,0.15)	(0.80,0.58,0.48)	(0.70,0.48,0.30)	(0.80,0.69,0.48)	(0.69,0.58,0.50)	(0.69,0.53,0.38)	(0.79,0.38,0.27)
f_4	(0.48,0.42,0.27)	(0.48,0.15,0.15)	(0.60,0.48,0.27)	(0.48,0.38,0.13)	(0.48,0.30,0.10)	(0.44,0.38,0.27)	(0.48,0.38,0.15)	(0.58,0.58,0.15)
f_5	(0.58,0.58,0.15)	(0.70,0.38,0.27)	(0.70,0.58,0.30)	(0.53,0.40,0.15)	(0.70,0.38,0.38)	(0.79,0.48,0.38)	(0.44,0.30,0.20)	(0.34,0.27,0.20)
f_6	(0.69,0.27,0.20)	(0.58,0.38,0.30)	(0.40,0.30,0.15)	(0.60,0.38,0.30)	(0.38,0.27,0.10)	(0.69,0.60,0.30)	(0.79,0.60,0.48)	(0.38,0.27,0.15)
f_7	(0.42,0.30,0.10)	(0.27,0.15,0.10)	(0.69,0.58,0.38)	(0.70,0.58,0.30)	(0.60,0.27,0.27)	(0.48,0.38,0.17)	(0.79,0.38,0.20)	(0.69,0.38,0.38)
f_8	(0.69,0.53,0.30)	(0.58,0.30,0.30)	(0.58,0.38,0.20)	(0.27,0.27,0.15)	(0.38,0.27,0.13)	(0.69,0.58,0.40)	(0.48,0.15,0.15)	(0.48,0.38,0.27)

$$V(C) = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{matrix} & \begin{bmatrix} 0.31 & 0.29 & 0.36 & 0.49 & 0.37 & 0.40 & 0.43 & 0.18 \\ 0.28 & 0.57 & 0.50 & 0.43 & 0.32 & 0.33 & 0.31 & 0.53 \\ 0.41 & 0.48 & 0.60 & 0.49 & 0.67 & 0.59 & 0.53 & 0.43 \\ 0.40 & 0.21 & 0.46 & 0.35 & 0.30 & 0.37 & 0.36 & 0.51 \\ 0.51 & 0.41 & 0.56 & 0.38 & 0.43 & 0.51 & 0.31 & 0.27 \\ 0.33 & 0.40 & 0.29 & 0.40 & 0.26 & 0.56 & 0.61 & 0.27 \\ 0.29 & 0.16 & 0.57 & 0.56 & 0.32 & 0.36 & 0.41 & 0.43 \\ 0.52 & 0.35 & 0.38 & 0.25 & 0.26 & 0.57 & 0.21 & 0.37 \end{bmatrix} \end{matrix}$$

Decisions are made under the following circumstances:

1. Laplace Criterion method analysis:

$$\text{Laplace Criterion} = 1 \div n(C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7 + C_8)$$

Therefore, f_3 (**max** = **0.52**) attains maximum profit.

2. Optimism Criterion method analysis:

Therefore, f_3 (**minimax** = **0.41**), (**maximax** = **0.67**) attains maximum profit

3. Savage Criterion method analysis:

a) Maximum Regret

$$\text{Regret payoff} = \text{maximum payoff from } C_j - \text{payoff}(j = 1, 2, 3, 4, \dots, 8)$$

Therefore, f_3 (**min** = **0.11**) attains maximum profit.

b) Minimum Regret

$$\text{Regret payoff} = \text{payoff} - \text{minimum payoff from } C_j, (j = 1, 2, 3, 4, \dots, 8)$$

Therefore, f_3 (**max** = **0.41**) attains maximum profit.

VI. To find the Crisp Value by using Neutrosophic Weighted Harmonic Operator (NH_w)

$$C = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{matrix} & \begin{bmatrix} (0.50, 0.30, 0.20) & (0.46, 0.25, 0.20) & (0.39, 0.36, 0.25) & (0.67, 0.50, 0.25) & (0.46, 0.40, 0.17) & (0.66, 0.36, 0.25) & (0.81, 0.36, 0.25) & (0.36, 0.14, 0.10) \\ (0.29, 0.27, 0.14) & (0.67, 0.56, 0.39) & (0.81, 0.50, 0.30) & (0.77, 0.36, 0.25) & (0.67, 0.25, 0.14) & (0.46, 0.30, 0.30) & (0.50, 0.30, 0.20) & (0.80, 0.50, 0.50) \\ (0.67, 0.40, 0.17) & (0.81, 0.50, 0.17) & (0.80, 0.60, 0.50) & (0.70, 0.50, 0.33) & (0.80, 0.71, 0.50) & (0.67, 0.56, 0.50) & (0.71, 0.57, 0.39) & (0.81, 0.36, 0.25) \\ (0.46, 0.39, 0.25) & (0.46, 0.14, 0.14) & (0.60, 0.46, 0.29) & (0.46, 0.36, 0.14) & (0.50, 0.30, 0.10) & (0.43, 0.36, 0.25) & (0.50, 0.39, 0.17) & (0.60, 0.60, 0.14) \\ (0.56, 0.56, 0.14) & (0.70, 0.36, 0.25) & (0.70, 0.60, 0.30) & (0.50, 0.40, 0.14) & (0.70, 0.39, 0.39) & (0.77, 0.50, 0.39) & (0.43, 0.30, 0.20) & (0.33, 0.25, 0.20) \\ (0.67, 0.25, 0.20) & (0.56, 0.36, 0.30) & (0.38, 0.30, 0.14) & (0.60, 0.39, 0.30) & (0.36, 0.25, 0.10) & (0.67, 0.60, 0.30) & (0.77, 0.60, 0.50) & (0.39, 0.29, 0.17) \\ (0.46, 0.30, 0.10) & (0.25, 0.14, 0.10) & (0.67, 0.56, 0.36) & (0.70, 0.60, 0.33) & (0.60, 0.29, 0.29) & (0.50, 0.39, 0.19) & (0.81, 0.36, 0.20) & (0.71, 0.36, 0.36) \\ (0.71, 0.57, 0.33) & (0.60, 0.33, 0.33) & (0.60, 0.39, 0.20) & (0.25, 0.25, 0.14) & (0.36, 0.25, 0.14) & (0.71, 0.60, 0.40) & (0.50, 0.14, 0.14) & (0.50, 0.39, 0.29) \end{bmatrix} \end{matrix}$$

$$V(C) = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{matrix} & \begin{bmatrix} 0.32 & 0.28 & 0.34 & 0.49 & 0.37 & 0.39 & 0.41 & 0.17 \\ 0.25 & 0.55 & 0.52 & 0.41 & 0.30 & 0.33 & 0.32 & 0.55 \\ 0.41 & 0.50 & 0.62 & 0.51 & 0.69 & 0.57 & 0.56 & 0.41 \\ 0.38 & 0.19 & 0.46 & 0.34 & 0.30 & 0.35 & 0.37 & 0.53 \\ 0.49 & 0.40 & 0.57 & 0.37 & 0.45 & 0.53 & 0.31 & 0.26 \\ 0.31 & 0.38 & 0.29 & 0.41 & 0.24 & 0.56 & 0.61 & 0.28 \\ 0.29 & 0.15 & 0.55 & 0.57 & 0.34 & 0.38 & 0.41 & 0.42 \\ 0.55 & 0.38 & 0.40 & 0.23 & 0.25 & 0.59 & 0.20 & 0.39 \end{bmatrix} \end{matrix}$$

Decisions are made under the following circumstances:

1. Laplace Criterion method analysis:

$$\text{Laplace Criterion} = 1 \div n(C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7 + C_8)$$

Therefore, f_3 (**0.53**) attains maximum profit.

2. Optimism Criterion method analysis:

Therefore, $f_3\{(\text{minimax} = 0.41), (\text{maximax} = 0.69)\}$ attains maximum profit.

3. Savage Criterion method analysis:

a) Maximum Regret

$$\text{Regret payoff} = \text{maximum payoff from } C_j - \text{payoff}, (j = 1, 2, 3, 4, \dots, 8)$$

Therefore, $f_3(\text{min} = 0.14)$ attains maximum profit.

b) Minimum Regret

$$\text{Regret payoff} = \text{payoff} - \text{minimum payoff from } C_j, (j = 1, 2, 3, 4, \dots, 8)$$

Therefore, $f_3(\text{max} = 0.45)$ attains maximum profit.

Result:

Ranking Methods	Laplace Criterion	Criterion of optimism	Savage Criterion
NA	f_3	f_3	f_3
NA_w	f_3	f_3	f_3
NG	f_3	f_3	f_3
NG_w	f_3	f_3	f_3
NH	f_3	f_3	f_3
NH_w	f_3	f_3	f_3

6. Comparison tables

Existing method	Proposed method	Result
Mary et al. 2015 [10]	Our method	f_3

Here, we have got the same result.

7. Discussion

To compare the neutrosophic soft sets, we developed score, certainty, and accuracy functions in this article. Then, we improved the neutrosophic arithmetic operator (NA), neutrosophic weighted arithmetic operator (NA_w), neutrosophic geometric operator (NG), neutrosophic weighted geometric operator (NG_w), neutrosophic harmonic operator (NH) and neutrosophic weighted harmonic operator (NH_w) to aggregate the neutrosophic information. Furthermore, based on the above operators and the score; certainty and accuracy functions, we developed a neutrosophic multiple criteria decision-making approach. Thus, a comparative analysis is also done using Laplace criterion, Optimism criterion and Savage criterion methods. It is observed that the N_{ss} with decision under uncertainty helps the decision makers to finalize the accurate reports. Hence, in reality N_{ss} paves a wave of change in the decision-making problems.

Ethical considerations

Not applicable.

Conflict of Interest

The authors declare no conflicts of interest.

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