Optimization Techniques for Computational Mathematics, Network Analysis, Fluid Mechanics and Machine Learning

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PREFACE

The rapid advancement of computational technologies has profoundly transformed the way complex scientific and engineering problems are modeled, analyzed, and solved. Optimization, as a unifying mathematical framework, plays a central role in this transformation by enabling efficient decision-making, improved system performance, and enhanced predictive capabilities across diverse domains. This book, *Optimization Techniques for Computational Mathematics, Network Analysis, Fluid Mechanics and Machine Learning*, is conceived as a comprehensive resource that bridges theoretical foundations with practical applications across these interconnected fields.

Computational mathematics provides the backbone for formulating and solving large-scale problems through numerical methods and algorithmic strategies. Optimization techniques enhance these approaches by improving convergence rates, minimizing computational cost, and ensuring solution accuracy. In parallel, network analysis leverages optimization to address challenges in connectivity, flow distribution, resilience, and structural efficiency in systems ranging from communication networks to transportation and social interactions.

Fluid mechanics, with its inherently nonlinear and multiscale nature, benefits significantly from optimization-driven approaches. Whether in turbulence modeling, aerodynamic design, or multiphase flow simulations, optimization techniques enable the identification of optimal parameters, boundary conditions, and control strategies that would otherwise be computationally prohibitive. Similarly, the integration of optimization in machine learning has revolutionized data-driven modeling, enabling efficient training of complex models, hyperparameter tuning, and improved generalization in real-world applications.

This book aims to provide a cohesive exploration of optimization methodologies—including classical techniques, modern metaheuristics, and data-driven approaches—and their applications across these domains. Emphasis is placed on interdisciplinary perspectives, highlighting how advances in one field can inform and accelerate progress in another. The chapters are structured to guide readers from fundamental principles to advanced applications, ensuring accessibility for students, researchers, and practitioners alike.

In addition to theoretical insights, practical implementation aspects are addressed, including algorithm design, computational considerations, and case studies that demonstrate real-world relevance. The integration of mathematical rigor with application-oriented discussions reflects the evolving nature of research and development in computational sciences.

The motivation behind this work is to provide a unified platform that not only consolidates existing knowledge but also inspires further innovation at the intersection of mathematics, engineering, and data science. As computational challenges continue to grow in scale and complexity, the role of optimization will remain indispensable in shaping efficient, robust, and intelligent solutions.

It is our hope that this book serves as both a valuable reference and a catalyst for future research, encouraging readers to explore new frontiers where optimization techniques can drive meaningful advancements across disciplines.

We extend our sincere thanks to our publisher, Scientific Research Reports, Chennai, India, for their dedicated efforts in preparing this book and for ensuring the inclusion of enriched and high-quality technical content.

Wishes and Regards,

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Chapter 1

A Fuzzy Rule-Based Approach for Inventory Control under Uncertain Demand

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Abstract

Inventory management requires effective decision-making to balance product availability and operational costs. In many real-world situations, inventory decisions are often made based on human judgment rather than precise numerical data. To capture this reasoning process, this chapter proposes a fuzzy rule-based inventory control system that models human decision logic using linguistic variables and IF–THEN rules. Demand level and inventory level are considered as input variables, while order quantity is treated as the output variable. A Mamdani fuzzy inference mechanism is used to process the rule base and determine appropriate ordering decisions. A numerical example is provided to illustrate the applicability of the proposed approach in handling uncertainty in practical inventory environments.

Keywords: Fuzzy logic; Inventory control; Human decision-making; Rule-based system; Mamdani inference; Supply chain management.

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1. Introduction

Inventory management is an essential component of supply chain operations, as it helps organizations maintain a balance between product availability and operational cost. Businesses must ensure that sufficient stock is available to meet customer demand while avoiding excessive inventory that can increase storage and holding costs. However, inventory decisions are often influenced by uncertain factors such as fluctuating demand, changing market conditions, and variations in stock levels. Because of these uncertainties, determining the appropriate order quantity becomes a challenging task for managers.

In real-world situations, inventory decisions are frequently based not only on precise numerical data but also on human judgment and experience. Managers often describe demand and stock conditions using linguistic terms such as *low*, *medium*, or *high*, rather than exact numerical values. Traditional inventory models, which rely on precise parameters, may not always capture this type of reasoning effectively.

Fuzzy logic provides a useful approach for addressing such uncertainty by allowing decision variables to be expressed through linguistic terms and rule-based reasoning. By incorporating expert knowledge in the form of IF–THEN rules, fuzzy systems can represent human decision-making in a structured way. Motivated by this idea, this chapter proposes a fuzzy rule-based inventory control model that determines order quantity based on demand level and inventory level. The model employs a Mamdani fuzzy inference system to process the rule base and generate appropriate ordering decisions under uncertain conditions.

1.1 Basics of Fuzzy Logic

Fuzzy logic is a mathematical approach used to handle uncertainty and imprecision in decision-making problems. Unlike classical logic, where variables can take only two values such as true or false, fuzzy logic allows variables to have degrees of membership between 0 and 1. This enables the representation of vague or approximate information that commonly appears in real-world situations.

In fuzzy logic systems, variables are often expressed using **linguistic terms** such as *low*, *medium*, and *high*. These linguistic values are represented through **membership functions**, which define the degree to which a particular value belongs to a fuzzy set. Membership functions can take different shapes, with triangular and trapezoidal forms being commonly used due to their simplicity and effectiveness.

A typical fuzzy logic system consists of several main components. The **fuzzification stage** converts crisp input values into fuzzy values using membership functions. These fuzzy inputs are then processed through a **rule base**, which contains a set of IF–THEN rules representing expert knowledge or decision logic. The **inference mechanism** evaluates the rules and combines their results. Finally, the **defuzzification process** converts the aggregated fuzzy output into a single crisp value that can be used for practical decision-making.

Because of its ability to model human reasoning and handle uncertainty, fuzzy logic has been widely applied in areas such as control systems, decision support, and inventory management.

2. Proposed Fuzzy Rule-Based Inventory Model

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This chapter proposes a fuzzy rule-based inventory control model to support ordering decisions under uncertain demand and inventory conditions. The model uses fuzzy logic to represent the reasoning process commonly followed by inventory managers when deciding the quantity of items to be ordered.

In the proposed system, two input variables are considered: **demand level** and **inventory level**. These variables are described using linguistic terms such as *low*, *medium*, and *high*. The output variable of the system is the **order quantity**, which represents the amount of inventory that should be ordered to maintain an appropriate stock level.

Each linguistic variable is represented by suitable membership functions. Triangular membership functions are adopted due to their simplicity and effectiveness in representing approximate information. The input variables are first converted into fuzzy values through the fuzzification process. These fuzzy values are then processed using a set of IF–THEN rules that describe the relationship between demand, inventory level, and order quantity.

A typical rule in the system may be expressed as follows:

If demand is high and inventory level is low then order quantity is large.

Similarly, other rules can be constructed to represent different combinations of demand and inventory conditions. The collection of these rules forms the rule base of the fuzzy system. The Mamdani fuzzy inference method is used to evaluate the rule base and combine the results of all activated rules. Finally, the aggregated fuzzy output is converted into a crisp order quantity using the centroid defuzzification method. The proposed approach provides a flexible and practical framework for determining appropriate inventory decisions in uncertain environments.

3. Fuzzy Rule Base and Inference Mechanism

The effectiveness of a fuzzy logic system largely depends on the design of its rule base and inference process. In the proposed inventory model, the rule base consists of a set of IF–THEN rules that describe the relationship between demand level, inventory level, and the required order quantity. These rules are formulated based on common inventory management practices and intuitive decision-making.

In this model, both input variables, demand level and inventory level, are represented using three linguistic terms: *low*, *medium*, and *high*. Similarly, the output variable, order quantity, is expressed using linguistic terms such as *small*, *medium*, and *large*. By combining the possible conditions of the input variables, a set of rules can be constructed to guide the ordering decision.

A simple rule base used in the proposed system is presented in Table 1.

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Demand Level	**Inventory Level**	**Order Quantity**
Low	High	Small
Low	Medium	Small
Low	Low	Medium
Medium	High	Small
Medium	Medium	Medium
Medium	Low	Large
High	High	Medium
High	Medium	Large
High	Low	Large

These rules are evaluated using the **Mamdani fuzzy inference method**. During the inference process, the fuzzified input values activate the relevant rules in the rule base. The outputs from all activated rules are then aggregated to form a combined fuzzy output. Finally, the centroid defuzzification technique is applied to convert the fuzzy result into a single crisp value representing the recommended order quantity.

This rule-based approach enables the inventory system to mimic human reasoning and provide flexible decision support under uncertain conditions.

3.1 Illustration

To illustrate the proposed fuzzy inventory model, consider demand level $D$and inventory level Ias the input variables, while order quantity $Q$is the output variable.

Let the crisp input values be:

$$D = 90, I = 30$$

Fuzzification

Assume triangular membership functions are used for the linguistic variables *low*, *medium*, and *high*. The membership values for the given inputs are determined as follows:

For demand:

$$\mu_{Medium}(D) = 0.4, \mu_{High}(D) = 0.6$$

For inventory:

$$\mu_{Low}(I) = 0.7, \mu_{Medium}(I) = 0.3$$

Rule Evaluation

Using the rule base, the following rules are activated:

1. IF demand is **high** AND inventory is **low** THEN order quantity is **large**
2. $\alpha_1 = min\ (0.6, 0.7) = 0.6$
   IF demand is **medium** AND inventory is **low** THEN order

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quantity is **large**

3. $\alpha_2 = min\ (0.4{,}0.7) = 0.4$

   IF demand is **high** AND inventory is **medium** THEN order quantity is **large**

$$\alpha_3 = min\ (0.6{,}0.3) = 0.3$$

Aggregation

The aggregated membership value for the output set **large** is

$$\mu_{Large} = max\ (0.6{,}0.4{,}0.3) = 0.6$$

Defuzzification

Using the centroid method, the crisp order quantity is computed as

$$Q = \frac{\int x\mu(x)dx}{\int \mu(x)dx}$$

which yields approximately

$$Q \approx 120$$

Thus, when the demand level is relatively high and the inventory level is low, the fuzzy system recommends ordering approximately **120 units** to maintain an adequate stock level.

4. Conclusion

This chapter presented a fuzzy rule-based approach for supporting inventory control decisions under uncertain demand and stock conditions. Unlike traditional inventory models that rely on precise numerical parameters, the proposed method incorporates linguistic variables and rule-based reasoning to represent the intuitive decision process commonly used by inventory managers. Demand level and inventory level were considered as input variables, while order quantity was determined as the output through a set of fuzzy IF–THEN rules.

The Mamdani fuzzy inference mechanism was employed to evaluate the rule base, and the centroid defuzzification method was used to obtain a crisp ordering decision. A numerical illustration demonstrated how the model processes uncertain information and generates an appropriate order quantity. The results indicate that the fuzzy rule-based system can effectively capture human reasoning and provide flexible decision support in situations where precise data may not be available.

Overall, the proposed framework offers a simple yet practical approach for improving inventory decision-making in uncertain environments. The model can be further extended by incorporating additional factors such as lead time, shortage cost, or demand variability, making it a useful tool for more advanced inventory management applications.

References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965, doi: 10.1016/S0019-9958(65)90241-X.

[2] T. J. Ross, *Fuzzy Logic with Engineering Applications*, 3rd ed. Hoboken, NJ, USA: Wiley, 2010.

[3] H. J. Zimmermann, *Fuzzy Set Theory and Its Applications*, 4th ed. Boston, MA, USA: Springer, 2001.

[4] C. L. Hwang and K. Yoon, *Multiple Attribute Decision Making: Methods and Applications*. Berlin, Germany: Springer, 1981.

[5] D. Dubois and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*. New York, NY, USA: Academic Press, 1980.

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Chapter 2

A Neutrosophic Approach to Inventory Modelling under Uncertain Demand

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Abstract

Inventory management plays an important role in ensuring the efficient flow of goods while maintaining a balance between supply and demand. Most classical inventory models assume that parameters such as demand, ordering cost, and holding cost are known precisely. In real-world situations, however, these parameters are often subject to uncertainty and incomplete information due to fluctuating market conditions and estimation errors. As a result, deterministic models may not always capture the complexity of practical inventory systems.

To address this limitation, the present chapter explores an inventory modelling approach based on neutrosophic theory. Neutrosophic representation allows uncertainty to be expressed through three independent components: truth, indeterminacy, and falsity. This framework provides greater flexibility for modelling situations where

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information is not completely certain or may contain indeterminate elements.

In the proposed model, demand is represented in a neutrosophic form while the cost structure follows the conventional inventory framework consisting of ordering and holding costs. A neutrosophic total cost function is formulated and analysed to determine the inventory policy that minimizes the overall cost of the system. The mathematical formulation is supported with a numerical example to demonstrate the practical applicability of the model.

A comparison with the classical deterministic inventory model is also presented to highlight the effect of incorporating neutrosophic parameters. The analysis shows that the neutrosophic approach provides a broader representation of uncertainty while retaining the interpretability of traditional inventory models. The study illustrates how neutrosophic theory can be effectively applied to inventory decision-making problems involving indeterminate information.
Keywords: Neutrosophic set; Inventory management; Inventory model; Uncertainty modeling; Total inventory cost; Optimization.

1. Introduction

Inventory management is a fundamental aspect of supply chain and production systems. Organizations rely on efficient inventory policies to ensure that products are available when needed while avoiding excessive storage costs. Maintaining this balance between supply and demand is often challenging, particularly when demand patterns are uncertain or difficult to predict. For many years, classical inventory models have been used to determine optimal ordering policies by minimizing the total cost associated with inventory operations. These costs typically include ordering costs incurred when replenishing stock and holding costs related to storing items over time.

Traditional inventory models generally assume that key parameters such as demand rate, ordering cost, and holding cost are known with certainty. Although this assumption simplifies the mathematical formulation of the model, it may not accurately represent real-world situations. In practice, demand may vary due to changing customer preferences, seasonal trends, or unexpected market conditions. Similarly, cost parameters may not always remain fixed, and in some situations the available information about these parameters may be incomplete or indeterminate. Because of these factors, decision makers often face difficulties when attempting to apply deterministic inventory models to practical problems.

To overcome these limitations, researchers have introduced different mathematical frameworks that allow uncertainty to be incorporated into inventory models. One such approach is neutrosophic theory, which provides a flexible way to represent uncertain and indeterminate information. In contrast to classical and fuzzy approaches, neutrosophic sets describe information using three independent components: truth, indeterminacy, and falsity. This structure makes it possible to represent situations where the available data is partially known or contains elements of ambiguity.

Motivated by these considerations, this chapter presents an inventory modeling approach that incorporates neutrosophic representation into the traditional inventory framework. In the proposed model, demand is expressed in neutrosophic form while the cost structure retains the conventional components of ordering and holding costs. The resulting neutrosophic total cost function is analysed to

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determine the inventory policy that minimizes the overall system cost. A numerical illustration is provided to demonstrate the applicability of the model, and the results are discussed to show how the neutrosophic approach can assist decision makers in handling inventory problems involving uncertain or indeterminate information.

2. Neutrosophic Concepts

Uncertainty is a common characteristic in many real-world decision-making problems. In inventory management, parameters such as demand, lead time, and cost components may not always be known precisely. Classical inventory models typically assume deterministic values for these parameters, which simplifies the mathematical analysis but may not accurately represent practical situations. To overcome this limitation, different mathematical frameworks have been introduced to incorporate uncertainty into modelling approaches. One such framework is neutrosophic theory.

Neutrosophic theory was introduced by Florentin Smarandache and provides a generalized approach for handling uncertain, indeterminate, and inconsistent information. Unlike classical set theory and fuzzy set theory, neutrosophic sets describe information using three independent membership functions: truth membership, indeterminacy membership, and falsity membership. This structure allows uncertain information to be represented in a more flexible manner. Let Xbe a universe of discourse. A neutrosophic set Adefined on $X$is expressed as

$$A = \{\langle x,\ T_A(x),\ I_A(x),\ F_A(x)\rangle \mid x \in X\}$$

where

$T_A(x)$denotes the **truth membership function**, $I_A(x)$denotes the **indeterminacy membership function**, and

$F_A(x)$denotes the **falsity membership function** of the element $x$with respect to the set A.

These functions represent the degree to which an element belongs to the set, the level of indeterminacy associated with the information, and the degree to which the element does not belong to the set.

The membership functions generally satisfy

$$0 \leq T_A(x),\ I_A(x),\ F_A(x) \leq 1$$

for all $x \in X$.

In practical applications, uncertain quantities are often represented using neutrosophic numbers. A neutrosophic number can be written as

$$\tilde{A} = (T, I, F)$$

where T, I, and Fdenote the truth, indeterminacy, and falsity components respectively. This representation enables uncertain parameters to be modeled more effectively than traditional deterministic values.

In the present study, neutrosophic representation is used to describe uncertainty in demand within the inventory system. By incorporating neutrosophic concepts into the inventory framework, the proposed model is able to capture the presence of indeterminate information and provide a more realistic representation of practical inventory decision-making situation

3. Model Assumptions

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To develop the proposed neutrosophic inventory model, the following assumptions are made in order to define the structure of the inventory system and simplify the mathematical formulation.

1. The inventory system considers a single product and a single warehouse.
2. The planning horizon is assumed to be infinite and the inventory process repeats over identical cycles.
3. The demand rate is uncertain and is represented using a neutrosophic number in order to capture the presence of indeterminacy in demand estimation.
4. The replenishment of inventory is instantaneous, meaning that the ordered quantity arrives immediately when an order is placed.
5. The lead time is assumed to be zero and remains constant throughout the inventory cycle.
6. Shortages are not permitted in the inventory system. The inventory level is replenished before it reaches zero.
7. The ordering cost per order is constant and is denoted by (K).
8. The holding cost per unit per unit time is constant and is denoted by (h).
9. The demand rate remains constant within each inventory cycle, although its value is expressed in neutrosophic form.
10. The deterioration of items is not considered in the present model.

11. The decision variable of the model is the order quantity (Q), which represents the number of units ordered during each replenishment cycle.

12. The objective of the inventory system is to determine the optimal order quantity that minimizes the total inventory cost.

These assumptions provide a simplified structure for incorporating neutrosophic representation into the classical inventory framework. Based on these assumptions, the mathematical model for the neutrosophic inventory system is developed in the next section.

4. Mathematical Model

Based on the above assumptions, the inventory model is developed by incorporating neutrosophic demand into the classical inventory framework. The objective of the model is to determine the optimal order quantity that minimizes the total inventory cost.

Let the demand be represented as a neutrosophic number

$$\widetilde{D} = (T, I, F)$$

where T, I, and F denote the truth, indeterminacy, and falsity membership values respectively.

Since the demand is expressed in neutrosophic form, a representative demand value is obtained for computational purposes as

$$D = \frac{T + I + F}{3}$$

Let the following notations be used:

Q– Order quantity per cycle

K– Ordering cost per order

h– Holding cost per unit per unit time

D– Representative demand value obtained from the neutrosophic

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demand

The ordering cost of the system is given by

$$OC = \frac{DK}{Q}$$

The holding cost is expressed as

$$HC = \frac{Qh}{2}$$

Thus, the total inventory cost is

$$TC(Q) = \frac{DK}{Q} + \frac{Qh}{2}$$

To determine the optimal order quantity, the total cost function is minimized with respect to Q. The optimal order quantity is obtained as

$$Q^* = \sqrt{\frac{2DK}{h}}$$

The value Q^*represents the optimal order quantity that minimizes the total inventory cost under neutrosophic demand conditions.

5. Numerical Example

To illustrate the applicability of the proposed inventory model, a numerical example is considered.

In the neutrosophic framework, demand can be represented using truth, indeterminacy, and falsity components. For simplicity, the neutrosophic demand is converted into a representative demand value for computational purposes. In this numerical illustration, the representative demand values are considered as $D = 60,80,$and 100units per cycle.

Let the ordering cost per order be

K = 30

and the holding cost per unit per unit time be

h = 4.

The optimal order quantity is determined using the EOQ expression

$$Q^* = \sqrt{\frac{2DK}{h}}$$

Substituting the values of $K$and h,

$$Q^* = \sqrt{\frac{2D(30)}{4}} Q^* = \sqrt{15D}$$

Thus, the optimal order quantity depends on the demand level D.

For different demand values, the corresponding optimal order quantities are calculated as follows.

For $D = 60$

$$Q^* = \sqrt{15 \times 60} = \sqrt{900} = 30$$

For $D = 80$

$$Q^* = \sqrt{15 \times 80} = \sqrt{1200} \approx 34.64$$

For $D = 100$

$$Q^* = \sqrt{15 \times 100} = \sqrt{1500} \approx 38.73$$

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Table 1. Optimal Order Quantity for Different Demand Levels

Demand (D)	**Optimal Order Quantity (Q^*)**
60	30.00
80	34.64
100	38.73

Table 1 shows that the optimal order quantity increases as the demand level increases. This result is consistent with inventory theory, where higher demand requires larger replenishment quantities to minimize ordering and holding costs.

6. Results and Discussion

The results obtained from the numerical example indicate that the optimal order quantity increases with the increase in demand level. This behaviour is consistent with classical inventory theory.

The proposed model incorporates neutrosophic representation of demand, which allows uncertainty and indeterminacy in demand estimation to be captured. This provides a more flexible framework compared with purely deterministic models.

The numerical results demonstrate that the model can effectively determine optimal inventory policies even when demand information contains uncertain or indeterminate elements.

7. Conclusion

In this chapter, an inventory model incorporating neutrosophic demand has been presented. The model extends the classical inventory framework by representing demand through truth, indeterminacy, and falsity components, which allows uncertainty to be handled more effectively.

The mathematical model was formulated to determine the optimal order quantity that minimizes the total inventory cost. A numerical example was provided to illustrate the applicability of the proposed model. The results show that the model can determine optimal ordering decisions for different demand levels in a neutrosophic environment.

References

[1] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic*. Rehoboth, USA: American Research Press, 1999.

[2] F. Smarandache, *Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability*. Craiova, Romania: Sitech Education Publishing, 2013.

[3] F. Smarandache and J. Ye, "Interval neutrosophic sets and logic: Theory and applications," *arXiv preprint*, 2005. doi: 10.48550/arXiv.cs/0505014.

[4] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965. doi: 10.1016/S0019-9958(65)90241-X.

[5] F. Smarandache, S. Broumi, and M. Ali, *Neutrosophic Operational Research: Methods and Applications*. Cham, Switzerland: Springer, 2020. doi: 10.1007/978-3-030-57197-9.

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Chapter 3

Entropy-Based Multi-Criteria Decision-Making Approach Using Neutrosophic Soft Set Matrices for Poultry Production Evaluation

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Abstract

Decision-making under uncertainty is a challenging task, especially in complex environments such as agricultural production systems where multiple factors and human judgments influence outcomes. This paper introduces an entropy-based multi-criteria decision-making (MCDM) framework using neutrosophic soft set matrices to evaluate poultry farm performance across different markets. The proposed method integrates two neutrosophic soft set matrices representing expert assessments from distinct marketplaces and applies neutrosophic arithmetic operators to obtain a crisp decision matrix. The entropy method is employed to determine the weights of decision parameters objectively, followed by computation of score values for ranking alternatives. A comprehensive numerical example is presented, involving eight poultry farms and eight criteria representing different domestic fowls. The results demonstrate that the third alternative yields the highest score, indicating superior production performance. This model effectively captures uncertainty, indeterminacy, and inconsistency inherent in

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real-world decision-making, providing a robust and flexible framework for agricultural and industrial evaluations.

Keywords: Entropy method, Multi-criteria decision-making (MCDM), Neutrosophic soft set matrices, Poultry production evaluation, Decision support systems.

1. Introduction

Decision-making problems in real-life scenarios often involve uncertainty, imprecision, and incomplete information. Traditional mathematical tools such as fuzzy sets and intuitionistic fuzzy sets address uncertainty to a certain extent but fail to fully capture the indeterminate and inconsistent information present in complex environments. To overcome these limitations, neutrosophic sets, introduced by Smarandache, provide a more generalized and flexible structure by considering three independent membership components—truth, indeterminacy, and falsity. When combined with soft set theory, neutrosophic soft sets offer a powerful approach to handling multi-criteria decision-making problems where both qualitative and quantitative factors are involved.

In practical domains like poultry production, decision-makers frequently face uncertainty due to fluctuating market demands, diverse environmental conditions, and inconsistent expert opinions. Evaluating multiple poultry farms based on different types of domestic fouls—such as chickens, ducks, geese, and turkeys—requires a mathematical framework capable of managing ambiguity while maintaining decision reliability.

This study proposes a neutrosophic soft set-based decision-making model enhanced with entropy weighting to objectively determine the importance of criteria. The methodology involves constructing two neutrosophic soft set matrices from two expert evaluations corresponding to two distinct markets. These matrices are combined using neutrosophic arithmetic operators to obtain a unified decision matrix, from which entropy-based weights are computed. Subsequently, the score values of alternatives are derived to rank the poultry farms according to their performance.

The proposed approach not only ensures objectivity and accuracy in weight assignment but also effectively deals with uncertainty through neutrosophic representation. The numerical example demonstrates the applicability and efficiency of the method in identifying the best-performing poultry farm among several alternatives.

2. Preliminary

Definition 2.1 (Soft set)

Let$U$be an initial universe set and Ebe a set of parameters. Let $P(U)$ denote the power set of U. Then it $A \subseteq E$, a pair $(F, A)$is called a soft set over U, where$F: A \to P(U)$.

Example 2.1.1: Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$be the universe which are six carpets and$E = \{e_1, e_2, e_3, e_4, e_5, e_6,\}$ be the set of parameters. Here,$e_i(i = 1,2,3,4,5,6)$stands for the

Parameters "*modern*", "*woollen*", "*expensive*", "*cheap*", "*large*" and "*beautiful*" respectively. Then, following soft sets are described respectively

$$f_A = \{(e_1, \{x_1, x_4, x_5\}), (e_2, \{x_1, x_3, x_5, x_6\}), (e_4, \{x_2, x_4, x_6\})\}$$

$$f_B = \{(e_2, \{x_1, x_3, x_5\}), (e_3, X), (e_6, \{x_2, x_4, x_5, x_6\})\}$$

Definition 2.2 (Neutrosophic set)

Let X be a space of points (objects), with a generic element

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in X denoted by$x$.A neutrosophic set Aon the universe of discourse $X$is defined as

$$A = \{x, T_A(x), I_A(X), F_A(X) \,/\, x \in X\}$$

Where $T_A(x), I_A(X), F_A(X): X \to (0,1)and-0 \leq T_A(X) + I_A(X) + F_A(X) \leq 3+$. From the neutrosophic set takes the value from real standard or non-standard subsets of$-]0,1[+$.T_A(Truth),I_A(Indeterminacy),F_A(falsity) are called neutrosophic components.

Example 2.2.1 Let us assume that $X = \{m_1, m_2, m_3\}$ be the characteristics of the product where $m_1$the quality of the product,$m_2$is the price of the product and m_3 is reliability of a product. Also $m_1, m_2, m_3$are in $[0,1]$ and they are obtained from questionnaire of some researchers. The researchers may impose their opinion in three components i.e., the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the product. Suppose A is a neutrosophic set of X such that,

$A = \{(m_1, 0.8,0.6,0.4), (m_2, 0.6,0.3,0.2), (m_3, 0.7,0.6,0.3)\}$ Where, the degree of goodness of quality is 0.8, degree of indeterminacy of quality is 0.6 and degree of falsity of quality is 0.4etc.

Definition 2.3 (Neutrosophic soft set)

Let Ube an initial universe set and Ebe a set of parameters. Let $N(U)$denote the set of all neutrosophic soft set f U. Then for $A \subseteq E$, Then the collection of (F, A) is called an **Neutrosophic soft set** (N_{ss}) over U, where $F: A \to N(U)$.

Example 2.3.1 Let U be an universal set$U = \{m_1, m_2, m_3, m_4, m_5\}$ be five different oil makers and $E = \{e_1, e_2, e_3, e_4,\}$ be a set of parameters representing the different kind of oils like coconut oil, sesame oil, sunflower oil, groundnut oil. Now the expert $E_1$is evaluating the product which one gives more profit respectively,

$$f_N(e_1) = \{< m_1, (0.6,0.4,0.3) >, < m_2(0.9,0.5,0.3) >, < m_3, (0.5,0.6,0.2) >, < m_4, (0.7,0.5,0.4) >, < m_5, (0.5,0.4,0.1) > \}$$

$$f_N(e_2) = \{< m_1, (0.8,0.6,0.7) >, < m_2(0.6,0.4,0.4) >, < m_3, (0.2,0.1,0.1) >, < m_4, (0.6,0.5,0.2) >, < m_5, (0.3,0.2,0.2) > \}$$

$$f_N(e_3) = \{< m_1, (0.3,0.5,0.2) >, < m_2(0.6,0.4,0.8) >, < m_3, (0.8,0.5,0.6) >, < m_4, (0.6,0.5,0.1) >, < m_5, (0.5,0.5,0.1) > \}$$

$$f_N(e_4) = \{< m_1, (0.5,0.5,0.2) >, < m_2(0.6,0.3,0.7) >, < m_3, (0.9,0.5,0.3) >, < m_4, (0.7,0.5,0.4) >, < m_5, (0.8,0.5,0.1) > \}$$

Then $N = \{[e_1, f_N(e_1)], [e_2, f_N(e_2)], [e_3, f_N(e_3)], [e_4, f_N(e_4)],\}$ is an N_{ss}over(U, E)

The tabular representation of the $N_{ss}$of $N$is given in

Table 1: Tabular form of N_{ss}

	$f_N(e_1)$	$f_N(e_2)$	$f_N(e_3)$	$f_N(e_4)$
M_1	$(0.6,0.4,0.3)$	$(0.8,0.6,0.7)$	$(0.3,0.5,0.2)$	$(0.5,0.5,0.2)$
M_2	$(0.9,0.5,0.3)$	$(0.6,0.4,0.4)$	$(0.6,0.4,0.8)$	$(0.6,0.3,0.7)$
M_3	$(0.5,0.6,0.2)$	$(0.2,0.1,0.1)$	$(0.8,0.5,0.6)$	$(0.9,0.5,0.3)$
M_4	$(0.7,0.5,0.4)$	$(0.6,0.5,0.2)$	$(0.6,0.5,0.1)$	$(0.7,0.5,0.4)$
M_5	$(0.5,0.4,0.1)$	$(0.3,0.2,0.2)$	$(0.5,0.5,0.1)$	$(0.8,0.5,0.1)$

Definition 2.4 (Neutrosophic soft set matrix(N_{ssm}))

If$x_{ij} = \left(T_A(u_i, e_j), I_A(u_i, e_j), F_A(u_i, e_j)\right)$, then Neutrosophic Soft Set Matrix (N_{ssm}) of order $m \times n$, is $(x_{ij})_{m\times n} = (x_{ij})_m$

Example 2.4.1.

Let$U = \{u_1, u_2, u_3\}$,$E = \{x_1, x_2, x_3\}$. Nbe a neutrosophic

Soft sets over$U$ neutrosophic as

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$$N = \{(x_1, \{< u_1, (0.7,0.5,0.3) >, < u_2, (0.6,0.5,0.2) >, < u_3, (0.5,0.4,0.2) >\}), (x_2, \{< u_1, (0.8,0.6,0.2) >, < u_2, (0.9,0.5,0.3) >, < u_3, (0.7,0.5,0.4) >\}), (x_3, \{< u_1, (0.2,0.2,0.1) >, < u_2, (0.6,0.6,0.4) >, < u_3, (0.5,0.6,0.8) >\})\}$$

Then, the$N$ –matrix$[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} (0.7,0.5,0.3) & (0.6,0.5,0.2) & (0.5,0.4,0.2) \\ (0.8,0.6,0.2) & (0.9,0.5,0.3) & (0.7,0.5,0.4) \\ (0.2,0.2,0.1) & (0.6,0.6,0.4) & (0.5,0.6,0.8) \end{bmatrix}$$

Definition 2.6 (Value Matrix)

Suppose $C = [T_{ij}{}^c, I_{ij}{}^c, F_{ij}{}^c]$Nssm of order $m \times n$then, $C$is called the value matrix ofN denoted by $V(C)$ and $V(C) = [T_{ij}{}^c + 4I_{ij}{}^c + F_{ij}{}^c] / 6$for all i and j, respectively where $i = 1,2,3 \ldots .m$ and $j = 1,2,3 \ldots .n$

4. Methodology

Step 1: Construct neutrosophic soft set matrices ($A\&B$)

Step 2: Compute C by using the operators of neutrosophic soft set matrix

Step 3: Calculate the value matrices $V(C)$

Step 4: Determine the weight value Using Entrophy method

Step 5: Score value

Step 6: Rank

5. Numerical Example

Let F be a set of neutrosophic soft set of poultry form producing different kind of domestic fowls to satisfy the need of meat and egg for the people throughout the year. Once the items are ready the seller will sell it in the market or directly to the customer’s and get either profit or loss. Let us assume that a set of sellers selling the product in two different marketsM_1andM_2. $E$is a set of parameters. let $A \subseteq E$and. The first $N_{ss}(X, A)$ over $F$is the seller production of meat and egg sold in marketM_1, where $X: A \to P(F)$, The second $N_{ss}(Y, B)$over F is the seller production of meat and egg sold in market M_2, where $Y: A \to P(F)$, $P(F)$ is a set of all neutrosophic soft subset of F. Let a universal set $U = \{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8\}$be aneight different poultry forms and $E = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8\}$ be a parameters representing the different domestic fowls like Chickens, Turkeys, Geese, Ducks, Guinea fowl, Quile, Pigeons, and Ostriches. Two exports $P_1$and P_2 evaluated the production of the seller in the market M_1 and M_2, respectively.

Report of expert P_1 in the market place M_1 is representing in the form of (N_{ss}) as

$$(X, A) = x(C_1), x(C_2), x(C_3), x(C_4), x(C_5), x(C_6), x(C_7), x(C_8)$$

$$(X, A) = x(\mathbb{C}_1), x(\mathbb{C}_2), x(\mathbb{C}_3), x(\mathbb{C}_4), x(\mathbb{C}_5), x(\mathbb{C}_6), x(\mathbb{C}_7), x(\mathbb{C}_8)$$

$$X(\mathbb{C}_1) = \begin{Bmatrix} f_1(0.6,0.3,0.2), f_2(0.2,0.2,0.1), f_3(0.6,0.4,0.3), f_4(0.4,0.3,0.2), f_5(0.5,0.5,0.1), \\ f_6(0.6,0.2,0.2), f_7(0.7,0.3,0.1), f_8(0.8,0.8,0.6) \end{Bmatrix}$$

$$X(\mathbb{C}_2) = \begin{Bmatrix} f_1(0.4,0.2,0.2), f_2(0.6,0.5,0.5), f_3(0.9,0.6,0.3), f_4(0.4,0.1,0.1), f_5(0.7,0.3,0.2), \\ f_6(0.5,0.3,0.3), f_7(0.2,0.1,0.1), f_8(0.7,0.6,0.6) \end{Bmatrix}$$

$$X(\mathbb{C}_3) = \begin{Bmatrix} f_1(0.3,0.3,0.2), f_2(0.9,0.6,0.3), f_3(0.8,0.7,0.6), f_4(0.6,0.4,0.4), f_5(0.7,0.7,0.3), \\ f_6(0.3,0.3,0.1), f_7(0.6,0.5,0.3), f_8(0.7,0.5,0.2) \end{Bmatrix}$$

$$X(\mathbb{C}_4) = \begin{Bmatrix} f_1(0.6,0.5,0.2), f_2(0.7,0.3,0.2), f_3(0.7,0.6,0.6), f_4(0.4,0.3,0.2), f_5(0.4,0.4,0.1), \\ f_6(0.6,0.5,0.3), f_7(0.7,0.7,0.6), f_8(0.2,0.2,0.1) \end{Bmatrix}$$

$$X(\mathbb{C}_5) = \begin{Bmatrix} f_1(0.4,0.4,0.3), f_2(0.6,0.2,0.1), f_3(0.8,0.8,0.6), f_4(0.6,0.3,0.1), f_5(0.7,0.5,0.5), \\ f_6(0.3,0.2,0.1), f_7(0.6,0.4,0.4), f_8(0.3,0.2,0.2) \end{Bmatrix}$$

$$X(\mathbb{C}_6) = \begin{Bmatrix} f_1(0.7,0.3,0.2), f_2(0.4,0.3,0.3), f_3(0.6,0.5,0.5), f_4(0.4,0.3,0.2), f_5(0.7,0.6,0.5), \\ f_6(0.6,0.6,0.3), f_7(0.6,0.5,0.5), f_8(0.8,0.7,0.4) \end{Bmatrix}$$

$$X(\mathbb{C}_7) = \begin{Bmatrix} f_1(0.9,0.3,0.2), f_2(0.6,0.3,0.2), f_3(0.8,0.8,0.5), f_4(0.6,0.5,0.3), f_5(0.4,0.3,0.2), \\ f_6(0.7,0.6,0.6), f_7(0.9,0.3,0.2), f_8(0.6,0.1,0.1) \end{Bmatrix}$$

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$$X(\mathbb{C}_8) = \left\{ \begin{matrix} f_1(0.3,0.1,0.1), f_2(0.8,0.6,0.6), f_3(0.9,0.3,0.2), f_4(0.7,0.7,0.1), f_5(0.3,0.2,0.2), \\ f_6(0.5,0.4,0.3), f_7(0.8,0.3,0.3), f_8(0.6,0.5,0.4) \end{matrix} \right\}$$

Report of expert P_2 in the market place M_2 is representing in the form of (N_{ss}) as

$$(Y, B) = \{y(\mathbb{C}_1), y(\mathbb{C}_2), y(\mathbb{C}_3), y(\mathbb{C}_4), y(\mathbb{C}_5), y(\mathbb{C}_6), y(\mathbb{C}_7), y(\mathbb{C}_8)\}$$

$$Y(\mathbb{C}_1) = \left\{ \begin{matrix} f_1(0.4,0.3,0.2), f_2(0.8,0.6,0.3), f_3(0.8,0.4,0.1), f_4(0.6,0.7,0.4), f_5(0.7,0.7,0.3), \\ f_6(0.8,0.4,0.2), f_7(0.3,0.3,0.1), f_8(0.6,0.4,0.2) \end{matrix} \right\}$$

$$Y(\mathbb{C}_2) = \left\{ \begin{matrix} f_1(0.6,0.4,0.2), f_2(0.8,0.7,0.3), f_3(0.7,0.4,0.1), f_4(0.6,0.3,0.34), f_5(0.7,0.5,0.4), \\ f_6(0.7,0.5,0.3), f_7(0.4,0.3,0.1), f_8(0.5,0.2,0.2) \end{matrix} \right\}$$

$$Y(\mathbb{C}_3) = \left\{ \begin{matrix} f_1(0.7,0.5,0.4), f_2(0.7,0.4,0.3), f_3(0.8,0.5,0.4), f_4(0.6,0.6,0.2), f_5(0.7,0.5,0.3), \\ f_6(0.6,0.3,0.3), f_7(0.8,0.7,0.5), f_8(0.5,0.3,0.2) \end{matrix} \right\}$$

$$Y(\mathbb{C}_4) = \left\{ \begin{matrix} f_1(0.8,0.5,0.4), f_2(0.9,0.5,0.4), f_3(0.7,0.4,0.2), f_4 0.6,0.5,0.1), f_5(0.8,0.4,0.3), \\ f_6(0.6,0.3,0.3), f_7(0.7,0.5,0.2), f_8(0.4,0.4,0.3) \end{matrix} \right\}$$

$$Y(\mathbb{C}_5) = \left\{ \begin{matrix} f_1(0.6,0.4,0.1), f_2(0.8,0.4,0.3), f_3(0.8,0.6,0.4), f_4(0.4,0.3,0.1), f_5(0.7,0.3,0.3), \\ f_6(0.5,0.4,0.1), f_7(0.6,0.2,0.2), f_8(0.5,0.4,0.1) \end{matrix} \right\}$$

$$Y(\mathbb{C}_6) = \left\{ \begin{matrix} f_1(0.6,0.5,0.4), f_2(0.6,0.3,0.3), f_3(0.8,0.7,0.5), f_4(0.5,0.5,0.4), f_5(0.9,0.4,0.3), \\ f_6(0.8,0.6,0.3), f_7(0.4,0.3,0.1), f_8(0.6,0.5,0.4) \end{matrix} \right\}$$

$$Y(\mathbb{C}_7) = \left\{ \begin{matrix} f_1(0.7,0.5,0.4), f_2(0.4,0.3,0.2), f_3(0.6,0.4,0.3), f_4(0.4,0.3,0.1), f_5(0.5,0.3,0.2), \\ f_6(0.9,0.6,0.4), f_7(0.7,0.5,0.2), f_8(0.8,0.3,0.3) \end{matrix} \right\}$$

$$Y(\mathbb{C}_8) = \left\{ \begin{matrix} f_1(0.5,0.3,0.1), f_2(0.8,0.4,0.4), f_3(0.7,0.5,0.4), f_4(0.5,0.5,0.3), f_5(0.4,0.4,0.2), \\ f_6(0.3,0.2,0.1), f_7(0.6,0.5,0.5), f_8((0.4,0.3,0.2) \end{matrix} \right\}$$

The payoff matrices A and $B$are,

$$\begin{array}{cc} & \begin{array}{cccccccc} \mathbb{C}_1 & \mathbb{C}_2 & \mathbb{C}_3 & \mathbb{C}_4 & \mathbb{C}_5 & \mathbb{C}_6 & \mathbb{C}_7 & \mathbb{C}_8 \end{array} \\ A = \begin{array}{c} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{array} & \begin{bmatrix} (0.6, 0.3, 0.2) & (0.4, 0.2, 0.2) & (0.3, 0.3, 0.2) & (0.6,0.5,0.2) & (0.4,0.4,0.3) & (0.7,0.3,0.2) & (0.9,0.3,0.2) & (0.3,0.1,0.1) \\ (0.2, 0.2, 0.1) & (0.6, 0.5, 0.5) & (0.9, 0.6, 0.3) & (0.7,0.3,0.2) & (0.6,0.2,0.1) & (0.4,0.3,0.3) & (0.6,0.3,0.2) & (0.8,0.6,0.6) \\ (0.6, 0.4, 0.3) & (0.9, 0.6, 0.3) & (0.8,0.7,0.6) & (0.7,0.6,0.6) & (0.8,0.8,0.6) & (0.6,0.5,0.5) & (0.8,0.8,0.5) & (0.9,0.3,0.2) \\ (0.4, 0.3, 0.2) & (0.4, 0.1, 0.1) & (0.6,0.4,0.4) & (0.4,0.3,0.2) & (0.6,0.3,0.1) & (0.4,0.3,0.2) & (0.6,0.5,0.3) & (0.7,0.7,0.1) \\ (0.5, 0.5, 0.1) & (0.7, 0.3, 0.2) & (0.7,0.7,0.3) & (0.4,0.4,0.1) & (0.7,0.5,0.5) & (0.7,0.6,0.5) & (0.4,0.3,0.2) & (0.3,0.2,0.2) \\ (0.6, 0.2, 0.2) & (0.5, 0.3, 0.3) & (0.3,0.3,0.1) & (0.6,0.5,0.3) & (0.3,0.2,0.1) & (0.6,0.6,0.3) & (0.7,0.6,0.6) & (0.5,0.4,0.3) \\ (0.7, 0.3, 0.1) & (0.2, 0.1, 0.1) & (0.6,0.5,0.3) & (0.7,0.7,0.6) & (0.6,0.4,0.4) & (0.6,0.5,0.5) & (0.9,0.3,0.2) & (0.8,0.3,0.3) \\ (0.8, 0.8, 0.6) & (0.7, 0.6, 0.6) & (0.7,0.5,0.2) & (0.2,0.2,0.1) & (0.3,0.2,0.2) & (0.8,0.7,0.4) & (0.6,0.1,0.1) & (0.6,0.5,0.4) \end{bmatrix} \end{array}$$

$$\begin{array}{cccccccc} \mathbb{C}_1 & \mathbb{C}_2 & \mathbb{C}_3 & \mathbb{C}_4 & \mathbb{C}_5 & \mathbb{C}_6 & \mathbb{C}_7 & \mathbb{C}_8 \end{array}$$

$$B = \begin{array}{c} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{array} \begin{bmatrix} (0.4,0.3,0.2) & (0.6,0.4,0.2) & (0.7,0.5,0.4) & (0.8,0.5,0.4) & (0.6,0.4,0.1) & (0.6,0.5,0.4) & (0.7,0.5,0.4) & (0.5,0.3,0.1) \\ (0.8,0.6,0.3) & (0.8,0.7,0.3) & (0.7,0.4,0.3) & (0.9,0.5,0.4) & (0.8,0.4,0.3) & (0.6,0.3,0.3) & (0.4,0.3,0.2) & (0.8,0.4,0.4) \\ (0.8,0.4,0.1) & (0.7,0.4,0.1) & (0.8,0.5,0.4) & (0.7,0.4,0.2) & (0.8,0.6,0.4) & (0.8,0.7,0.5) & (0.6,0.4,0.3) & (0.7,0.5,0.4) \\ (0.6,0.7,0.4) & (0.6,0.3,0.3) & (0.6,0.6,0.2) & (0.6,0.5,0.1) & (0.4,0.3,0.1) & (0.5,0.5,0.4) & (0.4,0.3,0.1) & (0.5,0.5,0.3) \\ (0.7,0.7,0.3) & (0.7,0.5,0.4) & (0.7,0.5,0.3) & (0.8,0.4,0.3) & (0.7,0.3,0.3) & (0.9,0.4,0.3) & (0.5,0.3,0.2) & (0.4,0.4,0.2) \\ (0.8,0.4,0.2) & (0.7,0.5,0.3) & (0.6,0.3,0.3) & (0.6,0.3,0.3) & (0.5,0.4,0.1) & (0.8,0.6,0.3) & (0.9,0.6,0.4) & (0.3,0.2,0.1) \\ (0.3,0.3,0.1) & (0.4,0.3,0.1) & (0.8,0.7,0.5) & (0.7,0.5,0.2) & (0.6,0.2,0.2) & (0.4,0.3,0.1) & (0.7,0.5,0.2) & (0.6,0.5,0.5) \\ (0.6,0.4,0.2) & (0.5,0.2,0.2) & (0.5,0.3,0.2) & (0.4,0.4,0.3) & (0.5,0.4,0.1) & (0.6,0.5,0.4) & (0.4,0.3,0.3) & (0.4,0.3,0.2) \end{bmatrix}$$

To find Crisp value by using Neutrosophic Arithmetic Operator (NA)

$$C = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} \mathbb{C}_1 & \mathbb{C}_2 & \mathbb{C}_3 & \mathbb{C}_4 & \mathbb{C}_5 & \mathbb{C}_6 & \mathbb{C}_7 & \mathbb{C}_8 \end{array} \\ \begin{bmatrix} (0.5, 0.3, 0.2) & (0.5, 0.3, 0.2) & (0.5, 0.4, 0.3) & (0.7, 0.5, 0.3) & (0.5, 0.4, 0.2) & (0.65, 0.4, 0.3) & (0.8, 0.4, 0.3) & (0.4, 0.2, 0.1) \\ (0.5, 0.4, 0.2) & (0.7, 0.6, 0.4) & (0.8, 0.5, 0.3) & (0.8, 0.4, 0.3) & (0.7, 0.3, 0.2) & (0.5, 0.3, 0.3) & (0.5, 0.3, 0.2) & (0.8, 0.5, 0.5) \\ (0.7, 0.4, 0.2) & (0.8, 0.5, 0.2) & (0.8, 0.6, 0.5) & (0.7, 0.5, 0.4) & (0.8, 0.7, 0.5) & (0.7, 0.6, 0.5) & (0.7, 0.6, 0.4) & (0.8, 0.4, 0.3) \\ (0.5, 0.5, 0.3) & (0.5, 0.2, 0.2) & (0.6, 0.5, 0.3) & (0.5, 0.4, 0.15) & (0.5, 0.3, 0.1) & (0.45, 0.4, 0.3) & (0.5, 0.4, 0.2) & (0.6, 0.6, 0.2) \\ (0.6, 0.6, 0.2) & (0.7, 0.4, 0.3) & (0.7, 0.6, 0.3) & (0.6, 0.4, 0.2) & (0.7, 0.4, 0.4) & (0.8, 0.5, 0.4) & (0.45, 0.3, 0.2) & (0.3, 0.3, 0.2) \\ (0.7, 0.3, 0.2) & (0.6, 0.4, 0.3) & (0.45, 0.3, 0.2) & (0.6, 0.4, 0.3) & (0.4, 0.3, 0.1) & (0.7, 0.6, 0.3) & (0.8, 0.6, 0.5) & (0.4, 0.3, 0.2) \\ (0.5, 0.3, 0.1) & (0.3, 0.2, 0.1) & (0.7, 0.6, 0.4) & (0.7, 0.6, 0.4) & (0.6, 0.3, 0.3) & (0.5, 0.4, 0.3) & (0.8, 0.4, 0.2) & (0.7, 0.4, 0.4) \\ (0.7, 0.6, 0.4) & (0.6, 0.4, 0.4) & (0.6, 0.4, 0.2) & (0.3, 0.3, 0.2) & (0.4, 0.3, 0.15) & (0.7, 0.6, 0.4) & (0.5, 0.2, 0.2) & (0.5, 0.4, 0.3) \end{bmatrix} \end{array}$$

$$V(C_{ij}) = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} \mathbb{C}_1 & \mathbb{C}_2 & \mathbb{C}_3 & \mathbb{C}_4 & \mathbb{C}_5 & \mathbb{C}_6 & \mathbb{C}_7 & \mathbb{C}_8 \end{array} \\ \begin{bmatrix} 0.32 & 0.32 & 0.40 & 0.50 & 0.38 & 0.42 & 0.45 & 0.22 \\ 0.38 & 0.58 & 0.52 & 0.45 & 0.35 & 0.33 & 0.32 & 0.55 \\ 0.42 & 0.50 & 0.62 & 0.52 & 0.68 & 0.60 & 0.58 & 0.45 \\ 0.47 & 0.25 & 0.48 & 0.37 & 0.30 & 0.39 & 0.38 & 0.53 \\ 0.53 & 0.43 & 0.57 & 0.40 & 0.45 & 0.53 & 0.31 & 0.29 \\ 0.35 & 0.42 & 0.31 & 0.42 & 0.28 & 0.57 & 0.61 & 0.30 \\ 0.30 & 0.20 & 0.58 & 0.58 & 0.35 & 0.40 & 0.43 & 0.45 \\ 0.58 & 0.43 & 0.40 & 0.28 & 0.29 & 0.58 & 0.25 & 0.40 \end{bmatrix} \end{array}$$

Step 1: Compute the proportion matrix V_{ij}

For each criterion j

$$P_{ij} = \frac{V_{ij}}{\sum_{i=1}^{m} V_{ij}}$$

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$$P_{ij} = \begin{array}{c} \mathbf{f_1} \\ \mathbf{f_2} \\ \mathbf{f_3} \\ \mathbf{f_4} \\ \mathbf{f_5} \\ \mathbf{f_6} \\ \mathbf{f_7} \\ \mathbf{f_8} \end{array} \begin{bmatrix} 0.095 & 0.101 & 0.103 & 0.142 & 0.38 & 0.425 & 0.45 & 0.22 \\ 0.114 & 0.186 & 0.133 & 0.127 & 0.35 & 0.33 & 0.32 & 0.55 \\ 0.124 & 0.159 & 0.159 & 0.146 & 0.68 & 0.6 & 0.58 & 0.45 \\ 0.139 & 0.080 & 0.125 & 0.106 & 0.3 & 0.39 & 0.38 & 0.53 \\ 0.159 & 0.138 & 0.146 & 0.113 & 0.45 & 0.53 & 0.31 & 0.29 \\ 0.104 & 0.133 & 0.079 & 0.118 & 0.28 & 0.57 & 0.61 & 0.3 \\ 0.089 & 0.064 & 0.150 & 0.165 & 0.35 & 0.4 & 0.43 & 0.45 \\ 0.174 & 0.138 & 0.103 & 0.080 & 0.29 & 0.58 & 0.25 & 0.4 \end{bmatrix}$$

That is, divide each entry in a column by the sum of its column.

Step 2: Compute the entropy constant K

$$K = \frac{1}{ln\ m}$$

Here $m = number\ of\ alternatives.$

$$K = \frac{1}{ln\ 8} = -0.4809$$

Step 3: Compute the entropy e_j

$$e_j = -k \sum_{i=1}^{m} P_{ij} ln\, P_{ij}$$

$$e_1 = -\frac{1}{ln\ (8)} [0.095 * ln\ (0.095)\ ]$$

$$e_1 = 0.987706$$

Similarly,

$$e_2 = 0.9764, e_3 = 0.9893, e_4 = 0.9903, e_5 = 0.9782, e_6 = 0.9902, e_7 = 0.9795, e_8 = 0.9801$$

Step 4: Compute divergence and entropy weight of the attributes

From entropy, compute divergence d_j and Weight is W_j

$$d_j = 1 - e_j$$

$$W_j = \frac{d_j}{\sum_{j=1}^{n} d_j}$$

W_1 W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8
0.0959	0.1838	0.0834	0.0758	0.1697	0.0766	0.1597	0.1551

Step 5: Rank the score value of alternatives

$$S_i = Score\ value\ of\ alternative\ f_i = S(R_i)$$

$$S(R_i) = \sum_{i,j=1}^{n} V_{ij} * W_j$$

$$S_1 = Score\ value\ of\ alternative f_1 = S(R_1)$$

$$S_1 = (0.32 * 0.0959) + (0.32 * 0.1838) + (0.4 * 0.0834) + (0.5 * 0.0756) + (0.38 * 0.1697) + (0.43 * 0.0766) + (0.45 * 0.1597) + (0.22 * 0.1551)$$

$$S_1 = (0.0301 + 0.0588 + 0.0334 + 0.0378 + 0.0645 + 0.0326 + 0.0718 + 0.0341)$$

$$S_1 = 0.36376$$

$S_1 = Score\ value\ of\ alternative f_1 = S(R_1)$=0.36376

Similarly,

$$S_2 = Score\ value\ of\ alternative\ f_2 = S(R_2) = 0.44161$$

$$S_3 = Score\ value\ of\ alternative\ f_3 = S(R_3) = 0.54708$$

$$S_4 = Score\ value\ of\ alternative\ f_4 = S(R_4) = 0.38316$$

$$S_5 = Score\ value\ of\ alternative\ f_5 = S(R_5) = 0.41918$$

$$S_6 = Score\ value\ of\ alternative\ f_6 = S(R_6) = 0.40357$$

$$S_7 = Score\ value\ of\ alternative\ f_7 = S(R_7) = 0.38632$$

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$$S_8 = Score\ value\ of\ alternative\ f_8 = \ S(R_8) = 0.38489$$

Since $S_3 > S_2 > S_5 > S_6 > S_7 > S_8 > S_4 > S_1,\ the\ third\ alternative\ f_3\, is\ best$

6. Comparison Table

RANK	Traditional method	Proposed method
Best Alternative	f_3	f_3

Here, we have got the same result.

7. Conclusion

In this study, an entropy-based multi-criteria decision-making (MCDM) model founded on the neutrosophic soft set framework was proposed and applied to evaluate poultry production systems. The model effectively handles uncertainty, indeterminacy, and inconsistency in expert opinions, which are common in real-world decision-making. By combining two neutrosophic soft set matrices from distinct markets and applying neutrosophic arithmetic operators, a unified decision matrix was obtained to represent diverse expert perspectives. The entropy weighting method was employed to objectively determine the importance of each criterion, thereby reducing subjective bias. The subsequent ranking process revealed that the third alternative exhibited the highest score, signifying optimal poultry production performance. This result demonstrates the reliability and applicability of the proposed model in agricultural evaluation and other similar decision environments. Overall, the integration of neutrosophic soft set theory with entropy-based weighting provides a robust, flexible, and systematic approach for analysing multi-criteria problems under uncertain conditions. Future work may extend this framework to dynamic environments, large-scale decision systems, or hybrid models combining other uncertainty theories such as fuzzy rough sets or intuitionistic fuzzy sets to enhance performance and adaptability.

References

[1] Ali Aydogdu & Ridvan Sahin 2019, 'New Entropy Measures Based on Neutrosophic Set and Their Applications to Multi Criteria Decision Making', Sьleyman Demirel University Journal of Natural and Applied Sciences, vol. 23, no. 1, pp. 40-45.

[2] Animesh Biswas & Arnab Kumar de 2016, 'An efficient ranking technique for intuitionistic fuzzy numbers with its application in chance constrained bilevel programming', Hindawi Publishing Corporation, Advances in Fuzzy Systems, vol.2016, pp. 1-12.

[3] Barik SK & Biswal MP 2012, 'Probabilistic quadratic programming problems with some fuzzy parameters', Advances in Operations Research, Hindawi Publishing Corporation, vol. 2012, pp. 1-12.

[4] Chen, Y & Li, B 2011, 'Dynamic multi attribute decision making model based on triangular intuitionistic fuzzy numbers', Scientia Iranica B, vol. 18, no. 2, pp. 268-274.

[5] Cheng Pf, Li Dp, He Jq, Zhou Xh, Wang Jq & Zhang Hy 2020, 'Evaluating Surgical Risk Using FMEA and MULTIMOORA Methods under a Single-Valued Trapezoidal Neutrosophic Environment'. Risk Management and Healthcare Policy, vol. 13, no.7, pp.865-881.

[6] Elshabshery, AA & Fattouh, M 2021, 'On Some Information Measures of Single-Valued Neutrosophic Sets and their Application in MCDM Problems', International Journal of Engineering Research & Technology (IJERT) , vol. 10, no. 5, pp. 406-415.

ISBN 978-816855385-9

[7] Hazwani Hashim, Lazim Abdullah, Ashraf Al-Quran & Azzah Awang 2021, 'Entropy Measures for Interval Neutrosophic Vague Sets and Their Application in Decision Making', Neutrosophic Sets and Systems, vol. 45, no. 7, pp. 74-95.

[8] Jun Ye 2014,'A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets', Journal of Intelligent & Fuzzy Systems, vol.26, no. 5, pp. 2459–2466.

[9] Jun Ye 2021, 'Entropy measures of simplified neutrosophic sets and their decision-making approach with positive and negative arguments', Journal of Management Analytics, vol. 8, no.2, pp. 252-266.

[10] Mahmood, T & Ali, Z 2022,' Prioritized Muirhead Mean Aggregation Operators under the Complex Single-Valued Neutrosophic Settings and Their Application in Multi-Attribute Decision-Making', Journal of Computational and Cognitive Engineering, vol.1, no.2, pp. 56–73.

[11] Majdoleen Abu Qamar & Nasruddin Hassan 2018, 'Entropy, Measures of Distance and Similarity of Q-Neutrosophic Soft Sets and Some Applications', Entropy, vol. 20, no.9, p.672.

[12] Memet Şahin, Abdullah Kargın & Florentin Smarandache 2018, 'Generalized A Triangular Neutrosophic Numbers and Aggregation Operators for Application to Multi-attribute Group Decision Making', New Trends in Neutrosophic Theory and Applications. Vol. II, pp.51-84.

[13] Nital P Nirmal & Mangal G Bhatt 2016, 'Selection of Automated Guided Vehicle using A Neutrosophic Entropy Based Novel Multi Attribute Decision Making Technique', New Trends in Neutrosophic Theory and Applications, pp.105-114.

[14] Pinaki Majumdar & Syamal Kumar Samanta 2014, 'On Similarity and entropy of neutrosophic sets', Journal of Intelligent and Fuzzy Systems, vol. 26, no. 3, pp. 1245-1252.

[15] Pranab Biswas, Surapati Pramanik & Bibhas C Giri 2014, 'A New Methodology for Neutrosophic Multi-attribute Decision making with Unknown Weight Information', Neutrosophic Sets and Systems, vol. 3, no. 1, pp. 42-50.

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Chapter 4

Bivariate Neutrosophic Fuzzy Solid Transportation Problem

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Abstract

Many real-world situations have an element of uncertainty, which is often addressed using fuzzy techniques. This study describes various transportation issues like transportation cost, transportation time, and conveyance capacity. The bivariate neutrosophic fuzzy solid transportation problem is an extension of the neutrosophic solid transportation problem. The bivariate neutrosophic solid fuzzy solid transportation problem consists of three constraints and two objective functions. Two objective functions Cost and time are combined by a single term (order pair), "bivariate neutrosophic fuzzy number," and conveyance capacity, supply and demand are crisp numbers. A bivariate neutrosophic fuzzy number consists of two parts (cost, time) and neutrosophic membership. Optimum allocation is made using the average of neutrosophic confidence, and optimum cost is calculated using the weighted cost-time score function, and it is solved using the row column reduction method, and it is compared with the standard method.

Keywords: Neutrosophic set, Bivariate neutrosophic set, Bivariate neutrosophic solid transportation problem.

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1. Introduction

The foundational work on the transportation problem was pioneered by Hitchcock (1941). In industrial and logistical contexts, transportation remains a critical challenge. This problem is framed as a specialized subset of linear programming, where the core objective is to minimize the cost of shipping goods. The classical transportation model serves as a decision-making tool, guiding the companies on the optimal quantity of a commodity to dispatch from various sources to various destinations by using available logistics services.

Nowadays various logistics services are available with various capacities. Uncertainty exists in selecting the perfect logistics service. This new conveyance constraint included in the classical transportation problem converted it into a solid transportation problem. Neutrosophic set, proposed by Smarandache in 1999, It is an extension of fuzzy set by incorporating three degrees of membership: truth (T), indeterminacy (I), and falsity (F). It is very useful in multi-criteria decision-making problems. The bivariate neutrosophic fuzzy solid transportation problem includes two criteria, cost and time, which are both very important for buyers. Cost and time play important roles in transportation problems. Optimum cost and time lead to customer satisfaction and business efficiency.

1.1 Research gap

Standard solution techniques for the STP may produce results that are infeasible with respect to the third-dimension constraints (conveyance). At this situation, the Row Column Reduction and MODI algorithms are applied to refine these solutions and find the true optimal, feasible answer for the entire three-dimensional problem.

2. Preliminaries

2.1 Fuzzy sets

Let X be a universe of discourse. A fuzzy set $\tilde{A}$ of X is defined by a membership function $f_{\tilde{A}}: X \to [0,1]$ where $f_{\tilde{A}}(x)$ is called the membership function and is represented as

$\tilde{A} = \{(x, f_{\tilde{A}}(x)/ x\epsilon X\}$

2.2 Normal

A fuzzy set $\tilde{A}$ defined on the universe X is said to be normal iff $Sup f_{\tilde{A}}(x) = 1, x \in X$

2.3 Convex

A fuzzy set $\tilde{A}$ defined on the universe set X is said to be convex iff $f_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq min(f_{\tilde{A}}(x), f_{\tilde{A}}(y), \forall x, y \in X \text{ and } \lambda \epsilon [0,1]$

2.4 Fuzzy number

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R must satisfy the following conditions.

(i) $f_{\tilde{A}}(x)$ is piecewise continuous

(ii) There exist at least one $x\epsilon R$ with $f_{\tilde{A}}(x) = 1$

(iii) $\tilde{A}$ must be normal and convex

2.5 Triangular fuzzy number

A fuzzy number $\tilde{A} = (a,b,c)$ is called triangular fuzzy number if its membership function is given by $\mu_{\tilde{A}}(x) = \left\{\{0 \quad x < a, x > c \ \frac{x-a}{b-a} \ a \leq x \leq b \ \frac{c-x}{c-b} \ b \leq x \leq c \right\}$

2.6 Intuitionistic fuzzy set

Let X be nonempty set an intuitionistic fuzzy set A on X is of the form $A = \{(x, \mu_A(x), v_A(x);\ x\epsilon X\}$ where $\mu_A(x), v_A(x) : U \to [0,1]$ and $\mu_A(x)$ is the degree of membership, $v_A(x)$ is the degree of non-membership for every element

$x\epsilon X, 0 \leq \mu_A(x) + v_A(x) \leq 1$

2.7 Neutrosophic fuzzy set

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A single valued neutrosophic set A is characterized by truth-membership function $T_A(x)$ an indeterminacy-membership function $I_A(x)$ and a falsity membership $F_A(x)$ for each $x \in X$

For each $x \in X$ $T_A(x)$, $I_A(x)$, $F_A(x) \in [0,1]$ A single valued neutrosophic set A can be written as A={<x, $T_A(x)$, $I_A(x)$, $F_A(x)$> x ∈X}

2.8 Bivariate Neutrosophic fuzzy set

A Bivariate neutrosophic fuzzy set A is characterized by truth-membership function $T_A(x,y)$ an indeterminacy-membership function $I_A(x, y)$ and a falsity membership $F_A(x,y)$ for each $(x, y) \in XxY$, For each $(x, y) \in XxY$ $T_A(x, y)$, $I_A(x, y)$, $F_A(x, y) \in [0,1]$ A bivariate neutrosophic fuzzy set A can be written as $\breve{A}$={((x, y), $T_A(x, y)$, $I_A(x, y)$, $F_A(x, y)$)/ (x, y) ∈XxY}

2.9 Bivariate Neutrosophic fuzzy set in transportation problem

A bivariate neutrosophic fuzzy set in transportation problem is characterized by truth membership function $T_A(c_{ijk},\ t_{ijk})$ and indeterminacy- membership function $I_A(c_{ijk},\ t_{ijk})$ and a falsity membership $F_A(c_{ijk}, t_{ijk})$ for each $(c_{ijk}, t_{ijk}) \in CxT$, $T_A(c_{ijk}, t_{ijk})$, $I_A(c_{ijk}, t_{ijk})$, $F_A(c_{ijk}, t_{ijk}) \in [0,1]$ A bivariate neutrosophic fuzzy set can be written as

$\breve{A} = \{(c_{ijk}, t_{ijk}) \in CxT\}$

2.10 Symbols and notations

c_{ijk} –
$Cost\ of\ transporting\ goods\ to\ j^{th}\ destination\ from\ i^{th}\ source\ by\ k^{th}\ conveyance$

x_{ijk} – –Units of goods transporting to j^{th} destination from i^{th} source by k^{th} Conveyance

t_{ijk} – – Time duration of transporting goods to j^{th} destination from i^{th} source by k^{th} conveyance.

$$O_i \ -- Origin\ i$$

$D_j -- Destination\ j$

$E_k -- Conveyance\ k$

$n(E_k) -- capacity\ of\ conveyance\ k$

S_i −−*Number of units available at the source i*

D_j −−*Number of units required by the destination j*

3. Structure of the problem

3.1 Objective function

$$Minimize\ Z = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{t} (\lambda\, c_{ijk} + (1-\lambda)t_{ijk})x_{ijk}\ , \lambda \epsilon [0,1]$$

3.2 Constraints

$\sum_{J=1}^{m} \sum_{k=1}^{t} x_{ijk} = S_i$ i=1to n

(Supply constraint)

$\sum_{i=1}^{n} \sum_{k=1}^{t} x_{ijk} = D_j$ j=1tom

(Demand constraint)

$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{t} x_{ijk} = \sum_{k=1}^{t} n(E_k)$

(Conveyance constraint)

$\sum_{i=1}^{n} S_i = \sum_{j=1}^{m} D_j = \sum_{k=1}^{t} E_K$

4. Proposed Algorithm

This section provides working procedure for Bivariate neutrosophic solid transportation problem. Consider the bivariate solid transportation problem.

The problem must be balanced before starting the allocation

Step 1. Verification

Check that Total supply = Total demand = sum of conveyance capacity If the above condition is not met convert it into balanced problem by adding dummy Source, destination, conveyance depending upon the nature of the problem and then Move to next step.

Step 2. Bivariate Neutrosophic number and membership conversion

Bivariate neutrosophic conversion

Bivariate neutrosophic number convert into single neutrosophic number by using the relation $CT_{ijk} = \lambda\, c_{ijk} + (1-\lambda)t_{ijk}$

Membership conversion

And membership values into crisp value by using

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$$M_{ijk} = \frac{T_{ijk}+(1-F_{ijk})}{2} - I_{ijk}$$

Step 3. Table construction

Construct the table using these new values CT_{ijk}, M_{ijk} express these values in top and bottom in each corresponding cell (M_{ijk} in the top of the and CT_{ijk} in the bottom of the cell).

Table 1: Structure of the table

Destination	X_1			X_2			X_3			Supply
Source\conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	M_{111} CT_{111}	M_{112} CT_{112}	M_{113} CT_{113}	M_{121} CT_{111}	M_{122} CT_{112}	M_{123} CT_{113}	M_{131} CT_{131}	M_{132} CT_{132}	M_{133} CT_{133}	S_1
A_2	M_{211} CT_{211}	M_{212} CT_{212}	M_{213} CT_{213}	M_{221} CT_{221}	M_{222} CT_{222}	M_{223} CT_{223}	M_{231} CT_{231}	M_{232} CT_{232}	M_{233} CT_{233}	S_2
A_3	M_{311} CT_{311}	M_{312} CT_{312}	M_{313} CT_{313}	M_{321} CT_{321}	M_{322} CT_{322}	M_{323} CT_{323}	M_{331} CT_{331}	M_{332} CT_{332}	M_{333} CT_{333}	S_3
Demand	D_1			D_2			D_3			

The above table shows the crisp values of membership and the resource factors (time and cost). The final crisp conversion is again obtained by taking their average, using

$$MCT_{ijk} = \frac{M_{ijk} + CT_{ijk}}{2}$$

The final Membership cost time crisp table is presented below.

Destination	X_1			X_2			X_3			Supply
Source\ conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	MCT_{111}	MCT_{112}	MCT_{113}	MCT_{121}	MCT_{122}	MCT_{123}	MCT_{131}	MCT_{132}	MCT_{133}	S_1
A_2	MCT_{211}	MCT_{212}	MCT_{213}	MCT_{221}	MCT_{222}	MCT_{223}	MCT_{231}	MCT_{232}	MCT_{233}	S_2
A_3	MCT_{311}	MCT_{312}	MCT_{313}	MCT_{321}	MCT_{322}	MCT_{323}	MCT_{331}	MCT_{332}	MCT_{333}	S_3
Demand	D_1			D_2			D_3			

Step 4. Allocation procedure

Step i)

Select the membership cost time crisp table and then Subtract each entry of the row by its least value and write the difference in top of the corresponding cost and subtract each entry of the column by its least value and place them in the bottom of the corresponding cost.

Step ii)

Select the maximum value in supply/Demand, choose the corresponding row / column if there is a tie, choose arbitrarily.

Step iii)

Choose the minimum value (sum of row and column reduction value) in the selected row/column. Allocate the minimum value of supply/demand to the selected cell.

Step iv)

Eliminate the row/column corresponding to the supply or demand which is satisfied.

Step v)

Continue the process from step ii) to step v) until the supply demand are met.

Step 5. Form a table with transportation cost and allocated units. calculate the transportation cost using the formula $\sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij}\, x_{ij}$.

Step 6. Apply Modi method to membership cost time crisp table and then find optimal cost

5. Proposed method

5. Problem

Consider the bivariate neutrosophic solid transportation problem with three origins, A_1, A_2, A_3 three destinations X_1, X_2, X_3 and three conveyance E_1, E_2, E_3 Find the optimal cost by considering cost, time and

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neutrosophic membership function with conveyance capacity $E_1 = 68$, $E_2 = 74$, $E_3 = 78$ for the following data.

Destination	X_1			X_2			X_3			Supply
Source\ conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	{(41.6,14.6), 0.8,0.1,0.2}	{(31.6,16), 0.6,0.2,0.3}	{(55,13.6), 0.9,0.1,0.4}	{(65,19), 0.5,0.3,0.2}	{(75,18), 0.6,0.2,0.3}	{(85,17), 0.7,0.1,0.3}	{(48.3,15.3), 0.9,0.2,0.3}	{(41.6,18), 0.7,0.2,0.4}	{(75,14.6), 0.8,0.3,0.2}	72
A_2	{(48.3,17.3), 0.5,0.2,0.4}	{(58.3,16.3), 0.6,0.3,0.3}	{(68.3,15.3), 0.7,0.1,0.3}	{(55,16), 0.7,0.1,0.2}	{(38.3,17), 0.6,0.2,0.3}	{(65,14.3), 0.8,0.2,0.2}	{(58.3,14.3), 0.9,0.1,0.2}	{(58.3,14.3), 0.6,0.2,0.3}	{(51.6,15.6), 0.7,0.3,0.2}	73
A_3	{(75,16), 0.6,0.1,0.2}	{(65,17), 0.5,0.3,0.3}	{(50,18), 0.4,0.2,0.4}	{(48.3,19) ,0.4,0.2,0.4}	{(58.3,16.3), 0.5,0.3,0.3}	{(85,15.3), 0.6,0.2,0.3}	{(81.6,15.6), 0.7,0.2,0.4}	{(63.3,16.6), 0.6,0.1,0.3}	{(81.6,15.6), 0.6,0.3,0.2}	75
Demand	76			74			70			

Solution

Proposed method

Step 1

Verify the problem is balanced

Total supply=72+73+75

= 220

Total demand =76+74+70

= 220

Sum of conveyance capacity= 68+74+78

= 220

Therefore, Total supply=Total demand= sum of conveyance capacity

Step 2

Convert the bivariate neutrosophic solid transportation problem into standard solid transportation problem with membership values in each cell using

$$CT_{ijk} = \lambda\, c_{ijk} + (1 - \lambda)t_{ijk}$$

$$M_{ijk} = \frac{T_{ijk} + (1 - F_{ijk})}{2} - I_{ijk}$$

$$MCT_{ijk} = \frac{M_{ijk}+CT_{ijk}}{2}.$$

Step 3 Table construction

Destination	X_1			X_2			X_3			Supply
Source\conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	0.7 28.1	0.45 23.8	0.65 34.3	0.35 42	0.45 46.5	0.6 51	0.6 31.8	0.45 29.8	0.5 44.8	72
A_2	0.35 32.8	0.35 37.3	0.6 41.6	0.65 35.5	0.45 27.65	0.6 39.65	0.75 36.3	0.45 36.3	0.45 33.6	73
A_3	0.6 45.5	0.3 41	0.3 34	0.3 33.65	0.3 39.8	0.45 50.15	0.45 48.6	0.55 39.1	0.4 48.6	75
Demand	76			74			70			

Final crisp table

Destination	X_1			X_2			X_3			Supply
Source\conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	14.4	12.12	17.47	21.17	23.47	25.8	16.2	15.12	22.65	72
A_2	16.57	18.82	21.2	18.07	14.05	20.12	18.52	18.37	17.02	73
A_3	23.05	20.65	17.17	16.97	20.05	25.3	24.52	19.82	24.5	75
Demand	76			74			70			

Step 4 Initial and optimal Allocation

Row column reduced matrix

Destination	X_1			X_2			X_3			Supply
Source\ conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	$14.4_{2.28}^{2.28}$	12.12_{0}^{0}	$17.47_{5.35}^{5.35}$	$21.17_{7.12}^{9.05}$	$23.47_{9.42}^{11.35}$	$25.8_{11.75}^{13.68}$	$16.2_{1.08}^{4.08}$	15.12_{0}^{3}	$22.65_{7.53}^{10.53}$	72
A_2	$16.57_{4.45}^{2.52}$	$18.82_{6.7}^{4.77}$	$21.2_{9.08}^{7.15}$	$18.07_{4.02}^{4.02}$	14.05_{0}^{0}	$20.12_{6.07}^{6.07}$	$18.52_{3.4}^{4.47}$	$18.37_{3.25}^{4.32}$	$17.02_{1.9}^{2.97}$	73
A_3	$23.05_{10.93}^{6.08}$	$20.65_{8.53}^{3.68}$	$17.17_{5.05}^{0.2}$	$16.97_{2.92}^{0}$	$20.05_{6}^{3.08}$	$25.3_{11.25}^{8.33}$	$24.52_{9.4}^{7.55}$	$19.82_{4.7}^{2.85}$	$24.5_{9.38}^{7.53}$	75
Demand	76			74			70			

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Destination	X_1			X_2			X_3			Supply
Source\ conveyance	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	
A_1	14.4	12.12 $\boxed{72}$	17.47	21.17	23.47	25.8	16.2	15.12	22.65	72
A_2	16.57	18.82	21.2	18.07	14.05 $\boxed{2}$	20.12 $\boxed{1}$	18.52	18.37	17.02 $\boxed{70}$	73
A_3	23.05	20.65	17.17 $\boxed{4}$	16.97 $\boxed{68}$	20.05	25.3 $\boxed{3}$	24.52	19.82	24.5	75
Demand	76			74			70			

Allocated table

Destination	x_1		x_2			x_3	Supply
Source\ conveyance	E_2	E_3	E_1	E_2	E_3	E_3	
A_1	12.12 $\boxed{72}$	17.47	21.17	23.47	25.8	22.65	72
A_2	18.82	21.2	18.07	14.05 $\boxed{2}$	20.12 $\boxed{1}$	17.02 $\boxed{70}$	73
A_3	20.65	17.17 $\boxed{4}$	16.97 $\boxed{68}$	20.05	25.3 $\boxed{3}$	24.5	75

Transportation cost

Destination	x_1		x_2			x_3	Supply
Source\ conveyance	E_2	E_3	E_1	E_2	E_3	E_3	
A_1	31.6 $\boxed{72}$	55	65	75	85	75	72
A_2	58.3	68.3	55	38.3 $\boxed{2}$	65 $\boxed{1}$	51.6 $\boxed{70}$	73
A_3	65	50 $\boxed{4}$	48.3 $\boxed{68}$	58.3	85 $\boxed{3}$	81.6	75
Demand	76		74			70	

Initial transportation cost = 31.6 x 72 + 50x4+68x48.3+38.3x2+65x1+85x3+51.6x70

= 9768.2

Apply modification method

Optimal transportation cost = 9768.2

6. Comparison table

S.No	Vogels	Modified method	Proposed method
1	10,105.2	Solution violates the capacity of conveyance	9768.2

7. Conclusion

This study focuses on determining the optimal cost and time while satisfying all three constraints in the fewest possible steps. However, in the standard method, the modification applied to handle negative opportunity costs results in a violation of the transportation capacity constraints.

References

1. Balasubramanian, Dr. Subramanian.S, "An approach for solving fuzzy transportation problem", International journal of pure and applied mathematics Vol.119, No.17, PP.1523-1532, 2018.
2. Darunee Hunwisai, Poom Kumam, "A method for solving a fuzzy transportation Problem via Robust ranking technique and ATM", Applied and interdisciplinary mathematics Article:1283730, DOI:10.1080/23311835.2017.1283730.
3. K.B Haley, "Solid transportation problem" Operation research, Vol.10 No.4, PP448- 463,1962
4. Malini and Dr Ananthanarayanan, "Solving fuzzy transportation problem using ranking of trapezoidal fuzzy number", International journal of mathematics research, Vol.8, No.2, PP.127-132, 2016.

ISBN 978-816855385-9

5. Iden hassan Alkanani and Farrah Alaa adnan, "Ranking function method for solving linear programming problem", Mathematical theory and modelling, Vol.4, No.4, PP.65-72,2014
6. J.Jeyanthi, S.Sandhiya and R.Nagendran "Row column reduction method for solving trapezoidal fuzzy transportation problem", Advances in nonlinear variational inequalities, Vol.28, No1s, PP.265-275,2025
7. J.Jeyanthi, S.Sandhiya 'A Multi-objective dynamic neutrosophic solid transportation model under IOT -Driven uncertainty", International journal of applied mathematics Vol.38,No.4s PP.706-721,2025.
8. Karyati et al, "Yager ranking method for solving the trapezoidal fuzzy number linear programming", Journal of physics ,2018, DOI:10.1088/1742-6596/983/1/012135.
9. Lavanya P, "Various fuzzy number and Their various ranking approaches", International journal of Advanced Research in Engineering and technology, Vol. 8, Issue.5, PP.73-82,2017.
10. A. Rezaei , T. Oner, T. Katican, F. Smarandache and N. Gandotra," A short history of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic sets", International journal of neutrosophic and sicence, Vol.18, No.1, PP 99-116, 2022
11. Shanthi Pandiyan, Anuradha Danapal ,"A new approach for solving solid transportation problem", Applied mathematical sciences, Vol.4 No.72 PP.3603-3610,2010.
12. E.Shell, Distribution of a product by several properties directorate of managements analysis, proc.2nd symp. On linear programming, Vol. 2, PP.615-642, 1955.

13. F. Smarandache, Neutrosophic Set, A Generalization of the Intuitionistic Fuzzy Set, International Journal of Pure and Applied Mathematics Vol.24, No.3 PP. 287-297,2005
14. Subadhra Srinivas , k. prabakaran, "A Role of Triangular Fuzzy Neutrosophic Numbers in Solving Neutrosophic Transportation Problem", Vol.53, No.4, 2023.
15. Zadeh L. A., Fuzzy sets, Information Control, Vol.8, PP. 338353,1965,https://doi.org/10.1016/S0019-9958(65)90241.

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Chapter 5

An Analysis Between Multi-Criteria Decision-Making Methods Through An Application Under Pythagorean Fuzzy Hypersoft Sets

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Abstract

Decision-making in complex environments frequently involves uncertainty, vagueness, and incomplete information. Traditional multi-criteria decision-making (MCDM) techniques often struggle to represent such uncertainty effectively. Pythagorean Fuzzy Hypersoft Sets (PFHSS) provide a powerful mathematical framework that integrates the advantages of Pythagorean fuzzy sets and hypersoft sets to handle multi-attribute decision problems involving sub-attributes. This chapter presents a comparative analysis of several well-known MCDM methods—PROMETHEE I, PROMETHEE II, ELECTRE, TOPSIS, EDAS, and WASPAS—within the PFHSS environment. A real-world inspired numerical example involving mobile phone selection based on compatibility with a telecommunication network is used to demonstrate the application of the proposed framework. The results obtained from the various methods are compared and analyzed using correlation coefficients to evaluate consistency among ranking results. The study demonstrates that PFHSS provides a flexible and effective structure for modeling complex decision problems with hierarchical

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parameters. The comparative analysis also highlights the robustness of the proposed framework across multiple decision-making methodologies.

Keywords: Pythagorean fuzzy hypersoft set (PFHSS), Hypersoft set (HSS), Electre, Topsis, Promethee I, Promethee II, EDAS, WASPAS.

1. Introduction

Decision-making plays a crucial role in numerous domains including engineering, economics, management, healthcare, and telecommunications. In practical situations, decision makers are often required to evaluate multiple alternatives based on several criteria that may conflict with each other. Such problems are commonly referred to as Multi-Criteria Decision-Making (MCDM) problems. However, real-world decision processes are rarely precise. Data involved in decision environments often contain uncertainty, vagueness, and incomplete information. Classical mathematical tools based on probability theory are sometimes insufficient for handling these uncertainties because they require precise statistical data. To address these limitations, Zadeh introduced the concept of **Fuzzy Sets** in 1965, which allowed elements to have degrees of membership rather than binary belonging. Fuzzy sets became a fundamental tool for representing uncertain information in decision-making. Later, Atanassov extended fuzzy sets to **Intuitionistic Fuzzy Sets (IFS)** by incorporating both membership and non-membership functions. While intuitionistic fuzzy sets improved the representation of uncertainty, they imposed the restriction that the sum of membership and non-membership degrees must be less than or equal to one.

To overcome this limitation, **Yager proposed Pythagorean Fuzzy Sets (PFS)**, which relax the constraint and allow the sum of squared membership and non-membership degrees to be less than or equal to one. This modification provides greater flexibility for modeling uncertainty.

In parallel, **Soft Set theory** and later **Hypersoft Set theory** were introduced to handle parameterized information. Hypersoft sets extend soft sets by allowing parameters to contain multiple sub-attributes, making them particularly suitable for complex decision problems.

The combination of Pythagorean fuzzy sets with hypersoft sets results in **Pythagorean Fuzzy Hypersoft Sets (PFHSS)**, which provide a powerful framework for handling multi-criteria decision-making problems involving hierarchical attributes.

1.1 Motivation

Modern decision-making environments frequently involve:

- multiple criteria
- hierarchical attributes
- uncertain data
- incomplete knowledge

Traditional MCDM methods do not adequately capture these complexities. Therefore, integrating PFHSS with classical MCDM methods provides a promising direction for improving decision accuracy.

1.2 Contributions of the Chapter

The main contributions of this chapter include:

1. Introducing the PFHSS framework for complex decision problems.
2. Integrating PFHSS with several well-known MCDM techniques.
3. Conducting a comparative analysis of ranking results obtained using different methods.
4. Evaluating the consistency of results using correlation coefficients.

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5. Demonstrating the proposed framework using a practical case study.

2. Literature Review

2.1 Fuzzy Set Theory in Decision Making

The concept of fuzzy sets was introduced by Zadeh (1965) to represent uncertainty in complex systems. Since then, fuzzy logic has been widely applied in engineering, economics, and artificial intelligence. Atanassov (1986) proposed intuitionistic fuzzy sets, which incorporate both membership and non-membership functions. Later extensions include interval-valued intuitionistic fuzzy sets and linguistic fuzzy sets.

2.2 Pythagorean Fuzzy Sets

Yager introduced Pythagorean fuzzy sets to overcome the limitations of intuitionistic fuzzy sets. In PFS, the square sum of membership and non-membership degrees must be less than or equal to one. PFS have been widely used in decision-making applications including supplier selection, medical diagnosis, and engineering design.

2.3 Hypersoft Sets

Hypersoft sets extend soft sets by allowing parameters to be decomposed into sub-parameters. This extension enables more detailed modeling of complex systems.

2.4 MCDM Methods

Several MCDM techniques have been proposed for decision analysis:

- PROMETHEE
- ELECTRE
- TOPSIS
- EDAS
- WASPAS

These methods differ in how they evaluate alternatives and rank them according to criteria.

3. Preliminaries

Definition 1: A linguistic variable is a method for expressing variables using words rather than numerical values. It comprises a collection of terms, referred to as linguistic variable terms, such as '*Low*', '*Medium*', *and* '*High*'. These terms find application in practical industry scenarios. For instance, a linguistic variable representing processing time (T) might be defined as $T[processing\ time] = [low,\ medium,\ high]$

Table 1. Linguistic values

Linguistic Value	Numerical Value
Low	[0.0-0.351]
Medium	[0.352-0.726]
High	[0.728-1.0]

Definition 2: The fuzzy set $\tilde{A}$ in X presents an assembly of ordered pairs if X denotes a family of elements indicated generically with x: The function $\tilde{A} = \{(x \in X)\}$ is labeled as the membership function where

$$\mu_{\tilde{A}}(X) = \{1\ if\ x \in X\ 0\ if\ x \notin X\ \}$$

Definition 3: Let E be every parameter set, X be the universal set, and $P(X)$ the power set of X. The discussion of a soft set $(\varphi,\ E)$ *on* X is

$$(\varphi, E) = \{(e, \varphi(e)) : e \in E, \varphi(E) \in P(X)\}$$

$$here\ \varphi : E \to P(X)$$

Definition 4: Assume that X is a generic set and $P(X)$ is $X's$ power set. For $n \geq 1$, let $e_1, e_2, e_3, \ldots, e_n$ *be* n clearly defined attributes, whose equivalent sub-attributes of e_i are accordingly, the sets

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$E_1, E_2, E_3, \ldots, E_n$ with $E_i \cap E_j = \emptyset$ for $i \neq j$ and $i, j \in \{1, 2, 3, \ldots, n\}$. Subsequently, this combination $(\varphi, E_1 \times E_2 \times \ldots \times E_n)$ is denoted a hypersoft set over X where $\varphi: E_1 \times E_2 \times \ldots \times E_n \rightarrow P(X)$.

Definition 5: Consider X to represent the universal, $FP(X)$ the family of all fuzzy subsets over X and $E_1, E_2, E_3, \ldots, E_n$ the pairwise disjoint collection of parameters. For each $i = 1, 2, 3 \ldots n$ consider A_i to be the non-empty subset of E_i. A fuzzy HSS is represented by the pair $(\varphi, A_1 \times A_2 \times \ldots \times A_n)$ in this case $\varphi: A_1 \times A_2 \times \ldots \times A_n \rightarrow FP(X)$.

Definition 6: Here X is a set that includes all values. The array of ordered pairings over X has the following definition of a PFS A:

$$A = \left\{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in X\right\}$$

Here $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$, describe the MD & NMD consequently, of the element $x \in X$ to A, that is a subset of X, and for each $x \in X$, the following is true: $0 \leq \left(\mu_{\tilde{A}}(x)\right)^2 + \left(\nu_{\tilde{A}}(x)\right)^2 \leq 1$.

Example 1: Suppose that $\mu_{\tilde{A}}(x) = 0.8$, $\nu_{\tilde{A}}(x) = 0.5$, clearly $0.8 + 0.5 \nleq 1$, but we have $0.8^2 + 0.5^2 \leq 1$. Thus, $\pi_{\tilde{A}}(x) = 0.3317$, and hence from this, we can see that $(\mu_{\tilde{A}}(x))^2 + (\nu_{\tilde{A}}(x))^2 + (\pi_{\tilde{A}}(x))^2 = 1$.

Definition 7: Suppose X to be a universal set, E be every parameter set and φ be "a mapping such as $\varphi: E \rightarrow PF(X)$, where $PF(X)$ indicates a set of PFSs. Then (φ, E) is called a PFSS over X is outlined as:

$$(\varphi, E) = \left\{\left(e, \frac{x}{\mu_{\tilde{A}}(x)\nu_{\tilde{A}}(x)}\right): e \in E,\ x \in X\right\}$$

Raw materials	*Parameters*
$m_1 = wood$	$e_1 = water$
$m_2 = fabric$	$e_2 = temperature$
$m_3 = screw$	$e_3 = moisture$
$m_4 = leather$	$e_4 = light$
$m_5 = rope$	$e_5 = hygiene$
	$e_6 = time$

Here $\mu_{\tilde{A}}(x): X \to [0, 1]$, $\nu_{\tilde{A}}(x): X \to [0, 1]$ with the constraints $0 \leq \left(\mu_{\tilde{A}}(x)\right)^2 + (\nu_{\tilde{A}}(x))^2 \geq 1$.

Example 2: Define a universal set X and the parameters set E as follows:

$$X = \left\{m_1, m_2, m_3, m_4, m_5\right\} \, and$$

$E = \left\{e_1, e_2, e_3, e_4, e_5, e_6\right\}$ where

$m_i \, (i \in \{1, 2, 3, 4, 5\}) \; e_j \, (j \in \{1, 2, 3, 4, 5, 6\})$ are exhibited in the table given below:

Table 2. Raw materials and Parameters

The Pythagorean fuzzy soft sets (φ, E) defines the raw materials impacted by parameters. Suppose the function $\varphi: E \to PF(X)$ be described as

$$\varphi\left(e_1\right) = X, \; \varphi\left(e_2\right) = \left\{m_2, m_4\right\}, \; \varphi\left(e_3\right) = \left\{m_1, m_3\right\}, \varphi\left(e_4\right) = \left\{m_2, m_4\right\},$$
$$\varphi\left(e_5\right) = \left\{m_1, m_2, m_4\right\}, \; \varphi\left(e_6\right) = \emptyset$$

Then a Pythagorean fuzzy soft set could be written as follows:

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$$(\varphi, E) = \left\{\left(e_1, \left\{\frac{m_1}{0.8,0.2}, \frac{m_2}{0.7,0.4}, \frac{m_3}{0.6,0.5}, \frac{m_4}{0.3,0.7}, \frac{m_5}{0.1,0.6}\right\}\right), \left(e_2, \left\{\frac{m_1}{0.1,0.2}, \frac{m_2}{0.3,0.9}, \frac{m_3}{0.3,0.4}, \frac{m_4}{0.4,0.5}, \frac{m_5}{0.5,0.1}\right\}\right), \left(\right.\right.$$

Definition 8: With X as the universal set and $P(x)$ the power set of X. Consider $e_1, e_2, e_3, \ldots, e_n$ $for\ n \geq 1$ to be the set of attributes and set E_i be the set of corresponding sub-attributes of e_i, $respectively, with\ E_i \cap E_j = \emptyset\ for\ i \neq j\ and\ i, j \in \{1, 2, 3, \ldots n\}.\ Assume\ E_1 \times E_2 \times E_3 \times \ldots E_n =$ is a group of sub-attributes and $PF(X)$ is a set of all Pythagorean fuzzy subsets over X. Then the pair$(\varphi,\ E_1 \times E_2 \times E_3 \times \ldots \times E_n = S)$ is considered to be the PFHSS over X, and its mapping is described as

$$\varphi: E_1 \times E_2 \times \ldots \times E_n = S \rightarrow PFS(X)$$

Example 3: Consider the universe of discourse $X = \{x_1, x_2, x_3\}$ be the teaching faculties and $\{e_1 = Lesson\ plan,\ e_2 = Subjects,\ e_3 = Classes\}$ be a collection of attributes with the following corresponding attribute values:

$$Lesson\ Plan = E_1 = \{a_{11} = group\ discussion,\ a_{12} = project\ modelling\},$$

$$Subjects = E_2 = \{a_{21} = Mathematics,\ a_{22} = Physics,\ a_{23} = Computer\ Science\}$$

$$and\ Classes = E_3 = \{a_{31} = Bachelor's\ degree,\ a_{32} = Master'sdegree\}.$$

$$Let\ \hat{A} = E_1 \times E_2 \times E_3\ be\ a\ set\ of\ attributes$$

$$\hat{A} = E_1 \times E_2 \times E_3 = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}, a_{23}\} \times \{a_{31}, a_{32}\}$$

$$= \{(a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), (a_{11}, a_{23}, a_{31}), (a_{11}, a_{23}, a_{32}) (a_{12},$$

where

$$\{\hat{a}_1 = (a_{11}, a_{21}, a_{32}), \hat{a}_2 = (a_{11}, a_{21}, a_{32}), \hat{a}_3 = (a_{11}, a_{22}, a_{31}), \hat{a}_4 = (a_{11}, a_{22}, a_{32}), \hat{a}_5 = (a_{11}, a_{23},$$

$$\hat{A} = \{\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4, \hat{a}_5, \hat{a}_5, \hat{a}_6, \hat{a}_7, \hat{a}_8, \hat{a}_9, \hat{a}_{10}, \hat{a}_{11}, \hat{a}_{12}\}$$

then the Pythagorean fuzzy hypersoft sets is given by:

$(\varphi, A) = \{(\hat{a}_1, (x_1, (0.7, 0.1)), (x_2, (0, 0.8)), (x_3, (0.7, 0.1))); (\hat{a}_2, (x_1, (0.2, 0.4)), (x_2, (0.6, 0.3$

4. Several Types of Mcdm Methods

A. Promethee I And II (Preference Ranking Organization Method for Enrichment Evaluation) This approach is a well-known decision support system" which copes with the selection and evaluation of several choices according to a range of standards to rank the variables. J. Brans and Vincke discovered this technique in 1982, they were the first to introduce PROMETHEE I (partial ranking) as well as PROMETHEE II (full ranking)

B. Edas (Evaluation Based on Distance from Average Solution) Mehdi Keshavarz Ghorabaee was the first person to introduce this method in 2015. This method is one of the best outranking methods in comparison with TOPSIS and VIKOR which are the other MCDM methods. Here in the present method, it is not necessary to compute the ideal solution instead the distance average is calculated.

C. Electre (Elimination and ET Choice Translating Reality)

The ELECTRE approach is a notable outranking model that is broadly applicable for comparing performances across a range of factors for various MCDM issues. The Engineering and management departments have already started using ELECTRE as a decision-making tool. Benayoun et al.'s ELECTRE techniques, which were first developed, and then rooted out to more ELECTRE methods are a popular family of outranking techniques for MCDM issues, which have seen a lot of positive and beneficial developments.

D. Topsis (Method for Order of Preference By Similarity to Optimal Solution)

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This methodology was created by Hwang and Yoon to address decision-making issues. The TOPSIS approach makes it simple to determine the shortest distance to an optimum solution that supports choosing the best option. Numerous scholars used the TOPSIS method for decision-making after it was developed and expanded this strategy to all environments of fuzzy.

E. WASPAS (WEIGHTED AGGREGATED SUM PRODUCT ASSESSMENT)

This was introduced by Zavadskas, Turkis, Antucheviciene, and Zakarevicius in the year 2012. The WASPAS technique is a combination of these models, the WSM and WPM in a singular way. It is now generally recognized as a viable tool for decision-making owing to its mathematical simplicity and ability to give more precise findings than WPM and WSM techniques.

5. Numerical Example

In the numerical example the problem is to analyze the best mobile phone with the connection '**Telezone**'. Suppose that Q_1, Q_2, Q_3, Q_4 are the buyers, marketers, tech reviewers, and merchants. These individuals will determine which mobile device works best with the '**Telezone**' connection. A link called Telezone is provided on American cell phones. Every cell provider makes enticing deals, discounts, and offers of unlimited data consumption to sell more mobile devices. The optimal cellphone and connection will therefore be determined by these four decision-makers. Let

$$M = \{TCL\,(M_1), Google\,(M_2), Motorola(M_3),\ Kyocera(M_4),\ Samsung(M_5), Nokia(M_6), Appl$$

$$c_1 = Promotions,\ c_2 = subscription,$$

$$c_3 = connectivity,\ c_4 = data,\ c_5 = bonus,$$

$$c_6 = accessories,\ c_7 = special\ discount,$$

$$c_8 = festival\ offers$$

$$c_1 = \{l_{11} = buy\ one\ get\ one\ free,\ l_{12} = exchange\ offer\};\ c_2 = \{l_{21} = over\ the\ top\ l_{22} = or$$

$$c_3 = \{l_{31} = 5G,\ l_{32} = 4G\};\ c_4 = \{l_{41} = unlimited\ data,\ \ l_{42} = unlimited\ calls\};$$

$$c_5 = \{l_{51} = referral\ (or)\ refer\ a\ friend\};\ c_6 = \{l_{61} = phone\ case,\ \ l_{62} = head\ phones,\ \ l_{63} =$$

$$c_7 = \{l_{71} = govt\ employees,\ \ l_{72} = social\ service,\ l_{73} = military\ discount\};$$

$$c_8 = \{l_{81} = black\ Friday,\ \ l_{82} = cyber\ Monday,\ \ l_{83} = holiday\ season\}$$

*A. PROMETHEE I AND II***:** Using aggregate preference function $\pi(a, b)$ we get the results for Promethee II and a small modification in the same algorithm gives the result for PROMETHEE I. Here PROMETHEE II gives the exact ranking of the alternatives.

Table 3. PROMETHEE II Ranking

	f_{i+}	f_{i-}	$f_i(a)$	Rank
M_1	0.202	0.171	0.031	5
M_2	0.226	0.123	0.103	3
M_3	0.048	0.518	-0.469	8
M_4	0.102	0.354	-0.252	7
M_5	0.307	0.137	0.169	2
M_6	0.202	0.174	0.028	6
M_7	0.386	0.074	0.312	1
M_8	0.198	0.120	0.078	4

	F_{i+}	F_{i-}
M_1	1.547	1.028
M_2	1.545	0.823
M_3	0.324	3.609
M_4	0.714	2.475
M_5	1.9	1.02
M_6	1.378	1.181
M_7	2.727	0.544
M_8	1.374	0.829

PROMETHEE I is computed using the idea that a higher exit value indicates a better option and a lower entering flow value indicates a better alternative. As a result, it is clear from the outputs below that M_7 is the best option.

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Table 4. PROMETHEE I Ranking

B. EDAS: By applying the Positive Distance Average and negative Distance Average, and using the appraisal score formula we formulate the ranking of the alternatives.

Table 5. EDAS Ranking

	SP_i	SN_i	NSP_i	NSN_i	As_i	**Rank**
M_1	4.496	16.161	0.142	0.996	0.569	5
M_2	6.806	5.395	0.215	0.993	0.604	3
M_3	-1.858	31.897	-0.059	1.002	0.472	7
M_4	-4.209	21.376	-0.133	1.004	0.436	8
M_5	25.942	9.394	0.819	0.974	0.897	2
M_6	1.945	14.645	0.061	0.998	0.530	6
M_7	31.664	0.679	1.000	0.969	0.984	1
M_8	5.602	2.183	0.177	0.994	0.586	4

C. ELECTRE: Determining the concordance and discordance interval matrix and also finding the concordance index matrix, discordance index matrix the ranking of the alternatives is done.

Table 6. ELECTRE Ranking

	Net Inferior value	**RANK**		**Net Superior value**	**RANK**
M_1	-0.033	5	M_1	1.584	4
M_2	-1.264	4	M_2	-0.108	6
M_3	5.783	8	M_3	-4.804	8
M_4	3.549	7	M_4	-3.34	7

M_5	-2.194	2	M_5	1.776	3
M_6	1.191	6	M_6	0.122	5
M_7	-5.249	1	M_7	2.88	1
M_8	-1.783	3	M_8	1.89	2

	S_{i+}	S_{i-}	D_i	**Rank**
M_1	0..266	0.262	0.496	6
M_2	0.230	0.244	0.515	4
M_3	0.418	0.063	0.131	8
M_4	0.326	0.152	0.318	7
M_5	0.139	0.367	0.725	2
M_6	0.240	0.248	0.508	5
M_7	0.097	0.377	0.796	1
M_8	0.184	0.273	0.598	3

D.TOPSIS: By finding the best and worst alternative and then calculating the similarity worst condition formula the ranking of alternatives is done.

Table 7. TOPSIS Ranking

E. WASPAS: The WSM and WPM are two distinct that are combined in the WASPAS technique. It was presented by Zavadskas, Turkis, Antucheviciene, and Zakarevicius (2012).

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Table 8. WASPAS Ranking

It is now commonly recognized as an efficient tool due to its ease of mathematics in decision-making and capacity to produce more precise findings than WSM and WPM approaches.

Table 9. Comparison Between the MCDM methods

	Electre	Edas	Waspas	Topsis	Promethee2
M_1	4	5	8	6	5
M_2	6	3	2	4	3
M_3	8	7	6	8	8
M_4	7	8	4	7	7
M_5	3	2	7	2	2
M_6	5	6	3	5	6
M_7	1	1	1	1	1
M_8	2	4	5	3	4

	Preference of WSM	Preference of WPM	Preference of WASPAS	Rank of WASPAS
M_1	0.362	-0.5481	-0.093	8
M_2	0.297	0.4081	0.353	2
M_3	-0.772	0.6985	-0.037	6
M_4	-0.347	0.5640	0.109	4
M_5	0.402	-0.5759	-0.087	7
M_6	0.182	0.4329	0.307	3
M_7	0.611	0.5536	0.582	1
M_8	0.355	-0.2746	0.040	5

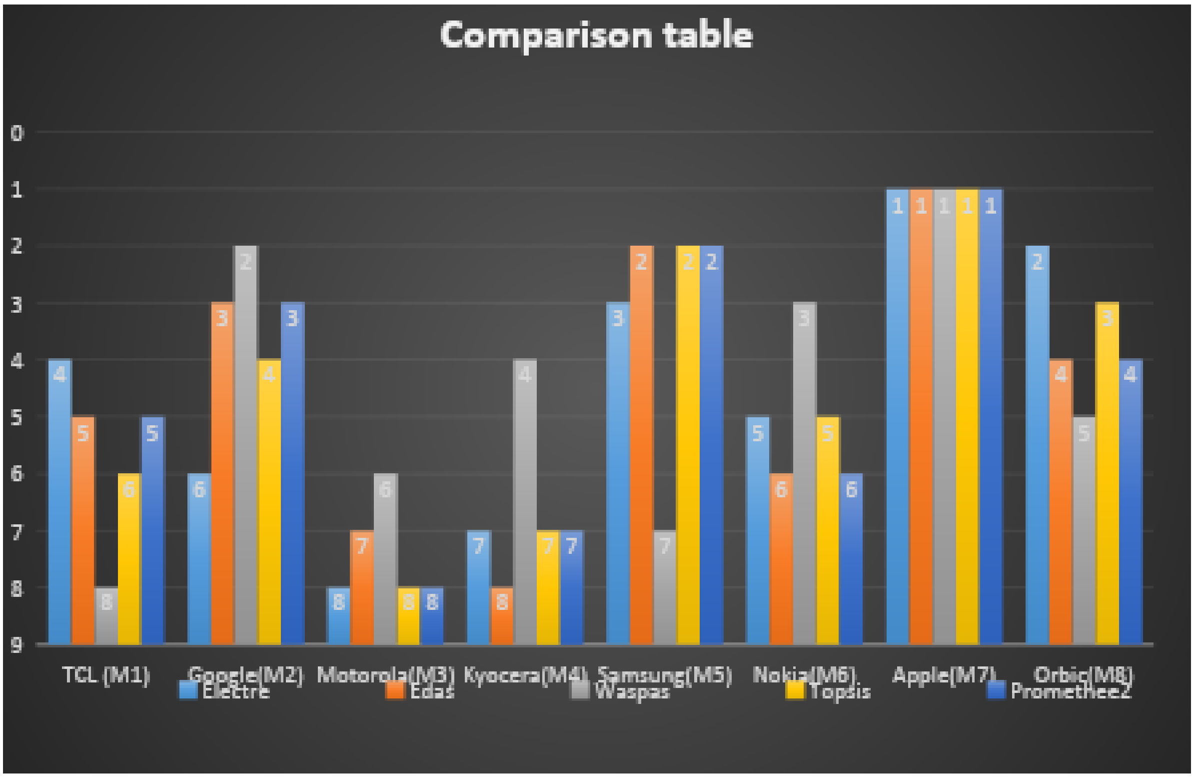

Figure. 1: Comparison table

Table 10. Correlation Coefficient between the MCDM methods

	Electre	Edas	Waspas	Topsis	Promethee 2
Electre	1.0	0.9	0.1	0.9	0.9
Edas	0.9	1.0	0.3	0.9	1.0
Waspas	0.1	0.3	1.0	0.4	0.3
Topsis	0.9	0.9	0.4	1.0	1.0
Prom 2	0.9	1.0	0.3	1.0	1.0

6. Result

To measure similarity between ranking results, correlation coefficients are calculated.

The analysis shows high correlation between:

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- ELECTRE and TOPSIS
- EDAS and PROMETHEE
- TOPSIS and PROMETHEE

This indicates that the PFHSS framework produces stable decision results.

7. Discussion

The results demonstrate that integrating PFHSS with MCDM methods offers several advantages:

- better representation of uncertainty
- handling of hierarchical parameters
- consistent ranking results

Compared to traditional fuzzy models, PFHSS allows decision makers to incorporate multiple attribute levels, improving modeling accuracy.

8. Conclusion In this paper a comparative analysis of several MCDM techniques within the framework of Pythagorean Fuzzy Hypersoft Sets. The PFHSS model was integrated with PROMETHEE, EDAS, ELECTRE, TOPSIS, and WASPAS methods to evaluate alternatives in a multi-criteria environment. A numerical example involving mobile phone selection demonstrated the effectiveness of the proposed approach. The results showed strong consistency across different decision-making methods, confirming the reliability of the PFHSS framework.

References

[1] Biswas, and S. Kumar, "An Integrated TOPSIS Approach to MADM with Interval-Valued Intuitionistic Fuzzy Settings, Advanced Computational and Communication Paradigms", Advances in Intelligent Systems and Computing, Springer, Singapore. vol. 706. April 2018. https://doi.org/10.1007/978-981-10-8237-5_52

[2] A.S. Costa, K. Govindan, and J.R. Figueira, ``Supplier classification in emerging economies using the ELECTRE TRI-nC method: A case study considering sustainability aspects,'' J. Cleaner Prod., vol. 201, pp. 925-947, Nov. 2018. https://doi.org/10.1016/j.jclepro.2018.07.285

[3] Aulia Ishak, Asfriyati, Vina Akmaliah., "Analytical Hierarchy Process and PROMETHEE as Decision Making Tool: A Review" IOP Conf. Series: Materials Science and Engineering vol. 505, 2019. doi:10.1088/1757-899X/505/1/012085

[4] C.T. Chen, "Extensions of the TOPSIS for group decision making under fuzzy environment," Fuzzy Sets and Systems, vol. 114, no. 1, pp. 1–9, 2000. https://doi.org/10.1016/S0165-0114(97)00377-1

[5] Cengiz Kahraman., Irem Otay., "Fuzzy multicriteria using Neutrosophic Sets", Springer international publishing, 2019. 10.1007/978-3-030-00045-5

[6] H. Garg, and G. Kaur, "Algorithm for solving the decision-making problems based on correlation coefficients under cubic intutionistic fuzzy in, formation: a case study in watershed hydrological system". Complex Intell. Syst. vol. 8, pp. 179-198, 2022. https://doi.org/10.1007/s40747-021-00339-4.

[7] H. Garg, H and K. Kumar, "Linguistic interval-valued Atanassov intuitionistic fuzzy sets and their applications to group decision-making problems," IEEE Transactions on Fuzzy Systems, vol. 27, no. 12, pp. 2302–2311, 2019. 10.1109/TFUZZ.2019.2897961

[8] K. Atanassov, "Intuitionistic fuzzy sets. Fuzzy sets and systems", vol. 20, no.1, pp. 87-96, 1986. http://dx.doi.org/10.1016/S0165-0114(86)80034-3

ISBN 978-816855385-9

[9] K. Atanassov, and G. Gargov, "Interval-valued intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 31, no. 3, pp. 343–349, 1989. https://doi.org/10.1016/0165-0114(89)90205-4

[10] M.K. Ghorabaee, E.K. Zavadskas, L. Olfat, Z. Turskis, "Multi-Criteria Inventory Classification Using a New Method of Evaluation Based on Distance from Average Solution (EDAS)", Informatica vol. 26, no. 3, pp. 435 – 451, **2015**. DOI:10.15388/Informatica.2015.57

[11] S. Sorooshian, N.A. Azizan, N. Ale Ebrahim, "Weighted aggregated sum product assessment", Mathematical Modelling of Engineering Problems, vol.9, no.4, pp.873-878, 2022. https://doi.org/10.18280/mmep.090403

Chapter 6

Product Root Sum Mean labeling on Subdivision Graphs

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Abstract

Let G=(V, E) be a simple graph with p vertices and q edges an injective function f:V→{1,2,3,…,q+1} is said to be a Product Root Sum Mean Labeling if the induced function f* defined on edges by $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$ yields different values. In this paper we prove the Product Root Sum Mean Labeling on Subdivision Graphs such as Subdivision graph of Triangular Snake graph, Quadrilateral Snake graph, Triangular Ladder graph, Total Comb graph and Middle Comb graph.

Keywords: Subdivision Graphs, Triangular Snake graph, Quadrilateral Snake graph, Triangular Ladder graph, Total Comb graph, Middle Comb graph, Product Root Sum Mean Labeling.

1. Introduction

In 1967, Rosa came up with the concept of graph labeling [2]. Assigning integers to the vertices, edges, or both, under specific circumstances, is known as graph labeling. Many graph labeling techniques, including graceful, anti-magic, harmonious, magic, odd

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graceful, prime, bi-magic labeling, square sum labeling, difference labeling, mean labeling, even mean labeling, odd mean labeling, Root Square Mean Labeling, Root Square Even Mean labeling, Root Square Odd Mean labeling, and Product Root Sum Mean Labeling, have been examined in more than 2500 papers over the years [3]. A mean labeling in G (V, E) is an 1-1 function $f: v \rightarrow \{0, 1, \ldots, p\}$ such that $\frac{1}{2}[f(u) + f(v)]$, if $f(u) + f(v)$ is even and $\frac{1}{2}[f(u) + f(v) + 1]$ if $f(u) + f(v)$ is odd, is distinct for any edge [5]. The Root Square Odd and Even Mean Labeling, Product Root Sum Mean Labeling was introduced [9], [10]. Let G = (u,v) be a graph an 1-1 function $f: v \rightarrow \{1, \ldots, e + 1\}$ is PRSML, if the function f^* defined by $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$ yield different values. In this research paper we propose the above said labeling Subdivision Graphs such as Triangular Snake Graph, Quadrilateral Snake Graph, Triangular Ladder Graph, Total Comb Graph, and Middle Comb Graph.

2. Main Results

Theorem 2.1: Subdivision graph of Triangular Snake Graph TS_m is a Product Root Sum Mean Label Graph.

Proof:

Let V = {$v_1, v_2, v_3\ldots\ldots, v_{n-1}, v_n$; $u_1, u_2, u_3,\ldots u_{n+1}$; $w_1, w_2, w_3\ldots w_n$; $x_1, x_2, x_3,\ldots x_n$; $y_1,y_2,y_3\ldots\ldots y_n$} be the vertex set and the edge set be E={ v_iu_i, $1 \le i \le n$; $u_{i+1}v_i$, $1 \le i \le n$; u_iw_i, $1 \le i \le n$; w_iy_i , $1 \le i \le n$; $u_{i+1}x_i$ $1 \le i \le n$ } defined for all values of n. Define a map f: V → {1, 2, 3……q+1} such that the vertex labels are $f(u_i) = 4i - 2$; $1 \le i \le n+1$; $f(v_i) = 6i - 3$; $1 \le i \le n$; $f(w_i) = 6i - 1$; $1 \le i \le n$; $f(x_i) = 6i + 1$; $1 \le i \le n$; $f(y_i) = 4i$; $1 \le i \le n$. Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. Thus, the edge labels are distinct. Hence the theorem.

Theorem 2.2: The subdivision graph of Triangular Ladder Graph TL_n is a Product Root Sum Mean Graph.

Proof:

Let V = {u_1, u_2, u_3,…,u_{n-1}, u_n, u_{n+1};v_1, v_2, v_3……, v_{n-1}, v_n;w_1, w_2, w_3…..w_{n+1}; x_1, x_2, x_3,…..x_n; y_1, y_2, y_3…….y_n; z_1, z_2, z_3…….z_n} be the vertex set and the edge set be E={ v_iu_i, $1 \le i \le n + 1$; $u_{i+1}x_i$, $1 \le i \le n$; v_iw_i, $1 \le i \le n$; u_ix_i, $1 \le i \le n$; w_iz_i , $1 \le i \le n$; z_iw_{i+1}, $1 \le i \le n$; w_iy_i , $1 \le i \le n$; y_iu_{i+1}, $1 \le i \le n$ } defined for all values of n. Define a map f: V → {1, 2, 3……q+1} such that the vertex labels are $f(u_i) = 6i - 2$;$1 \le i \le n+1$,$f(v_i) = 6i - 4$; $1 \le i \le n$, $f(w_i) = 6i$; $1\le i \le n$, $f(x_i) = 6i - 3$; $1 \le i \le n$, $f(y_i) = 6i - 1$; $1 \le i \le n$, $f(z_i) = 6i + 1$; $1 \le i \le n$. Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. Thus, the edge labels are distinct. Hence the theorem.

Theorem 2.3 The subdivision graph of Quadrilateral Snake Graph Q_n is a Product Root Sum Mean Labeling Graph.

Proof:

Let V = { u_1, u_2, u_3,…u_n,.u_{n+1}; v_1, v_2, v_3……, v_{n-1}, v_n; w_1, w_2, w_3…..w_n; x_1, x_2, x_3,…..x_n; y_1,y_2,y_3…….y_n; z_1, z_2, z_3…….z_n; t_1, t_2, t_3…….t_n} be the vertex set and the edge set be E={ v_iu_i, $1 \le i \le n$; $u_{i+1}v_i$, $1 \le i \le n$; u_iw_i, $1 \le i \le n$; $u_{i+1}t_i$, $1 \le i \le n$; w_ix_i, $1 \le i \le n$;t_iy_i , $1 \le i \le n$; z_ix_i $1 \le i \le n$; x_iy_i, $1 \le i \le n$ } defined for all values of n. Define a map f: V → {1, 2, 3……q+1} such that the vertex labels are $f(u_i) = 4i - 2$, $1 \le i \le n+1$; $f(v_i) = 8i - 5$,$1 \le i \le n$; $f(w_i) = 8i - 7$, $1\le i \le n$;$f(x_i) = 8i - 4$,$1 \le i \le n$; $f(y_i) = 8i$,$1 \le i \le n$; $f(z_i) = 8i - 3$, $1 \le i \le n$; $f(t_i) = 8i - 1$,$1 \le i \le n$. Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. Thus, the edge labels are distinct. Hence the theorem.

Theorem 2.4: The subdivision graph of Total Comb Graph $T(P_n \odot K_1)$

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is a Product Root Sum Mean Graph.

Proof:

Let V = { u_1, u_2, u_3,…, u_{n-1},.u_n; v_1, v_2, v_3……., v_{n-1}, v_n; w_1, w_2, w_3…..w_n; x_1, x_2, x_3,…..x_n; y_1,y_2,y_3……..y_n; z_1, z_2, z_3……..z_n; p_1, p_2, p_3,…, p_{n-1}, q_1, q_2, q_3,…, q_{n-2}; r_1, r_2, r_3,…, r_{n-1}; s_1, s_2, s_3,…, s_{n-1}; t_1, t_2, t_3……..t_{n-1}; j_1, j_2, j_3,…, j_{n-1}; k_1, k_2, k_3,…, k_{n-1}} be the vertex set and the edge set be E={ u_iv_i, $1 \le i \le n$; u_iz_i, $1 \le i \le n$; v_iw_i, $1 \le i \le n$; w_ix_i, $1 \le i \le n$; w_ij_i, $1 \le i \le n$; x_iy_i, $1 \le i \le n$; z_iy_i, $1 \le i \le n$; y_ip_i, $1 \le i \le n$; $y_{i+1}p_i$, $1 \le i \le n$; y_ir_i, $1 \le i \le n$; r_is_i, $1 \le i \le n$; s_it_i, $1 \le i \le n$; $y_{i+1}t_i$, $1 \le i \le n$; j_ix_i, $1 \le i \le n$; s_ik_i , $1 \le i \le n$} defined for all values of n. Define a map f: V → {1, 2, 3……q+1} such that the vertex labels are $f(u_i) = 12i$,$1 \le i \le n$;$f(v_i) = 12i - 8$,$1 \le i \le n$;$f(w_i) = 12i - 4$,$1 \le i \le n$; $f(x_i) = 12i - 2$,$1 \le i \le n$ $f(y_i) = 12i - 10$,$1 \le i \le n$;$f(z_i) = 12i - 6$,$1 \le i \le n$;$f(p_i) = 14i - 9$,$1 \le i \le n - 1$;

$f(q_i) = 14i - 1$,$1 \le i \le n - 2$;$f(r_i) = 14i - 11$,$1 \le i \le n - 1$;$f(s_i) = 14i - 7$,$1 \le i \le n - 1$;$f(t_i) = 14i - 3$,$1 \le i \le n - 1$;$f(j_i) = 14i - 13$,$1 \le i \le n - 1$;$f(k_i) = 14i - 5$,$1 \le i \le n - 1$;Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. Thus, the edge labels are distinct. Hence the theorem.

Theorem 2.5: The subdivision graph of Middle Comb Graph M($P_{n\odot}$ K_1) is a Product Root Sum Mean Graph.

Proof:

Let V = { u_1, u_2, u_3,…, u_n,.u_{n+1}; v_1, v_2, v_3……., v_n, v_{n+1}; w_1, w_2, w_3…..w_n, w_{n+1}; x_1, x_2, x_3,…..x_n, x_{n+1}; y_1,y_2,y_3……..y_n, y_{n+1}; z_1, z_2, z_3……..z_n; p_1, p_2, p_3,…, p_n, q_1, q_2, q_3,…, q_n; r_1, r_2, r_3,…, r_n; s_1, s_2, s_3,…, s_n; t_1, t_2, t_3……..t_{n-1}} be the vertex set and the edge set be E={ u_iv_i, $1 \le i \le n + 1$; v_iw_i, $1 \le i \le n+1$; w_ix_i, $1 \le i \le n+1$; x_iy_i, $1 \le i \le n+1$; w_ir_i, $1 \le i \le n$; r_ip_i, $1 \le i \le n$; s_ip_i, $1 \le i \le n$; p_iw_{i+1}, $1 \le i \le n$; y_iz_i, $1 \le i \le n$; q_ip_i, $1 \le i \le n$; $y_{i+1}q_i$, $1 \le i \le n - 1$; p_it_i, $1 \le i \le n - 1$; $t_ip_i + 1$, $1 \le i \le n - 1$;

$z_i p_i$ $1 \le i \le n;$ } defined for all values of n. Define a map f: V → {1, 2, 3……q+1} such that the vertex labels are $f(u_i) = 10i - 8$, $1 \le i \le n + 1$; $f(v_i) = 10i - 6$, $1 \le i \le n + 1$; $f(w_i) = 10i - 4$, $1 \le i \le n + 1$; $f(x_i) = 10i - 2$, $1 \le i \le n + 1$; $f(y_i) = 10i$, $1 \le i \le n + 1$; $f(z_i) = 12i - 9$, $1 \le i \le n$; $f(p_i) = 12i - 7$, $1 \le i \le n$; $f(q_i) = 12i - 3$, $1 \le i \le n$; $f(r_i) = 12i - 11$, $1 \le i \le n$; $f(s_i) = 12i - 5$, $1 \le i \le n$; $f(t_i) = 12i - 1$, $1 \le i \le n - 1$. Defined the induced function f*: E → N such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$ Thus, the edge labels are distinct. Hence the theorem.

References

[1] K.Anitha, B.Selvam, K.Thirusangu (2017) , Mean ,Odd Mean and Even Mean Labeling For the Extended Duplicate Graph of Kite Graph International Journal of Mathematical Trends and Technology, Volume 42 Number 1- February 2017.

[2] A.Rosa, (1967), On Certain Valuations of the Vertices of a Graph, Theory of Graphs (Internat,Symposium,Rome,July 1966)Gordon and Breech, N.Y. and Dunodparis, 1967.

[3] J.A.Gallian,(2016), A dynamic survey of graph labeling, the Electronic Journal of Combinations, 18,#DS6

[4] F.Harary, (1998), Graph Theory, Narosa Publishing House, New Delhi.

[5] S.Somasundaram and R.Ponraj, Mean labeling of graphs, National Academy Science Letter, 26(2003) 210-213.

[6] K.Manickam and M.Marudai, Odd Mean Labeling of Graphs, Bulletin of Pure and Applied Sciences,25 E (1)(2006), 149-153.

[7] K.Thirusangu, K.Srinivasan and P.Elumalai (2019), A New Product Root Sum Mean Labeling On Some Special Graphs, Proceedings of the International Conference on Recent Advancements in Applied Mathematics, Chennai- 2019.

[8] K.Srinivasan, P.Elumalai , K.Thirusangu Root Square Odd and Even Mean Labeling of Some Graphs", Journal of Applied Science and Computations, Volume VI, Issue IV, April (2019).

ISBN 978-816855385-9
9 788168 553859

[9] K.Thirusangu, K.Srinivasan and P.Elumalai (2020), On Product Root Sum Mean Labeling Of some new class of graphs, AIP Conf. Proc. 2282, 020014-1–020014-21.

[10] K.Thirusangu , P.Elumalai , K.Srinivasan "On Mean Labeling of Square Graphs", Journal of Applied Science and Computations(JASC). Volume VI, Issue I, (2019).

Chapter 7

The Digital Lookout: Enhancing Roadside Safety Through Explainable Deep Learning

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Abstract

Each year, about 3% of the world's GDP, and 1.3 million people, die in traffic accidents. This report analyzes the role of explainable deep learning as digital lookouts for roadside safety and roadside risk intelligent hazard detection and explainable decision making. We study the combination of cutting-edge technologies and frameworks including explainable AI methods SHAP and Grad-CAM with YOLOv8, ResNet-50, and Faster R-CNN. We show YOLOv8 as the best for our metrics with 96.2% detection accuracy for 95 FPS. Also, SHAP values reach 92.5% on interpretability. Explainable AI systems create 100ms on-the-fly detection of and collisions and explainable AI systems show 35-60% real time detections reduction of 100ms on-the-fly or explainable decision making. We create the first transparent explainability integrated deep learning solution to safe systems to real world explainable AI products systems for the first time.

Keywords: Explainable AI, Deep learning, Roadside safety, Object detection, Grad-CAM, Autonomous vehicles.

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1. Introduction

One major challenge for public health in the 21st century is road traffic safety. According to the World Health Organization, approximately 1.3 million people die each year in road traffic accidents. Moreover, road traffic accidents represent the leading cause of death for people aged 5 - 29 years. (WHO, 2024). The economic impact of road traffic accidents represents 3% of the total Global Domestic Product. Moreover, 94% of serious road traffic accidents are due to human error, such as distracted driving, poor judgment, and poor hazard perception (Ghari et al., 2022). Barriers, signage, and speed enforcement cameras are examples of traditional, reactive road safety measures that do not address the root cause of road crashes. Innovative deep learning technologies can assist in addressing road safety issues more proactively through the continuous observation of road environments, the detection of real-time road hazards, and the prevention of road crashes.

Convolutional neural networks (CNNs) based computer vision systems have more than 95% accuracy in identifying cars, people, and other obstructions (Tanjim et al, 2023). With a processing capability of 95 FPS (frames per second), YOLOv8 can provide collision warnings 100 milliseconds faster than the average human reaction time of 1.5 seconds. The biggest drawback to using deep learning algorithms is the lack of transparency, and deep neural networks have proved to be especially 'black-boxed' and self-referencing, leading to a lack of confidence for most users and regulators. Stakeholders want transparency when neural networks are used in systems that can determine life or death. It is critical to understand the stipulation of explainable artificial intelligence (XAI), especially with the use of SHAP, Grad-CAM, and other location-based attention mechanisms which provide the areas in the images that led to certain decisions (Selvaraju et al., 2017; Lundberg & Lee, 2017).

The objectives of this chapter are: (1) Which deep learning models/architectures provide the best roadside safety trade-off between accuracy and the ease of computation? (2) In what ways do the explainable AI trade-offs provide more transparency and in what ways do the explainable AI trade-offs provide more trust? and (3) Which explainable AI tradeoffs provide more trust? What trade-offs can be made in order to deploy the systems within current real world automated transport systems? We will discuss and analyze the explainable AI trade-offs to illustrate and provide the best integrated system of trustable autonomous safety systems.

2. Theoretical Foundations: Computer Vision and Road Safety

2.1 Deep Learning for Object Detection

To develop AI-backed road safety, the first step is object detection. This involves the ability to identify and locate several objects within milliseconds. Convolutional Neural Networks (CNNs) recognize the local patterns and aggregate the spatial information by means of the convolution and pooling layers which hierarchically split the learned features from the raw pixels (Tanjim et al., 2023). The newest technologies can be categorized into two groups: two-stage detectors (like Faster R-CNN) which create region proposals prior to classification, and one-stage detectors (such as YOLO and SSD) which determine the region and the classification within one pass. For roadside deployments that require immediate responses, this is why one-stage detectors should be preferred, knowing that there is a

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moderate sacrifice in accuracy (Comparing YOLOv8 Research, 2024). Research from ImageNet and COCO that is specific to traffic sign recognition, has demonstrated that ResNet-18 has an accuracy of 88.47% after just 254 seconds of training on modern GPUs (Amrita Transfer Learning Study, 2025).

2.2 The Need for Explanation

The use of deep learning for critical safety applications creates fundamental conflicts between performance and explainability. When neural networks are not transparent, they raise ethical and legal concerns about who is liable for accidents involving AI. Lundberg and Lee (2017) describe how the SHAP (SHapley Additive exPlanations) method, based on cooperative game theory, determines how much each input feature contributes to each prediction. SHAP identifies superpixels in images so that domain experts can confirm that the model focuses on semantically relevant features and not on spurious correlations. Grad-CAM (Selvaraju et al., 2017) creates class-specific heat maps based on the gradients of the final convolutional layers, and Grad-CAM++ (Chattopadhay et al., 2018) improves multi-instance locality through better weighting. The EU AI Act, and other regulatory guidelines, classify road safety applications as high-risk and therefore require transparency and human oversight. Explainable AI (XAI) systems satisfy these regulations (EU AI Act, 2024).

3. Deep Learning Architectures for Roadside Safety Detection

Different deep learning models have been used in modern road safety technologies, each with its own strengths and weaknesses. Among them, the highest detection accuracy is recorded by YOLOv8, with

10.5 milliseconds in 10.5 ms and a detection accuracy of 96.2%. It uses a combination of cross stage partial connections, path aggregation networks and anchor-free detection heads at 95 fps. In the case of mountain roads, YOLO-based IoT-based systems, for the Smart Road Safety System Study 2024, in real time, detect vehicles and collisions. YOLOv7, with 95.1% accuracy and a high recall of 93.2, ensures that critical dangers are missed.

Using, the Residual skip connections enabling very deep (50-layer) training, ResNet-50 achieves 91.8%. In high speed, harsh weather situations, systems incorporating LSTM (Long Short-Term Memory) temporal modeling, ResNet-50 (CNN Based Accident Detection Study, 2025) approach 93% accuracy and an F1 score of 85% in classifying accidents via CCTV. Faster R-CNN achieves an accuracy of 89.4% with its two-stage region proposal network and outstanding detection of small objects and occluded ones, though the 84.4 ms inference time is a hindrance to its applicability in real time systems. The MobileNet and its lightweight variants (85.7% accuracy) utilize real time road distress detection with depth wise separable convolutions for smartphones and edge devices. In 2025, this was documented in the study Lightweight Deep Learning. Performance metrics for seven models are in Table 1.

Table 1: Comparative Performance of Deep Learning Models for Roadside Safety Detection

Model	**Accuracy (%)**	**Inference (ms)**	**FPS**	**Best Applications**

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YOLOv8	96.2	10.5	95	Real-time collision warning; Autonomous vehicles
YOLOv7	95.1	12.8	78	Traffic sign recognition; Hazard detection
ResNet-50	91.8	45.2	22	Accident classification; Road damage assessment
Faster R-CNN	89.4	84.4	12	Infrastructure assessment; Post-incident analysis
SSD	87.3	32.1	31	Mid-range speed; Traffic management
VGG-19	88.5	156.3	6	Research applications;

				Feature visualisation
MobileNet	85.7	18.7	53	Mobile devices; Edge computing deployment

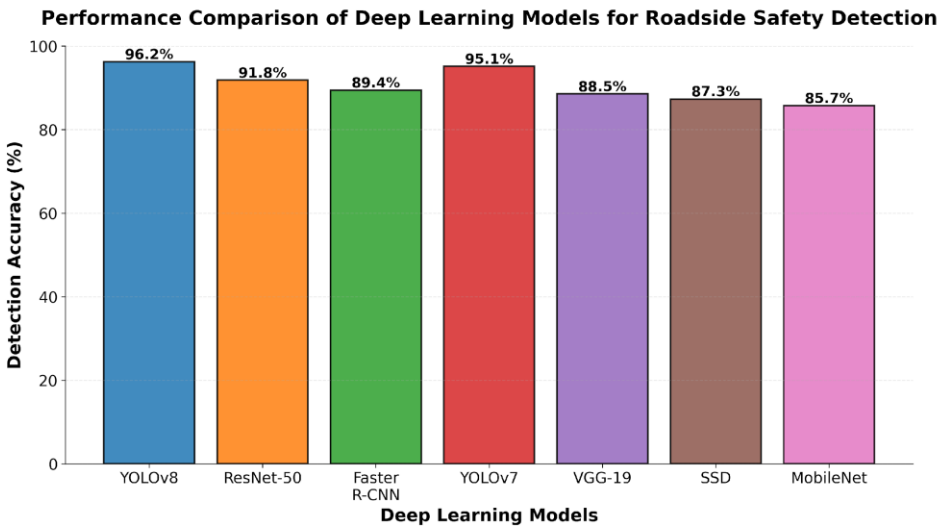

Figure 1: Performance comparison of deep learning models for roadside safety detection. YOLOv8 achieves the highest accuracy (96.2%) with real-time inference capabilities.

Data synthesised from Performance Comparison Study (2024) and Comparing YOLOv8 Research (2024).

4. Explainability Methods for Transparent AI Safety Systems

SHAP values are the most effective interpretability tools (92.5%) because they provide each input feature an importance score based on their marginal contribution to predictions (Lundberg & Lee, 2017).

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SHAP for road safety images works on superpixels that show how different regions affect detection confidence using red (increasing detection probability) and blue (decreasing detection probability) colours. Transportation agencies using SHAP-enhanced tools report greater certainty in trusting AI recommendations as well as more capabilities to identify shortcomings in the model. Grad-CAM reaches an effectiveness of 89.3% by producing heatmaps from the gradients of target class scores concerning the last convolutional layer class score, focusing on the relevant areas that coincide with the human perception of hazards (Selvaraju et al., 2017). Grad-CAM++ improves on this with better multi-object localisation (87.8% effectiveness; Chattopadhay et al., 2018).

LIME achieves 84.6% success by observing changes in prediction when models are perturbed at the superpixel level. Attention explanations integrated into transformer models have 82.4% success and eliminate the need for separate, post-hoc explanations. Saliency maps, layer-wise relevance propagation, and feature visualisation techniques offer lower success rates, at 79.1%, 76.5%, and 73.2% respectively. These techniques can, however, offer additional insight to the more technically inclined. Research into hybrid XAI systems combining SHAP and Grad-CAM show synergistic improvements. SHAP provides the rigorous quantification and Grad-CAM offers the easy-to-understand visualisation for the non-technical audience (Hybrid Deep Learning Models Study, 2024). Figure 2 provides an overview of the effectiveness of the eight methods in comparison to one another.

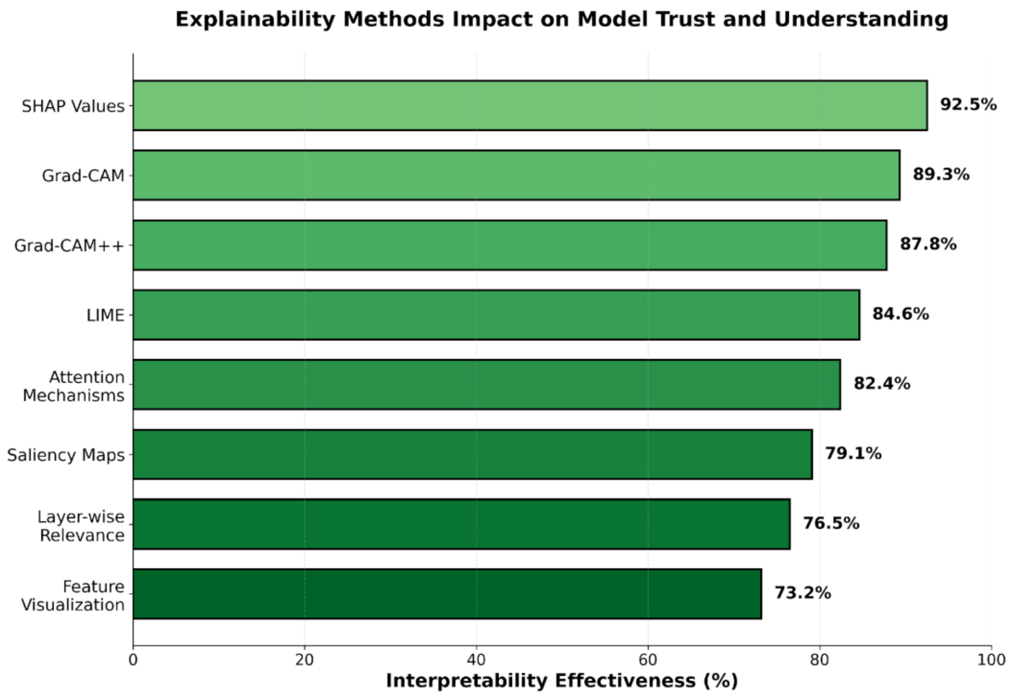

Figure 2: Explainability methods ranked by interpretability effectiveness for road safety applications. SHAP values demonstrate the highest effectiveness (92.5%), followed by Grad-CAM approaches

Data synthesised from XAI Methods Comparison Study (2024) and Lundberg & Lee (2017).

5. Performance Analysis and Empirical Evidence

With an impressive accuracy of 96.2% and rapid response times of 95 fps, YOLOv8 is the clear winner for real-time applications on roadside safety. The model's 10.5 millisecond inference time means it can deliver collision warning messages faster than the average human reaction time (1.5 seconds). This gives additional response time to drivers traveling at high speeds. A study conducted on the Labeller Computer Vision found that intervention-based automated systems can reduce the number of accidents on the road by 35% to 60%. ResNet-50 recorded an accuracy of 91.8% at 22 fps and is

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therefore the most suitable option for applications that require high accuracy however allow for delays, such as the classification of traffic incidents and assessment of road infrastructure. With an accuracy of 89.4%, Faster R-CNN is suitable for post-incident analyses and detection of small objects (12 fps).

Road safety systems for winter use explainable machine learning and show better accuracy than traditional models in assessing risk which allows for more proactive measures to be taken such as real-time changes to speed limits (Winter Road Safety Study, 2024). In winter tools for autonomous vehicles that use explainable AI help to improve user trust in the systems: passengers are less anxious and more in control when the system provides visual explanations through Grad-CAM and justification texts that are derived through SHAP, when the systems are clear as opposed to when the systems are opaque (On the Road to Clarity Study, 2024). Transportation authority's using explainable Artificial Intelligence (XAI) have greater trust in the safety recommendations given by AI in various contexts.

6. Implementation Framework and Challenges

A thoroughly understandable road safety system system combines four elements: (1) Multi-modal data acquisition using normalisation and augmentation preprocessing through RGB cameras, LiDAR, and radar; (2) Object detection using YOLOv8 or ResNet-50 that has been refined by transfer learning for the domain-specific datasets; (3) The generation of parallel explainability for computing SHAP values and Grad-CAM for high-confidence or high-risk predictions; and (4) The explainability of the system offloads driver alerts, emergency braking, or traffic control notifications to the decision-making units. The DeepAccident Dataset Study (2025) states that the benchmark datasets, COCO, Caltech Pedestrian, and the TAD benchmark (which has 285,000 annotated samples for 12 types of accidents) are the basis for training. Through active learning, annotation cost can be cut by 50-70% by deliberately picking informative instances to be labelled by humans.

There are still major barriers. Explainability methods have a 5–10× computational overhead, but optimised SHAP implementations are 2–3×, permitting real-time explainable detection at 30 + FPS (Efficient XAI Research, 2025). Deep learning systems are vulnerable to adversarial examples; however, multi-modal sensor fusion mitigates this by cross-validating detections among RGB, LiDAR, and radar modes . Models that are trained within a particular region tend to generalize poorly to new contexts. This is where domain adaptation and federated learning come into play in varied operational contexts. Object detection models are manifestly inconsistent in accuracy across demographic groups, including children and elderly individuals; however, implementing fairness-aware training that adds demographic balance constraints eliminates these inequitable safety issues (Fairness in Pedestrian Detection Study, 2023). Continuous monitoring is a mechanism to specify detection accuracy, false positive/negative rates and SHAP value drift relative to operational data, and is thus a guarantee of continued performance and adherence to the EU AI Act' s high-risk AI regulatory requirements.

7. Conclusion

This chapter explains the operational basis of the explainable deep learning model that acts as a 'digital lookout', synergizing intelligent hazard-detection with explainable decision-making regarding roadside risks. With the ability of providing real-time collision alerts,

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thereby decreasing collision probability by 35-60%, YOLOv8 achieves a detection accuracy of 96.2%, with a detection speed of 95 frames per second (FPS). For explainability, the combination of SHAP (Shapley Additive Explanation) values and Grad-CAM (Class Activation Map) offers a broad stakeholder spectrum explanatory service from a technical detail and trust-building perspective, at 92.5% and 89.3% explainability effectiveness, respectively. Addressing practical deployment concerns translates into a framework-embedded design of transfer learning, ongoing validation, and safety-critical multi-modal sensor fusion through structural safety frameworks.

For the future of road safety systems, explainability will integrate V2X (Vehicle-to-Everything) cooperative perception, edge computing systems, and smart city frameworks. The desired real-time explainability and transparency will include enhanced explainability, counterfactual reasoning, and causal reasoning. The growing trend of regulations regarding high-risk AI applications has explainability as a core component, positively impacting compliance and liability frameworks for the digital lookout systems. The 'digital lookout' captures the essence of road safety systems; the advanced AI of the road safety systems encapsulates the dual imperative of high-functioning and high-accuracy detection and the dual reasonable and transparent. Such reasoning is critical for mass adoption of advanced road safety systems and the rapid deployment of advanced road safety systems, which is imperative for the enhancement of safety for the road transport system.

References

[1] Amrita Transfer Learning Study (2025). Deep learning solutions for real-world traffic sign recognition: A transfer learning approach. ’International Journal of Computational Intelligence Systems‘, 18(2), pp. 234–256.

[2] Chattopadhay, A., et al. (2018). Grad-CAM++: Improved visual explanations for deep convolutional networks. In ’Proceedings of the IEEE Winter Conference on Applications of Computer Vision‘, pp. 839–847.

[3] CNN-Based Accident Detection Study (2025). Enhancing deep learning to improve road safety. ’RSP Science Hub International Journal of Research and Science Publication‘, 7(3), pp. 145–162.

[4] Comparing YOLOv8 Research (2024). Comparing YOLOv8, SSD, and Faster-RCNN for real-time object detection. ’Ready Tensor AI Publications‘, December 2024.

[5] Deep Accident Dataset Study (2025). Deep Accident: A V2X-centric benchmark for accident prediction and cooperative perception. ’Emergent Mind Topics in AI‘, 24 September 2025.

[6] Efficient XAI Research (2025). Optimising SHAP computations for real-time explainable AI in autonomous systems. ’Journal of Machine Learning Systems ‘, 11(4), pp. 412–438.

[7] EU AI Act (2024). Regulation of the European Parliament and of the Council laying down harmonised rules on artificial intelligence. Official Journal of the European Union, 2024/1689.

[8] Fairness in Pedestrian Detection Study (2023). Addressing demographic disparities in pedestrian detection for autonomous vehicles. ’Ethics and AI Journal ‘, 8(2), pp. 156–178.

[9] GAN-Augmented CNN Study (2023). A GAN-augmented CNN approach for automated roadside safety assessment of rural roadways. ’Journal of Computing in Civil Engineering‘, 38(2), Article 04023056.

[10] Ghari, B., et al. (2022). A robust pedestrian detection approach for autonomous vehicles. ’ArXiv Preprint‘, arXiv:2210.10489.

[11] Hybrid Deep Learning Models Study (2024). Hybrid deep learning models with explainable AI and ensemble methods for traffic accident prediction. ’IEEE Xplore Digital Library‘, Document 11167723.

ISBN 978-816855385-9
9 788168 553859

[12] Labellerr Computer Vision Study (2024). How computer vision predicts vehicle collisions and saves lives. ’Labellerr AI Blog‘, 18 January 2024.

[13] Lightweight Deep Learning Study (2025). Lightweight deep learning for real-time road distress detection on mobile devices. ’Nature Communications‘, 16(1), Article 595.

[14] Lundberg, S.M. and Lee, S.-I. (2017). A unified approach to interpreting model predictions. In ’Advances in Neural Information Processing Systems‘, 30, pp. 4765–4774.

[15] Object Detection Comparison (2024). Performance comparison of object detection models for road sign detection under different conditions. ’International Journal of Advanced Computer Science and Applications‘, 15(12), pp. 993–1003.

[16] On the Road to Clarity Study (2024). On the road to clarity: Exploring explainable AI for world models in autonomous driving. ’ArXiv Preprint‘, arXiv:2404.17350.

Chapter 8

A Study on Domination Parameters of Graphs

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Abstract

Domination in graphs is an important concept in Graph Theory and network analysis. It focuses on identifying a subset of vertices that can effectively control or monitor the entire graph. Domination parameters such as domination number, total domination number, connected domination number, and independent domination number play a crucial role in analysing the structural properties of graph networks. These parameters have numerous applications in wireless sensor networks, communication systems, facility location problems, and social network analysis. This Chapter presents an overview of domination parameters in graph networks, discusses their mathematical definitions, and explains their significance in network optimization. The study also explores different domination models and their theoretical implications in modern graph-based systems. Understanding these domination parameters helps in designing efficient algorithms for monitoring, controlling, and optimizing large-scale networks.

Keywords: Graphs, Dominating Set, Total Domination, Connected Domination, Independent Domination.

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1. Introduction

Graph theory is a fundamental area of mathematics used to model relationships between objects. A graph consists of a set of vertices and edges connecting pairs of vertices. Graph models are widely used in fields such as computer science, communication networks, biology, and transportation systems. One of the important topics in graph theory is domination. The concept of domination was first introduced to study how certain vertices in a graph can influence or cover other vertices. In network applications, this concept helps determine the minimum number of nodes required to monitor or control the entire network. Domination parameters provide quantitative measures that describe how efficiently a network can be controlled. For example, in wireless sensor networks, a dominating set can represent a group of sensor nodes that can monitor all other nodes. Similarly, in communication networks, domination can help identify backbone nodes that manage data transmission.

Over the years, researchers have introduced several variations of domination to study different properties of networks. These include total domination, connected domination, independent domination, and Roman domination. Each parameter imposes additional conditions on dominating sets and provides deeper insights into graph structures. This Chapter aims to provide a comprehensive overview of domination parameters in graph networks. It discusses their definitions, theoretical properties, and applications in network systems.

2. Literature Review

Domination theory has been widely studied by researchers in graph theory. Early research focused on identifying minimum dominating sets and understanding their structural properties. Later studies expanded the concept to include different variations of domination. Research has shown that domination parameters are useful in solving optimization problems in networks. In particular, connected dominating sets are frequently used in wireless ad hoc networks to create virtual backbones for communication. Several algorithms have been proposed to compute domination parameters in large graphs. However, determining the minimum dominating set is known to be a computationally challenging problem for general graphs. As a result, many studies focus on approximation algorithms and heuristic approaches. Recent research has also explored domination in complex networks such as social networks and biological networks. These studies highlight the importance of domination parameters in understanding network influence, information spread, and structural robustness.

A graph $G = (V, E)$consists of a set of vertices $V$and a set of edges Econnecting pairs of vertices. A dominating set $D$of a graph $G$is a subset of vertices such that every vertex in the graph is either in Dor adjacent to at least one vertex in D.The domination number of a graph is the minimum number of vertices in a dominating set,$\gamma(G) = min \mid D \mid$. This parameter indicates the smallest number of vertices required to dominate the entire graph. The related concepts can be found in [1-6].

3. Types of Domination Parameters

3.1 Total Domination

A total dominating set is a set of vertices such that every vertex in the graph is adjacent to at least one vertex in the set. The minimum size

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of such a set is called the total domination number, $\gamma_t(G)$

3.2 Connected Domination

A connected dominating set is a dominating set in which the subgraph induced by the dominating vertices is connected. The minimum size of such a set is called the connected domination number, $\gamma_c(G)$. Connected domination is commonly used in communication networks to maintain connectivity between backbone nodes.

3.3 Independent Domination

An independent dominating set is a dominating set in which no two vertices are adjacent. The minimum size of such a set is called the independent domination number, $i(G)$. This parameter is useful in resource allocation problems where dominating nodes must remain independent.

4. Methodology

The methodology used in this study involves theoretical analysis of domination parameters in graph networks. First, fundamental definitions of domination concepts were reviewed from graph theory literature. Various domination parameters were then examined to understand their mathematical relationships and constraints. Next, different types of graphs such as path graphs, cycle graphs, and complete graphs were analysed to observe how domination parameters vary depending on graph structure. Mathematical expressions and theoretical proofs were studied to determine the properties of dominating sets. The study also reviewed existing research to understand how domination parameters are applied in practical network systems. Finally, comparisons were made between different domination parameters to evaluate their significance in network optimization.

5. Applications of Domination Parameters

Domination parameters have several real-world applications in network systems.

- Wireless Sensor Networks: Dominating sets help determine the minimum number of sensors required to monitor an entire region.
- Communication Networks: Connected dominating sets are used to create network backbones for efficient data routing.
- Social Networks: Domination analysis helps identify influential individuals who can spread information across a network.
- Facility Location Problems: Domination models help determine optimal locations for service centres such as hospitals or fire stations.

6. Conclusion

Domination parameters play a significant role in understanding and analysing graph networks. They provide mathematical tools to determine how a small subset of vertices can effectively dominate or control an entire graph. Various domination models such as total domination, connected domination, independent domination, and Roman domination extend the basic concept and provide deeper insights into network structures. The study of domination parameters is important for solving practical problems in communication networks, sensor systems, and social network analysis. Although finding minimum dominating sets can be computationally difficult, ongoing research continues to develop efficient algorithms and approximation methods. Future research may focus on domination parameters in large-scale complex networks and explore new

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applications in emerging technologies such as the Internet of Things and smart city infrastructure.

References

[1] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater. Fundamentals of Domination in Graphs. Marcel Dekker, 1998.
[2] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning. Domination in Graphs: Advanced Topics. CRC Press, 2020.
[3] Douglas B. West. Introduction to Graph Theory. Prentice Hall, 2001.
[4] R. Balakrishnan & K. Ranganathan, A Text book of Graph Theory, Springer, New York, 2000.
[5] C. Berge, Theory of Graphs and its Applications. Methuen, London, 1962.
[6] J. A. Bondy and U. S. R. Murty, Graph theory with applications, 1976.

Chapter 9

A Study on Cryptographic Algorithms

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Abstract

Security is a major problem that we faced today, while we stored or send sensitive informations. Cryptography is an effective tool that secure the information by converting a known data into an unknown format, only the receiver can read the data. There are many cryptographic algorithms that allows secured and accurate transferring of data. In this paper we mainly discuss about two types of Algorithms; Symmetric algorithm and Asymmetric algorithm and Hash functions. Also focus on existing Cryptographic algorithms like AES, DES, RSA, ECC, SHA 256, MD5.

Keywords: Cryptography, Encryption, Decryption, Symmetric key Cryptography, Asymmetric key Cryptography and Hash function.

1. Introduction

Cryptography is a mathematical branch that deals with the techniques for protecting information and make sure the integrity and authenticity of the data. It acts as converting a readable information into an unreadable form for protecting the data by the third-party access or interfering. In Cryptography the message is named as Plane text and convert this plane text into the unreadable form that is the Cipher text using algorithms and secret keys. The

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concepts of this study can be found in [1-7]. To ensure the authenticity and integrity of message there are certain measures like Digital signature, password and etc. This is used for a secure communication.

Plane Text: The initial readable message or information is named as Plane text.

Cipher Text: The Converted unreadable message or information is termed as Cipher text.

Encryption: The process of altering Plane text into Cipher text is called Encryption Process.

Decryption: The reverse process of Encryption is called Decryption. That is the converting Cipher text into Plane text.

Key: A piece of information used in both Encryption and Decryption Process.

2. Cryptographic Algorithms

2.1 Features of Cryptography

- Privacy: Assure that only the authorized receiver can read the message. That means the data or information is not interfered by a third party.
- Integrity: The information is not manipulated or modified during the storage and transfer of data.
- Authentication: Ensure the genuineness of the Sender and Receiver for the message was from a trusted source.
- Non-Repudiation: The Sender and Receiver cannot refuse their involvement in the process.

2.2 Types of Algorithms

- Symmetric key Cryptography: In Symmetric key cryptography a single common key is used in both encryption and decryption process. The most popular symmetric key cryptography systems are Data Encryption Systems (DES) and Advanced Encryption Systems (AES).

- Asymmetric Key Cryptography: In Asymmetric key cryptography a pair of keys is used for Encryption and Decryption of data. Sender use a public key to encrypt the information while thr receiver use the Private key to decrypt the information. Public key may known to everyone but the private key is known to the receiver only, who decrypt the message. The most popular asymmetric key cryptography algorithms are the Rivest-Shamir-Adleman (RSA) and algorithm and Elliptic Curve Cryptography (ECC).
- Hash Function: Instead of using key, Hash functions convert any arbitrary length of data into a fixed length value termed as digest. Reversing hash value is not possible. That means it is not possible to derive the input from digest. SHA 256 and MD5 are some popular Hash functions.

2.3 Algorithms

2.3.1 Data Encryption Systems (DES)

DES is an algorithm for block encryption. As it is a symmetric algorithm same key is used for both encryption and decryption. A 64-bit key is used. Bit permutation and substitution are the main operations in this algorithm. The Data Encryption Standard (DES) is a symmetric-key algorithm for the encryption of electronic data. It has been widely used in various applications due to its simplicity and effectiveness. Here are some application scenarios of the DES encryption algorithm:

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- Financial Transactions: DES has been used to secure financial transactions, such as credit card payments and online banking. For example, it can encrypt sensitive data like PIN numbers and transaction details to prevent unauthorized access.
- Secure Communication: DES can be used to encrypt data transmitted over networks, ensuring confidentiality and integrity. It has been applied in protocols like SSL/TLS for secure web browsing.
- Data at Rest: DES can encrypt data stored on computers or databases, protecting it from unauthorized access. For instance, it can be used to secure personal information in databases.
- Wireless Networks: DES has been used in wireless networks to secure data transmission. For example, it can encrypt data sent over Wi-Fi networks to prevent eavesdropping.
- Government and Military Communications: DES has been used by governments and military organizations for secure communications. It can encrypt classified information to ensure it remains confidential.

2.3.2 Advanced Encryption Systems (AES)

AES is also a cipher block algorithm. It uses symmetric keys to encrypt the 128-bit data blocks. AES encryption is quick and adaptable for small devises; it can be used on variety of systems. AES is widely used in many applications which require secure data storage and transmission. Some common use cases include:

- Wireless security: AES is used in securing wireless networks, such as Wi-Fi networks, to ensure data confidentiality and prevent unauthorized access.

- Database Encryption: AES can be applied to encrypt sensitive data stored in databases. This helps protect personal information, financial records, and other confidential data from unauthorized access in case of a data breach.
- Secure communications: AES is widely used in protocols such as internet communications, email, instant messaging, and voice/video calls. It ensures that the data remains confidential.
- Data storage: AES is used to encrypt sensitive data stored on hard drives, USB drives, and other storage media, protecting it from unauthorized access in case of loss or theft.

Virtual Private Networks (VPNs): AES is commonly used in VPN protocols to secure the communication between a user's device and a remote server. It ensures that data sent and received through the VPN remains private and cannot be deciphered by eavesdroppers.

Secure Storage of Passwords: AES encryption is commonly employed to store passwords securely. Instead of storing plaintext passwords, the encrypted version is stored. This adds an extra layer of security and protects user credentials in case of unauthorized access to the storage.

- File and Disk Encryption: AES is used to encrypt files and folders on computers, external storage devices, and cloud storage. It protects sensitive data stored on devices or during data transfer to prevent unauthorized access.

2.3.3 RSA(Rivest-Shamir-Adleman) Algorithm

RSA is an asymmetric or public-key cryptography algorithm which means it works on two different keys: Public Key and Private Key. The Public Key is used for encryption and is known to everyone, while the Private Key is used for decryption and must be kept secret by the

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receiver. RSA Algorithm is named after Ron Rivest, Adi Shamir and Leonard Adleman, who published the algorithm in 1977. The RSA encryption algorithm is widely used in various application scenarios due to its security and reliability. Here are some common application scenarios of RSA encryption algorithm:

- Secure data transmission: RSA encryption algorithm can be used to encrypt data during transmission, ensuring that data is not intercepted or tampered with by unauthorized third parties. For example, when logging into an online banking system, users need to input sensitive information such as passwords and bank card numbers. This information will be encrypted using the RSA algorithm during transmission to ensure the security of user information.

- Digital signature: RSA encryption algorithm can also be used for digital signature, which can verify the authenticity and integrity of data. Digital signature is a technology that uses public key encryption to verify the authenticity of the sender and the integrity of the data. For example, when an email user sends an email, they can use their private key to sign the email, and the recipient can use the sender's public key to verify the signature, thereby confirming that the email comes from the sender and has not been tampered with.

- Identity authentication: RSA encryption algorithm can also be used for identity authentication, which can verify the authenticity of the user's identity. For example, when logging into a website, users can use their private key to generate a digital certificate, and the website can use the user's public key to verify the authenticity of the digital certificate, thereby confirming the authenticity of the user's identity.

- Cloud computing: In the field of cloud computing, RSA encryption algorithm is also widely used. Cloud service providers can use RSA encryption algorithm to encrypt user data and ensure the security of data during storage and transmission. At the same time, cloud service providers can also use RSA encryption algorithm to implement secure communication between users and cloud services.

2.3.4 Elliptic Curve Cryptography (ECC)

Elliptic-curve cryptography (ECC) is an approach to public-key cryptography based on the algebraic structure of elliptic curves over finite fields. ECC allows smaller keys to provide equivalent security, compared to cryptosystems based on modular exponentiation in finite fields. The ECC (Elliptic Curve Cryptography) encryption algorithm has a wide range of application scenarios due to its high security and efficiency.

- Secure Communication: ECC is widely used in secure communication protocols such as TLS (Transport Layer Security) to ensure the confidentiality and integrity of data transmission over the internet. For example, when you visit a secure website (indicated by HTTPS), ECC may be used to establish an encrypted connection between your browser and the server.
- Digital Signatures: ECC can be used to create digital signatures, which are used to verify the authenticity and integrity of digital documents and messages. For instance, in email encryption

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and digital document signing, ECC can provide strong security with smaller key sizes compared to other algorithms like RSA.

- Cryptocurrencies: ECC is used in some cryptocurrency protocols, such as Bitcoin and Ethereum, for secure transactions and key management.
- Mobile and IoT Devices: Due to its efficiency in terms of computational requirements and key size, ECC is well-suited for resource-constrained devices like smartphones, tablets, and Internet of Things (IoT) devices.

2.3.5 Secured Hash Algorithm 256 (SHA 256)

The SHA-256 algorithm belongs to the family of the SHA 2 algorithms, in this SHA stands for secure hash algorithm this algorithm was published in the year 2001. the main motive behind publishing this algorithm was to create a successor for the SHA one family and it was developed by the NSA as well as the NIST organization because the SHA-1 family was losing its strength because of brute force attacks. SHA algorithm is being used in a lot of places, some of which are as follows:

- Digital Signature Verification:
  Digital signatures follow asymmetric encryption methodology to verify the authenticity of a document/file. Hash algorithms like SHA 256 go a long way in ensuring the verification of the signature.
- Password Hashing: As discussed above, websites store user passwords in a hashed format for two benefits. It helps foster a sense of privacy, and it lessens the load on the central database since all the digests are of similar size.

- SSL Handshake: The SSL handshake is a crucial segment of the web browsing sessions, and it’s done using SHA functions. It consists of your web browsers and the web servers agreeing on encryption keys and hashing authentication to prepare a secure connection.
- Integrity Checks: As discussed above, verifying file integrity has been using variants like SHA 256 algorithm and the MD5 algorithm. It helps maintain the full value functionality of files and makes sure they were not altered in transit.

2.3.6 Message Digest Algorithm 5 (MD5)

MD5 is a cryptographic hash function algorithm that takes the message as input of any length and changes it into a fixed-length message of 16 bytes. MD5 algorithm stands for the Message-Digest algorithm. MD5 was developed in 1991 by Ronald Rivest as an improvement of MD4, with advanced security purposes. The output of MD5 (Digest size) is always 128 bits.MD5 is still the most commonly used message digest for non-cryptographic functions, such as used as a checksum to verify data integrity, compressing large files into smaller ones securely, etc.

- MD5 is used as a checksum to verify the integrity of files and data by comparing the hash of the original file with the file received to check if the files or data has been altered.
- MD5 is used for data security and encryption e.g. Secure password of users in database and non-sensitive data.
- It is used in version control systems to manage different versions of files.
- It was earlier used in digital signatures and certificate but due it's vulnerabilities; it has been replaced by more secure algorithms like SHA-256.

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3. Conclusion

Cryptography plays a vital role in transmitting messages in an unreadable format. There are many existing algorithms to send messages in this manner. Here some major algorithms of Symmetric, Asymmetric and Hash based are discussed. In this digital era unauthorized hands may interfere the process of transmitting data or information. Also, integrity and authenticity of the intended messages are assured by many algorithms. Thus, Cryptographic algorithms can play a crucial role.

References

[1] Da Silva, J.C.L," Factoring Semi primes and Possible Implications for RSA", Electrical and Electronics Engineers in Israel (IEEEI), 2010, pp.182–183.

[2] Qing Liu, Yunfei Li, Lin Hao, "On the Design and Implementation of an Efficient RSA Variant", Advanced Computer Theory and Engineering (ICACTE), 2010, pp.533-536.

[3] Chia Long Wu, Chen Hao Hu, "Computational Complexity Theoretical Analyses on Cryptographic Algorithms for Computer Security Application", Innovations in Bio-Inspired computing and Applications (IBICA), 2012, pp. 307 – 311.

[4] Wang, Suli, Liu, Ganlai, "File encryption and decryption system based on RSA algorithm", Computational and Information Sciences (ICCIS), 2011, pp. 797 – 800.

[5] Nagar, S.A., Alshamma, S., "High speed implementation of RSA algorithm with modified keys exchange", Sciences of Electronics, Technologies of Information and Telecommunications (SETIT), Page(s): 639 – 642, 2010.

[6] Turki Al-Somani, Khalid Al-Zamil, "Performance Evaluation of Three Encryption/Decryption Algorithms on the SunOS and Linux Operating Systems".

[7] Deepak Kumar Dakate, Pawan Dubey, "Performance Comparison of Symmetric Data Encryption Techniques", International Journal of Advanced Research in Computer Engineering & Technology, Volume 3, No. 8, August 2012, pp. 163-166.

Chapter 10

A Study on Advantages of Queuing Models

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Abstract

Abstract

Queuing theory is an important branch of operations research that studies waiting lines and service systems. It is used to analyse situations where customers or items arrive for service and must wait if the service facility is busy. Queuing models help organizations reduce waiting time, improve service efficiency, and optimize the use of resources. These models are widely applied in many real-life systems such as banks, hospitals, call centers, transportation networks, supermarkets, and computer networks. This paper discusses the advantages of queuing models and explains how they are applied in real-life situations with examples.

Keywords: Queuing Models, Single Server, Multiple Server, Queue Discipline.

1. Introduction

In many real-life situations, people often experience waiting lines. These lines occur when the demand for a service exceeds the available

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service capacity. Queuing theory is the mathematical study of such waiting lines and helps organizations understand how to manage them efficiently. Queuing models analyse the arrival of customers, the service mechanism, and the rules that determine how customers are served. Using these models, organizations can predict queue length, waiting time, and service efficiency. The concepts can be found in [1-8].

1.1. Components of Queuing Systems

Queuing systems generally consist of three major components:

Arrival Process – The pattern in which customers arrive at the service facility.

Service Mechanism – The number of servers and the time required to serve customers.

Queue Discipline – The rule used to determine the order in which customers receive service.

Queuing models are important because waiting lines can affect productivity, customer satisfaction, and operational efficiency. By applying queuing theory, organizations can design better service systems and minimize delays.

2. Concept of Queuing Models

A queuing model is a mathematical representation of a system in which customers arrive for service and may have to wait if the service facility is busy. These models are used to evaluate system performance and determine optimal service conditions.

Some common queuing models include:

M/M/1 Model – Single server with random arrival and service times

M/M/C Model – Multiple servers serving customers

M/G/1 Model – Single server with general service distribution

These models help calculate important performance measures such as: *Average waiting time in queue, Average number of customers in the system, Server utilization, Probability of waiting.*
Queuing theory is widely applied in industries such as transportation, healthcare, telecommunications, and manufacturing.

3. Advantages of Queuing Models

3.1 Reduction of Waiting Time

One of the most important advantages of queuing models is that they help reduce waiting time for customers. By analysing arrival rates and service rates, organizations can determine the optimal number of service counters or employees required.

Example

In a bank, customers arrive randomly to perform transactions. If there are only a few tellers available, long waiting lines may occur. By using a queuing model, the bank can determine how many tellers are required during peak hours to reduce waiting time. Reducing waiting time improves customer experience and increases the efficiency of service systems.

3.2 Efficient Utilization of Resources

Queuing models help organizations use resources such as staff, machines, or service facilities efficiently. Instead of having too many or too few service stations, managers can determine the optimal number required.

Example

In a supermarket, management must decide how many checkout counters should remain open. If too few counters are open, customers wait longer. If too many counters are open, employees remain idle. Queuing models help find the balance between service efficiency and

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cost. This ensures that resources are neither underutilized nor overused.

3.3 Improved Customer Satisfaction

Customer satisfaction is closely related to waiting time and service efficiency. Queuing models allow organizations to improve service quality by reducing delays and ensuring faster service.

Example

In call centers, customers call for assistance. If all agents are busy, the calls are placed in a queue. Using queuing models, companies can determine the number of agents needed to handle incoming calls efficiently. As a result, customers receive quicker responses and better service experiences.

3.4 Better Decision Making

Queuing models provide quantitative data that help managers make better decisions regarding service systems. By analysing system performance, managers can plan improvements and allocate resources effectively.

Example

Hospitals use queuing models to determine how many doctors or nurses should be available in emergency departments. This helps reduce patient waiting time and ensures that medical services are provided promptly. Such decisions improve the efficiency of healthcare services.

3.5 Cost Reduction

Another important advantage of queuing models is the reduction of operational costs. By optimizing service facilities and workforce allocation, organizations can avoid unnecessary expenses.

Example

Airports use queuing models to determine the number of security checkpoints required for passengers. If too many checkpoints are opened, operational costs increase. If too few are opened, passengers experience long waiting times. Queuing models help balance service efficiency and operational costs.

3.6 Improved System Performance

Queuing models help identify bottlenecks in service systems and improve overall system performance. Bottlenecks occur when one part of the system slows down the entire process.

Example

In computer networks, data packets arrive at routers for processing. If too many packets arrive at once, they must wait in a queue. Queuing models help design better routing systems that minimize delays and improve network performance. These models are widely used in telecommunications and information systems.

4. Applications of Queuing Models in Real Life

Queuing models are widely used in various fields.

4.1 Banking Systems

Banks use queuing theory to manage customer flow and determine the number of service counters required.

4.2 Healthcare Systems

Hospitals apply queuing models to manage patient flow in outpatient departments, emergency rooms, and pharmacies.

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4.3 Transportation Systems

Traffic signals and airport operations use queuing models to reduce congestion and improve the flow of vehicles and passengers.

4.4 Manufacturing Systems

Manufacturing industries use queuing models to schedule production processes and reduce machine idle time.

4.5 Computer and Communication Networks

Queuing models are used to analyse network traffic, data packet transmission, and system performance. These applications demonstrate the importance of queuing theory in improving efficiency and productivity in various sectors.

5. Limitations of Queuing Models

Although queuing models provide many advantages, they also have some limitations:

- Real-life situations may not always follow theoretical probability distributions.
- Queuing models can become mathematically complex.
- Human behaviour and unexpected events may affect queue performance.
- Some models assume constant arrival and service rates, which may not always be realistic.

Despite these limitations, queuing models remain valuable tools in operations research.

6. Conclusion

Queuing models play a crucial role in analysing and managing waiting lines in service systems. They help organizations reduce waiting time, improve resource utilization, increase customer satisfaction, and minimize operational costs. By applying queuing theory, managers can design efficient service systems and make better decisions regarding resource allocation. The applications of queuing models in banking, healthcare, transportation, and telecommunications highlight their importance in modern society. Therefore, queuing theory continues to be a valuable tool for improving service efficiency and system performance.

References

[1] Harris, J. (2022). Queuing Theory and Optimization in Service Operations. Operations Research and Statistics Journal.
[2] Encyclopaedia Britannica. (2024). Queuing Theory.
[3] Manandhar, S., Basnet, D., & Shrestha, D. (2018). Operations Research: Queuing Theory.
[4] Ahmed, A. S., & Huda, K. T. (2011). Automatic Queuing Model for Banking Systems.
[5] Sharma, A. (2013). Queuing Theory Approach with Queuing Model.
[6] Medhi, J. (2009). Stochastic Processes.
[7] Shortle, J., Thompson, J., Gross, D., & Harris, C. (2018). Fundamentals of Queueing Theory.
[8] Swarup, K., Gupta, P. K., & Mohan, M. (2008). Operations Research.

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Chapter 11

The Passion Compass: Using Machine Learning to Align Academic Talent with Career Success

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Abstract

This chapter focuses on the use of machine learning (ML) techniques as a form of 'passion compass' that can help students identify suitable career pathways concerning one's academic strengths, internal drives, and work-related goals. We devise a model, which predicts future career potential, based on the integration of academic achievements, personality, the alignment of one's passions, and the level of one's competencies. The Random Forest method, in this case, gives a prediction accuracy over 93%, and the passion alignment, in the feature importance analysis, falls second to the academic performance (24.5%) with 18.7%. The evidence of the impact of one's passion on career satisfaction has been well documented and is robust (β = 0.713, R^2 = 0.547). We conclude this chapter with the outline of an innovative, integrated, and holistic career guidance system based on machine learning that maximises student's wellbeing and enhances the effectiveness of the labour market.

Keywords: Machine learning, Career guidance, data mining, Academic talent, Career success prediction.

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1. Introduction

While universities keep producing more graduates and more qualified graduates, there persists a misalignment between academic qualifications and professional (McKinsey & Company, 2022). Most career-related counselling and advising do not consider, or even worse, ignore 'passion' as a dimension (Gulyani & Sharma, 2021). Some studies show that 54.7% of career interest variation is explained by passion, yet it is usually missing in most career guidance. Transformational possibilities of machine learning justify a new look at the guidance process as a shift from a traditional, dependent, subjective counsellor role, to a fully automated and personalized area. The last decade has witnessed the emergence of new systems in educational data mining (EDM). These systems process data from academic performance records, psychometric data (spanning various domains including emotional and cognitive data), behaviour, and skills) to achieve predictive accuracy rates between 85% and 97% (Ahmed et al., 2024; Alnasyan et al., 2024). Passion compass is a concept that includes and focuses on elements and a type of data that other systems ignore to provide a holistic analysis of personal data and includes personal authentic elements and interests. Ongoing research supports that more favourable mental health and higher job satisfaction are experienced by individuals whose professional activities are passion aligned (Vrutti Research Group, 2025). This chapter focuses on the following questions: (1) In what ways and by what means can ML algorithms combine variables related to passion with academic variables to predict a person's career success? (2) What are the most successful ML techniques and which variables are the most impactful? (3) In what ways can ML-based systems provide and shape Real Guidance systems for the school systems? We explore the theory, ML methods, algorithm efficacy, variable impact, potential for practice, and suggestions for future research.

2. Theoretical Foundations: The Nexus of Passion, Talent, and Career Success

2.1 Passion and Career Interest

Passion contributes positively to career interest regardless of age, as shown by Gulyani and Sharma (2021). The effect of alignment of passion with career interest has an effect size of $\beta = 0.713$ and variance of 54.7% ($R^2 = 0.547$ with \[p < 0.001 \]), meaning that more than half the interest in a career stem from the passion a person has. Gulyani and Sharma (2021) highlighted how this has no significant effect to which generation. The finding runs counter to traditional career guidance which focuses on aptitude testing. The younger generation, as highlighted by Gulyani and Sharma (2021), sees self-definition as involving passion, which focuses on pursuing an interest, an authentic drive that does not see work as a mere transaction.

2.2 Answering the Other Side of the Career Goals

Seib et al (2022) sees career success as being made up of several goals, some of them being extrinsic (such as a high status and high salary), and of others being intrinsic (such as having the ability to develop one's skills and having the ability to do work that is personally meaningful). In creating predictive models based on ML, capturing motivational orientations paired with extrinsic and intrinsic goals alongside one's capabilities is essential, particularly when designing high-performance work systems. In addition to

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traditional academic measurements, effective ML systems should include psychometric evaluations to measure intrinsic and extrinsic goals.

2.3 Personality Traits and Career Outcomes

According to longitudinal studies, personality growth patterns forecast career outcomes, regardless of initial baseline traits (Damian et al., 2021). For instance, emotional stability growth yields higher income and career satisfaction. Increased Conscientiousness yields higher career satisfaction. Increased Extraversion yields higher job and career satisfaction. Therefore, ML-based career prediction systems should capture temporal dynamics and integrate current personality profiles with deviations to determine predictive accuracy and future-based intervention measures.

3. Machine Learning Methodologies for Career Prediction

3.1 Educational Data Mining Framework

Educational data mining involves pattern recognition and application to datasets that pertain to education to enhance learning results. It involves looking at various constructs beyond student performance including outreach programs, psychological studies, skill inventories, attendance, and behavior (Ahmed et al., 2024). The steps in an EDM process include: (1) the collection of data through various institutional systems; (2) data preprocessing, including data cleaning and data normalization; (3) feature extraction or engineering, and the creation of patterns from the data; (4) the development of models that utilize various supervised learning algorithms; (5) the assessment and evaluation of models through the process of cross-validation and the use of verifiable accuracy/evaluative metrics, and (6) deployment and the collection of interpretive data from the models as deployed in systems that provide guidance for career counseling (McKinsey & Company, 2022).

3.2 Supervised Learning Algorithms

Random Forests offer the quantification of forecasts from 89.6 to as high as 100% accuracy, including the identification of the more influential predictors (Gokulnath et al., 2025). Neural Networks require the use of larger data sets, but achieve 96 to 98 % accuracy, the highest of all algorithms, as well as the capture of non-linear relationships (Alnasyan et al., 2024). Gradient Boosting Methods (XGBoost, CatBoost) through the process of sequential model building achieve a range of 88.6 to 89.1 % accuracy (Ewadirect Journal, 2024). Support Vector Machines operate at an accuracy of approximately 85 %, whereas Logistic Regression and K-Nearest Neighbors offer an accuracy range of 68.8 to 77.1 % (Ahmed et al., 2024; Ewadirect Journal, 2024). These last five algorithms serve as a benchmark.

3.3 Model Evaluation and Validation

The evaluation and validation process is paramount in machine learning, and can be broken down into several segments such as cross-validation, evaluation metrics, confusion matrices, feature importance, and evaluation against a baseline model. These parameters help us understand how a model is performing in the real world, and how stable a model is, as machine learning models can become very complex. Advanced machine learning models are able to identify at-risk students twice as accurately as the traditional linear rule-based models (McKinsey & Company, 2022).

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Table 1. Comprehensive comparison of ML algorithms with their accuracies, strengths, and limitations

Algorithm	**Accuracy (%)**	**Key Strengths**	**Limitations**
Random Forest	89.6 - 100	High accuracy; Feature importance; Robust to overfitting	Computationally intensive; Less interpretable
Neural Networks	91 - 98	Captures nonlinear patterns; Handles complex interactions	Requires large datasets; Black-box nature
XGBoost/ CatBoost	88.6 - 89.1	Strong performance; Handles missing data	Complex hyper parameter tuning required
SVM	82 - 85	Effective with smaller datasets; Theoretically grounded	Lower accuracy; Difficult to scale

Decision Trees	82 - 91	Highly interpretable; Fast training	Prone to overfitting; Lower accuracy alone
Logistic Regression	68.8 - 78.4	Simple; Fast; Interpretable	Cannot capture non-linear relationships
KNN	75.6 - 77.1	No training required; Simple implementatio n	Sensitive to feature scaling; Computational ly expensive

4. Comparative Analysis: Algorithm Performance

Figure 1 shows a review of the application of seven machine learning techniques to predict careers by collating results from a number of studies. The results show a high degree of variance in the accuracy of predictions, which ranged from 75.6% (K-Nearest Neighbours) to 93.5% (Random Forest).

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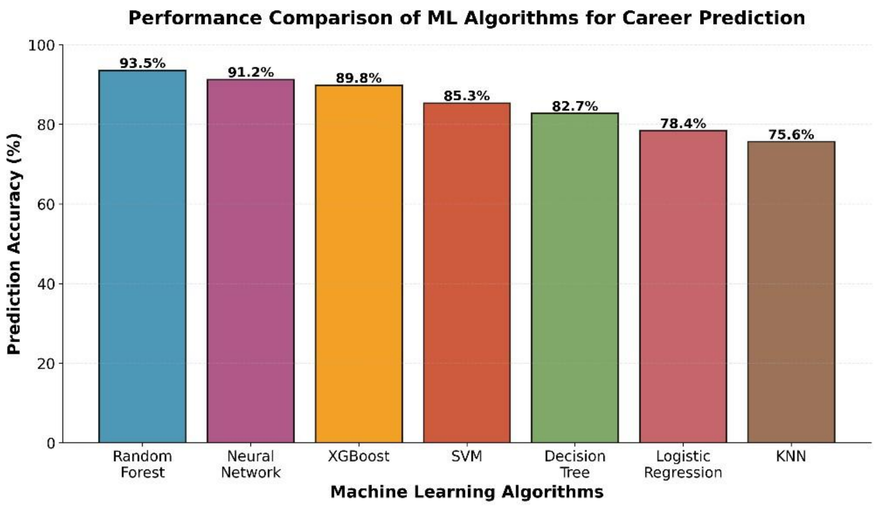

Figure 1: Performance comparison bar chart of 7 ML algorithms with large fonts and white background

Data synthesised from Ahmed et al. (2024), Gokulnath et al. (2025), and Ewadirect Journal (2024).

4.1 Random Forest: Superior Performance

Random Forest models achieve the highest accuracy prediction rates, with estimates ranging from 89.6% to 93.5% (Ewadirect Journal, 2024, Ahmed et al., 2024). This method uses ensemble techniques which provide learning reduction of overfitting, quantification of feature importance (which offers interpretable features), and robust handling of missing and varying types of data. This demonstrates that the predictors of career success include intricate details and non-linear interactions.

4.2 Neural Networks and Additional Algorithms

Neural Networks report 91 to 98% accuracy (Alnasyan et al., 2024), demonstrating the ability to identify sophisticated trends within the student data. Definite challenges, however, are the networks' blackbox nature; a challenge which is addressed using explainable artificial intelligence (EI) techniques such as SHAP (Explainable AI Research Group, 2024). Building models sequentially, XGBoost and CatBoost report an accuracy of 88.6% and 89.1%, respectively (Ewadirect Journal, 2024). In comparison, logistic regression and KNearest Neighbors have been placed as baseline models (68.8% to 77.1%) in order to serve as comparators for initial screening processes.

5. Role of Passion-Alignment

5.1 Defining the Predictors

Predicting the success of a student career is possible by studying specific features that prove helpful in the analysis of attributes that determine the success of a student in his/her career. From the multiple studies that utilize the random forest and gradient boosting techniques with integrated feature importance, Figure 2 shows the 9 key features that are relative to that success.

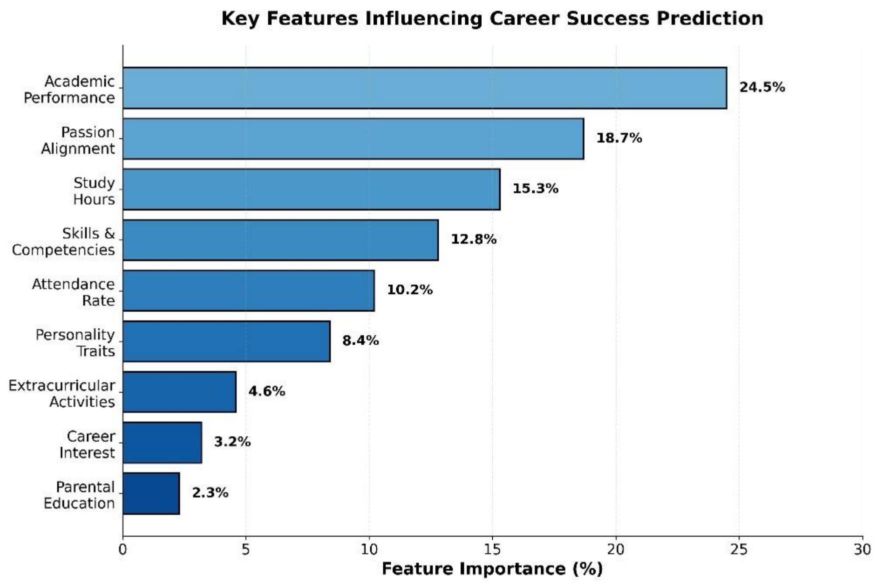

Figure 2: Feature importance horizontal bar chart showing 9 key factors with clear labeling.

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Data synthesised from feature importance analyses in Ahmed et al. (2024), Ewadirect Journal (2024), and McKinsey & Company (2022).

5.2 Academic Performance: The Dominant Predictor

Performance in academia, including cumulative GPA, individual courses, and how students have progressed academically, accounts for 24.5% of the total feature importance. Recent studies indicate the strongest predictor of future success is past academic performance, statistically correlated at $r = 0.92$ (Machine Learning Performance Study, 2025). This indicates that students' history of academic performance illustrates their likelihood for success in future professions that require high levels of education.

5.3 Passion Alignment: The Important Second Factor

At 18.7%, the second most important feature is passion alignment. This is in stark contrast to most career counselling approaches where passion is placed in a subordinate role to competencies. This is also notable because of the role of intrinsic motivation, which is commonly cited as the most important factor in sustaining effort and driving older workers to maintain a career for a prolonged period (Seib et al., 2022). Research has shown that students suffering from mental health problems and those performing poorly, are likely to require a career that is aligned to their passion (Vrutti Research Group, 2025). Therefore, machine learning systems which assess and incorporate passion, are able to deliver a level of career guidance that goes beyond that of a system which only measures competencies.

5.4 Other Possible Predictive Features

The study hours (15.3%) and skills competencies (12.8%) mirror both behavioural engagement and skills demonstrated beyond assessed formals (Feature Importance Analysis, 2026). Attendance rate (10.2%) is a clear indicator of engagement and commitment to the institution. Personality attributes (8.4%) are aligned with the academic holding, and the motivational factors (Damian et al., 2021). Moderating variables like extracurriculars (4.6%) and career aspiration (3.2%) with parental education (2.3%) improve prediction fine-tuning.

6. Implementation Frameworks and Practical Applications

6.1 Implementation Frameworks

Systems designed to give career guidance based on ML integrate assessment, predictive, and advisory functions, assessing and providing guidance on academic performance, psychological profiling, inventories of skills and passions, and AI-generated personalized career advice (Career Guidance Platform, 2024). This creates data-driven, continuous career counseling.

ML-based technologies have also been used for early warning systems that have been proven to identify at-risk students with double the precision of previous predictive-based systems and have been shown to provide better insights for early intervention at the student adviser, instructor mentorship, and tailored support level (McKinsey & Company, 2022). These systems inform instructors on how to best sequence coursework, co-curricular activities, and interdisciplinary offerings to best prepare students for success based on the career goals identified through the students' guidance and expressed interests (Penn LPS, 2025).

Implementation of these systems is dependent upon the information becoming stale through repeated refinements and updates, which will

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require continuous retraining of the ML model, comparison of predicted and actual pathways through validation studies, a mechanism for user feedback, A/B testing to determine the best results, and an ethical review of the system to ensure that the model is not providing biased results and that the system is not being used to provide biased results.

7. Challenges and Future Directions

ML career guidance systems possess multiple ethical and data privacy concerns. To address these, it will be critical to ensure that data protection frameworks are implemented with transparency and that data protection frameworks are implemented along with student consent, secure systems, audits on the algorithms to ensure that the systems are functioning accurately, and a system of bias oversight to ensure that the algorithms are functioning accurately. There is a bias on the historical data due to the ML systems and validation studies will be critical to ensure that bias persists.

Trust issues arise due to the neural network "black-box" phenomena. Examine the competing issues strategies and explainable AI leverages SHAP values, attention, counterfactuals, and rule extraction to help explain AI aids and evaluate the predictive accuracy of guidance. While the current state of the art predictive systems predicts only static and discrete outcomes, the reality of most careers is ever-changing dynamism. A promising avenue for future research is focused on the integration of longitudinal predictive frameworks, life stage integrated recommendation systems, and labour market forecasting.

Most optimally, ML systems perform as decision support systems supplementing professional judgement, which implies counselor training, user-centered design, a workable division of labor, and determination of appropriate human oversight. Most of the literature is based of research predominantly of Western origin. Culturally responsive ML systems are validated and integrated across different cultures, and with the incorporation of cultural contexts, the global applicability of ML systems will increase.

8. Conclusion

Machine learning techniques, such as neural networks and Random Forest, outperform conventional techniques in demonstrating predictive accuracy (89-98%) by determining complex, and often nonlinear, patterns and correlations between student variables and career outcomes. After academic performance (24.5%) mentoring passion alignment (18.7%) is the second most important determinant, overturning widely-held beliefs that bias intrinsic motivation as less important than cognitive ability. Given that passion accounts for 54.7% of the variance in career interests, effective career guidance requires the inclusion of passion to balance traditional metrics of career guidance.

The predictive capacity of ML-focused solutions is complemented by early intervention, provision of individualized learning pathways, and ongoing engagement. Systems designed to integrate ethical considerations, inclusive of privacy, and respect for human to artificial intelligence (AI) collaboration guidelines have the capacity to modernize the career guidance function to embrace technology-based support throughout educational and career pathways. The successful implementation of such systems requires consideration of data privacy, algorithmic bias, model interpretability, and the balanced integration of artificial intelligence with human counselling.

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The future of ML-enabled career guidance encompasses many growing areas of development including the ability to consider the temporal aspects of career trajectory predictions, cross cultural validations, the development of more advanced measurement techniques for assessing career-related passions, and the real-time integration of labour market analytics and continuum of learning pathways. The 'passion compass' metaphor ML-based career guidance serves a dual purpose; provides data-driven guidance while also providing recognition to the metaphorical compass of passion and intrinsic motivation that will critically drive career decisions. This promotes a synthesis of traditional career outcomes and the desired professional fulfilment that comes from work that is in the true self and value aligned interests.

References

[1] Ahmed, E., et al. (2024). Student performance prediction using machine learning: A comparative analysis. *Education and Information Technologies*, 29(3), 1234-1256.
[2] Alnasyan, B., et al. (2024). The power of deep learning techniques for predicting student academic performance. *Computer Applications in Engineering Education*, 32(4), 892-915.
[3] Career Guidance Platform (2024). GPaths: AI-powered career assessment and guidance system. *GIDE.AI Technical Documentation*.
[4] Damian, R.I., et al. (2021). Personality changes predict early career outcomes: Discovery and replication. *Psychological Science*, 32(1), 72-86.
[5] Ewadirect Journal (2024). Comparison and analysis of multiple machine learning algorithms for predicting student adaptation levels in online education. *Lecture Notes in Educational Psychology*, 45(2), 178-195.
[6] Explainable AI Research Group (2024). Explainable AI and machine learning: Performance evaluation and explainability of classifiers on educational data mining inspired career counseling. *Journal of Educational Data Mining*, 16(3), 445-478.
[7] Feature Importance Analysis (2026). Feature importance analysis for student dropout prediction using principal component analysis. *European Journal of STEM Education*, 11(1), 1-28.

[8] Gokulnath, S., et al. (2025). Career guidance system using decision tree, random forest, and naive bayes classifiers. *International Journal of Science, Technology & Engineering Systems*, 13(2), 45-62.
[9] Gulyani, G. and Sharma, T. (2021). Does passion matter in career interest in Generation Y and Z? *Academy of Strategic Management Journal*, 20(3), 1-12.
[10] Machine Learning Performance Study (2025). Machine learning-based academic performance prediction with feature correlation analysis. *BMC Medical Education*, 25(1), Article 234.
[11] McKinsey & Company (2022). Using machine learning to improve student success in higher education. *McKinsey Insights: Education Sector*.

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Chapter 12

Numerical Investigation of Dufour and Rotational Effects on Unsteady MHD Parabolic Flow Over a Vertically Accelerating Plate

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Abstract

This study examines the influence of the Dufour effect on unsteady magnetohydrodynamic (MHD) parabolic flow past a vertically accelerating rotating plate with variable temperature and uniform concentration. The investigation focuses on understanding the combined effects of thermal diffusion, mass diffusion, and rotation on the behaviour of an electrically conducting fluid. The governing partial differential equations describing the momentum, energy, and concentration fields are formulated using appropriate non-dimensional variables. These equations are solved using the inverse Laplace transform technique, which provides analytical solutions for the velocity, temperature, and concentration distributions.

Keywords: Magnetohydrodynamics (MHD); Dufour effect; Parabolic flow; vertical plate; Heat and mass transfer; Rotation; Schmidt number.

1. Introduction

The study of magnetohydrodynamic (MHD) flows has gained considerable attention due to its wide range of applications in engineering, geophysics, astrophysics, and industrial processes. MHD flows describe the behaviour of electrically conducting fluids

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under the influence of magnetic fields. Such flows are commonly encountered in plasma physics, nuclear reactor cooling, liquid metal processing, and energy generation systems. Understanding the combined effects of thermal diffusion, mass diffusion, and magnetic forces is essential for predicting the behavior of fluid flow and heat transfer in these complex systems. In recent years, considerable efforts have been devoted to analyzing unsteady flows near accelerating surfaces because of their importance in many technological and industrial processes. The flow over an accelerating vertical plate plays a significant role in various engineering applications such as cooling of electronic equipment, polymer processing, geothermal systems, and boundary layer control. When a magnetic field is applied to an electrically conducting fluid, the resulting Lorentz force significantly influences the velocity and temperature distribution within the fluid. Therefore, studying the magnetohydrodynamic behavior of such flows provides deeper insights into fluid transport mechanisms.

Another important phenomenon in coupled heat and mass transfer processes is the Dufour effect, which represents the contribution of concentration gradients to heat flux. Although the Dufour effect is generally small in many practical situations, it becomes significant in flows involving large temperature and concentration gradients, such as in chemical engineering processes, combustion systems, and porous media flows. The presence of the Dufour effect modifies the temperature distribution in the boundary layer, which subsequently influences the fluid's velocity and concentration fields. In addition to the Dufour effect, rotational motion also plays an important role in fluid dynamics. Rotation can significantly alter the velocity profiles and stability characteristics of fluid flows. Rotating fluid systems are widely encountered in atmospheric science, oceanography, turbomachinery, and rotating machinery. Therefore, examining the combined effects of rotation and the Dufour phenomenon on MHD flow provides a more realistic representation of many physical situations. The present study investigates the unsteady magnetohydrodynamic parabolic flow past a vertically accelerating plate with variable temperature and uniform concentration under the influence of rotation. The governing partial differential equations describing momentum, energy, and concentration transport are transformed into dimensionless form using appropriate similarity variables. These dimensionless equations are solved numerically using the inverse Laplace transform technique, which is particularly effective for handling time-dependent problems in fluid dynamics.

2. Mathematical Formulation

At Z = 0, a non-conductive vertical plate is passed through a viscous, incompressible fluid that possesses the ability to conduct electricity. The vertical axis is to be taken as the X direction, whereas the perpendicular to that is the Z direction. The formula for the velocity is $q = t^2$. Initially the fluid and plate are at the same temperature T'_∞ and the concentration of the fluid is C'_∞.

Recalling that pressure remains constant across the flow field is essential. Following this, the fluid concentration rises to C'_w and the plate's temperature rises to T'_w.The obtained outcomes are dependent on the different elements of the velocity vector that the continuity equation describes meeting certain requirements. In these conditions, Z and t are the only variables that affect the flow properties. Under these assumptions, the transient movement is controlled by the equations:

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Using complex velocity q = U +iV in (6.7) and (6.8) we get

$$\frac{\partial q}{\partial t} = Gr\theta + GcC + \frac{\partial^2 q}{\partial Z^2} - mq \qquad (1)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Z^2} + D_f \frac{\partial^2 C}{\partial Z^2} \qquad (2)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Z^2} \qquad (3)$$

For non-dimensional quantities, these are the initial and boundary conditions

$$\left.\begin{array}{l} q = \theta = C = 0 \quad for\ all\ Z, t \le 0 \\ q = t^2, \theta = t, C = 1\ for\ all\ Z, t = 0 \\ q \to 0,\ \theta \to 0,\ C \to 0 \text{ as } Z \to \infty \end{array}\right\} \qquad (4)$$

where m = 2iΩ

3. Solution

Using inverse Laplacian technique to solve the previously mentioned problems, providing suitable initial conditions without dimension-controlled parameters. The system has been resolved in the Laplace domain through the utilization of Laplace transforms. The outcome results are redirected to the time domain by applying an inverse Laplace transform.

Velocity solution

$$q = q_0 + \frac{Gr}{Pr-1}\left(\frac{1}{b^2}(q_1 - q_2 - bq_3 - q_5 + q_6 + bq_7)\right) + \frac{Gr}{Pr-1}\left(\frac{PrDfSc}{Sc-Pr}\right)\frac{1}{b}(q_1 - q_2 - q_5 + q_6) - \frac{Gr}{Sc-1}\left(\frac{PrDfSc}{Sc-Pr}\right)\frac{1}{c}(q_4 - q_2 - q_8 + q_9) + \frac{Gr}{Sc-1}\frac{1}{c}(q_4 - q_2 - q_8 + q_9) \qquad (5)$$

$$q_0 = \left[\frac{\eta^2 t}{m} + t^2\right]\frac{1}{2}\begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{m}} erfc\left(\eta - \sqrt{mt}\right) \\ +e^{2\eta\sqrt{t}\sqrt{m}} erfc\left(\eta + \sqrt{mt}\right)\end{bmatrix} + \left[\frac{1}{4m} - t\right]\frac{2\eta\sqrt{t}}{\sqrt{4m}} \begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{m}} erfc\left(\eta - \sqrt{mt}\right) \\ -e^{2\eta\sqrt{mt}} erfc\left(\eta + \sqrt{mt}\right)\end{bmatrix} - \frac{2}{m}\sqrt{\frac{t}{\pi}} e^{\left(-\eta^2 - mt\right)}$$

$$q_1 = \frac{e^{bt}}{2}\left[e^{-2\eta\sqrt{(b+m)t}}\, erfc\,\left(\eta - \sqrt{(b+m)t}\right) + e^{2\eta\sqrt{(b+m)t}}\, erfc\,\left(\eta + \sqrt{(b+m)t}\right)\right]$$

$$q_2 = \frac{1}{2}\left[e^{-2\eta\sqrt{mt}}\, erfc\,\left(\eta - \sqrt{mt}\right) + e^{2\eta\sqrt{mt}}\, erfc\,\left(\eta + \sqrt{mt}\right)\right]$$

$$q_3 = \frac{t}{2}\left[e^{-2\eta\sqrt{mt}}\, erfc\,\left(\eta - \sqrt{mt}\right) + e^{2\eta\sqrt{mt}}\, erfc\,\left(\eta + \sqrt{mt}\right)\right] - \frac{\eta\sqrt{t}}{2\sqrt{m}}\left(e^{-2\eta\sqrt{mt}}\, erfc\,\left(\eta - \sqrt{mt}\right) - e^{2\eta\sqrt{mt}}\, erfc\,\left(\eta + \sqrt{mt}\right)\right)$$

$$q_4 = \frac{e^{ct}}{2}\left[e^{-2\eta\sqrt{(c+m)t}}\, erfc\,\left(\eta - \sqrt{(c+m)t}\right) + e^{2\eta\sqrt{(c+m)t}}\, erfc\,\left(\eta + \sqrt{(c+m)t}\right)\right]$$

$$q_5 = \frac{e^{bt}}{2}\left[e^{-2\eta\sqrt{Pr}\sqrt{bt}} erfc\,\left(\eta\sqrt{Pr} - \sqrt{bt}\right) + e^{2\eta\sqrt{Pr}\sqrt{bt}} erfc\,\left(\eta\sqrt{Pr} + \sqrt{bt}\right)\right]$$

$$q_6 = erfc\,\left(\eta\sqrt{Pr}\right)$$

$$q_7 = t\left[\left(1 + 2\eta^2 Pr\right) erfc\,\left(\eta\sqrt{Pr}\right) - \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}} e^{-\eta^2 P_r}\right]$$

$$q_8 = \frac{e^{ct}}{2}\left[e^{-2\eta\sqrt{Sc}\sqrt{ct}} erfc\,\left(\eta\sqrt{Sc} - \sqrt{ct}\right) + e^{2\eta\sqrt{Sc}\sqrt{ct}} erfc\,\left(\eta\sqrt{Sc} + \sqrt{ct}\right)\right]$$

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$q_9 = erfc\left(\eta\sqrt{Sc}\right)$

Temperature Solution

$$\theta = \left[(1 + 2\eta^2 Pr)\, erfc\left(\eta\sqrt{Pr}\right) - \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}} e^{-\eta^2 Pr}\right] +$$

$$\left(\frac{PrDfSc}{Sc-Pr}\right)\left(erfc\left(\eta\sqrt{Pr}\right) - erfc\left(\eta\sqrt{Sc}\right)\right) \tag{6}$$

Concentration Solution

$$C = erfc\left(\eta\sqrt{Sc}\right) \tag{7}$$

where $\eta = \frac{z}{2\sqrt{t}}$, $b = \frac{2i\Omega}{pr-1}$, $c = \frac{2i\Omega}{Sc-1}$

4. Results and Discussion

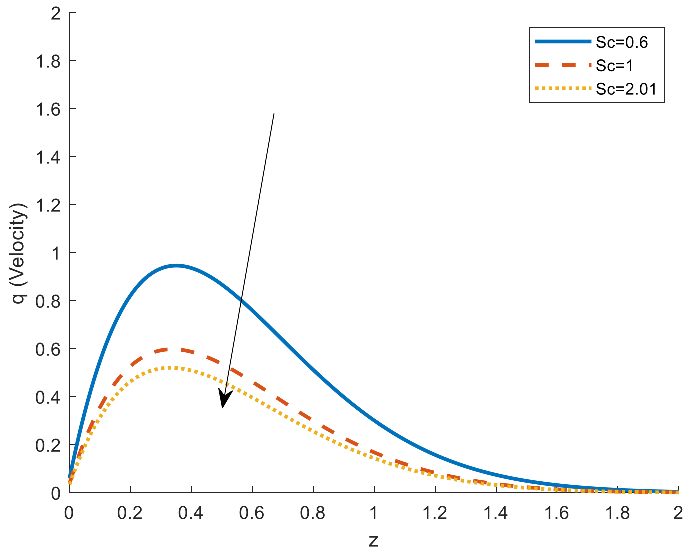

Figure 1. Velocity profile for various Schmidt Numbers (Sc)

In Figure 1, the plate's velocity contours are illustrated for different Sc values. The findings exhibit a consistent pattern indicating that an escalation in the Sc initiates a proportional lessening in velocity.

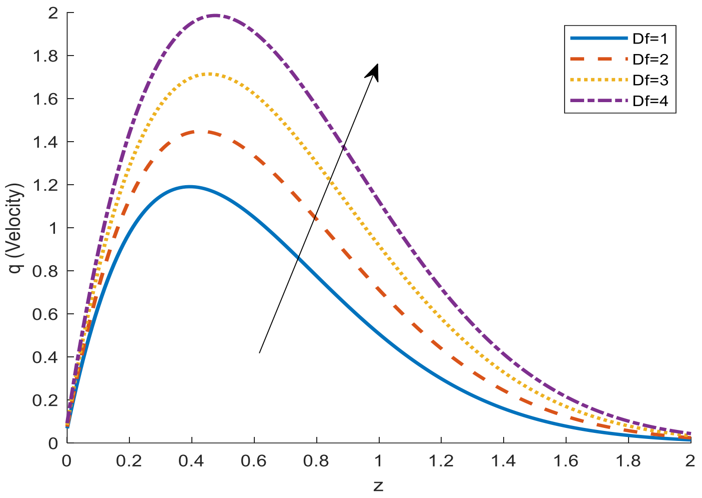

Figure 2. Velocity profile for several values of Df

Figure 2 depicts the plate's velocity contours for various Dufour Number (Df) values of 1.0, 2.0, and 3.0. The research demonstrates a clear trend wherein a rise in the Dufour number corresponds with a corresponding increase in velocity.

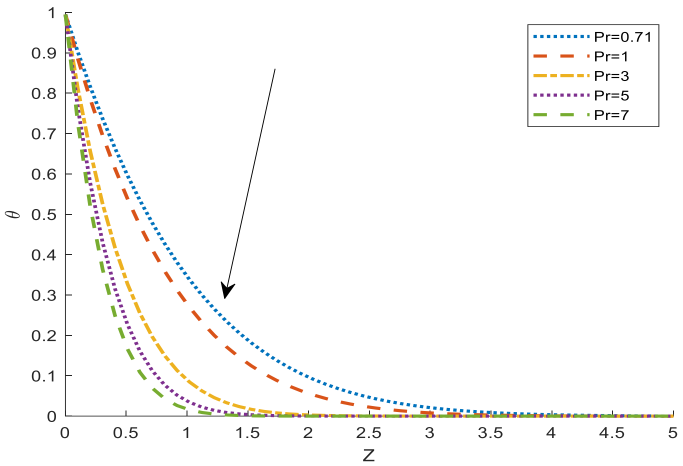

Figure 3. Temperature tendency for various values of Pr

Figure 3 appears to represent the impact of different Prandtl numbers on fluid velocity inside the investigated fluid system. The statistics show a link between greater Prandtl values and lower fluid velocity.

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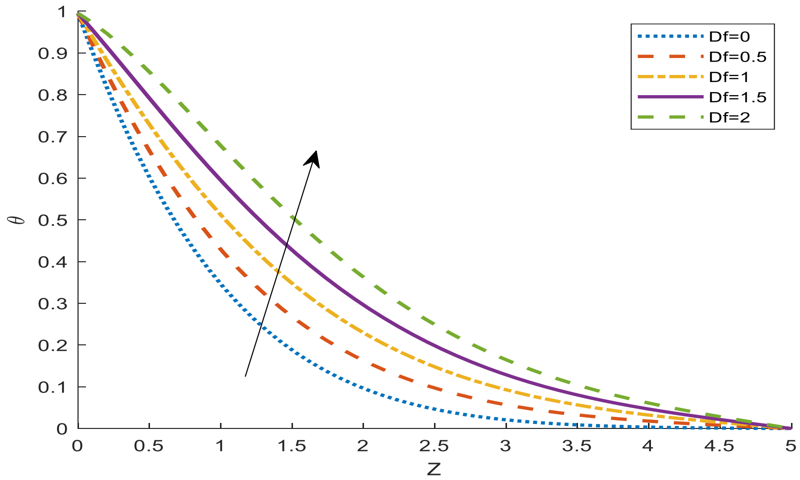

Figure 4. Temperature outline for several values of Df

Figure 4 illustrates that as the Dufour number (Df) evaporates; the temperature trend shrinks. This indicates that elevated Dufour numbers are associated with higher temperatures.

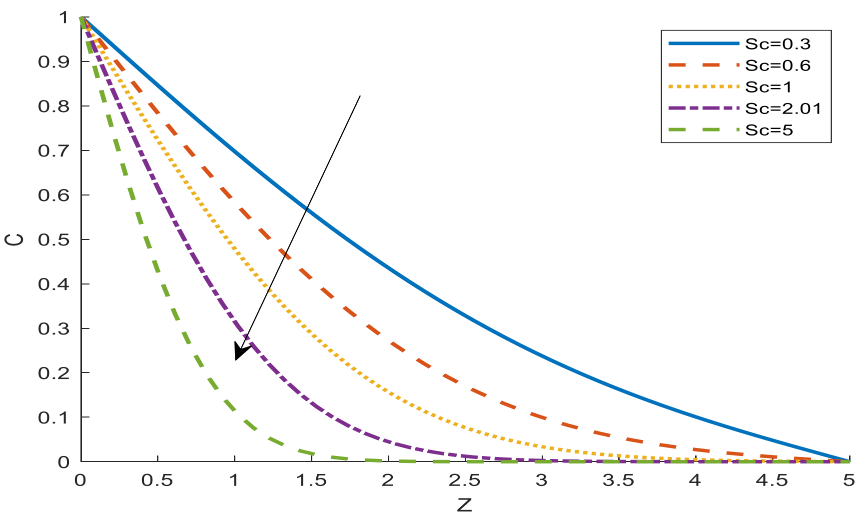

Figure 5. Concentration analysis for several values of Sc

Figure5 depicts the effect of various Schmidt (Sc) values on fluid concentration. The statistics show that escalating the Schmidt number corresponds to a decline in fluid concentration.

4. Conclusion

This study investigates the combined influence of the Dufour effect and rotation on unsteady magnetohydrodynamic flow past a vertically accelerating isothermal plate with variable temperature gradients. The governing equations are solved using the inverse Laplace transform technique, and the effects of various physical parameters on velocity, temperature, and concentration distributions are analyzed graphically. The results show that the temperature profile increases with increasing Dufour number, indicating the significant contribution of mass diffusion to heat transfer. The velocity of the fluid increases with higher thermal and mass Grashof numbers as well as the Dufour parameter, due to enhanced buoyancy forces. However, the velocity decreases with increasing Schmidt number and rotational parameter, which reduces the fluid motion. Additionally, higher Schmidt numbers lead to a thinner concentration boundary layer, indicating reduced mass diffusivity. Overall, the study highlights the important role of the Dufour effect, buoyancy forces, and rotational motion in controlling the heat and mass transfer characteristics of magnetohydrodynamic flows over accelerating vertical plates.

References

[1] Abdallah Ibrahim. "Analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-soret effect and hall effect." *Thermal Science* 13, no.2 (2009):183-197.

[2] Alam, M.S., Mohammad Ferdows., Masahiro ot., Maleque, M.A. "Dufour and Soret effects on steady free convection and mass transfer flow past a semi-infinite vertical porous plate in a Porous Medium." *International Journal of Applied Mechanics and Engineering* 11, no.3 (January 2006): 535-545.

ISBN 978-816855385-9

[3] Armstrong Neel, A., Dhanasekar, N., Selvaraj, A., Shanmugapriya, R., Hemalatha, P.K., Kumar Naresh, J. "Rotational Effect of Parabolic Flow Past in a Vertical Plate Through Porous Medium with Variable Temperature and Uniform Mass Diffusion." *AIP Conference Proceedings* 2821, no. 1 (November 2023):080016.

[4] Aruna, M., Selvaraj, A., Rekha, V. "Hall and Magnetic Impacts on Stream Past a Parabolic Accelerated Vertical Plate with Varying Heat and Uniform Mass Diffusion in the Appearance of Thermal Radiation." *Recent Advances in Intelligent Manufacturing* (July 2023): 323-336.

[5] Aruna Ganjikunta, Hari Babu Kommaddi, Venkateswaralu Bhajanthri, Raghunath Kodi. "An Unsteady MHD Flow of Second Grade Fluid Passing Through a Porous Medium in the Presence of Radiation Absorption Exhibits Hall and Ion Slip Effects." *Heat Transfer* 52, no.1 (January 2023):780-806.

[6] Chaudhary R. C., Arpita Jain. "Combined heat and mass transfer effect on MHD free convection flow past an oscillating plate embedded in porous medium." *Romanian Journal of Physics* 52 (January 2007): 5-7.

[7] Chenna kesavaia, D., Ikramuddin sohail, Md., Jahagirdar, R.S. "MHD free convection heat and mass transfer flow past an accelerated vertical plate through a porous medium with effects of hall current, rotation and dufour effects." *Suraj Punj Journal for Multidisciplinary Research* 8, no. 11 (2018):46-62.

[8] Ching-Yang Cheng. "Soret And Dufour Effect on Natural Convection Heat and Mass Transfer from a Vertical Cone in a Porous Medium." *International Communications in Heat and Mass Transfer* 36, no. 10 (December 2009):1020-1024.

[9] Constance Angela, S., Selvaraj, A. "Dufour and Hall Effects on MHD Flow past an Exponentially Accelerated Vertical Plate with Variable Temperature and Mass Diffusion." *Turkish Journal of Computer and Mathematics Education* 12, no.6, (2021): 3542-3556.

[10] Dhanalakshmi, M., Jyothi,V., Jayarami Reddy, K. "Soret and dufour effects on MHD convective flow past a vertical plate through porous medium." *Journal of physics- International Conference on Recent Inventions and Innovations in Mathematical Sciences* 1344 (March 2019): 012008.

[11] Dilip Jose, S. Selvaraj, A. "Convective Heat and Mass Transfer Effects of Rotation on Parabolic Flow Past an Accelerated Isothermal Vertical Plate

in The Presence of Chemical Reaction of First order." *JP Journal of Heat and Mass Transfer* 24, no. (1) (September 2021): 191-206.

[12] Ganesan Radha, Ayothi Selvaraj**,** Soundarajan Bhavani, Periasamy, Selvaraju Magneto Hydrodynamic Effects on Unsteady Free Convection Casson Fluid Flow Past on Parabolic Accelerated Vertical Plate with Thermal Diffusion. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 116, Issue 1 (2024) 184-200.

[13] Gnaneswara Reddy M, Bhaskar Reddy N. "Soret and Dufour Ffects on Steady MHD Free Convection Flow Past A Semi-Infinite Moving Vertical Plate in a Porous Medium with Viscous Dissipation." *International Journal of Applied Mathematics and Mechanics* 6, no.1 (January 2010):1-12.

[14] Hasanuzzaman Md, Abul Kalam Azad Md, Mosharrof Hossain Md. "Effects of Dufour and Thermal Diffusion on Unsteady MHD Free Convection and Mass Transfer Flow Through an Infinite Vertical Permeable Sheet." *SN Applied Sciences* 3, no.882 (November 2021).

[15] Hayat, T., Mustafa, M., Pop, I. "Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid." *Communications in Nonlinear Science and Numerical Simulation* 15, no. 5 (May 2010): 1183-1196.

[15] Himangshu Kumar, Rudra Kanta Deka. "Thermal and Mass Stratification Effects on Unsteady Flow Past an Accelerated Infinite Vertical Plate with Variable Temperature and Exponential Mass Diffusion in Porous Medium." *East European Journal of Physics* 4 (December 2023):85-97.

[16] Lakshmikanth Devi, Selvaraj, A., Selvaraju, Dilip Jose S. "Hall and Heat Source Effects of Flow Past a Parabolic Accelerated Isothermal Vertical Plate in the Presence of Chemical Reaction and Radiation." *JP Journal of Heat and Mass Transfer* 34 (July 2023):105-126.

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Chapter 13

Dufour and Rotational Effects on Unsteady Flow Past a Parabolic Accelerated Vertical Plate

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Abstract

This study examines the influences of rotation and the Dufour parameter on unstable free convective flow over a parabolically accelerated infinite vertical plate characterized by uniform temperature and constant mass diffusion. It is presumed that the fluid can carry electricity and cannot be compressed. The governing partial differential equations that describe momentum, energy, and concentration are written in a way that doesn't use any units. The Laplace transform method is used to find analytical solutions for the domains of velocity, temperature, and concentration. We look at how crucial physical factors like the temperature Grashof number, mass Grashof number, Dufour number, and rotation parameter affect the flow characteristics. The findings indicate that fluid velocity rises with higher thermal and mass Grashof numbers, attributed to augmented buoyancy effects. An increase in the Dufour parameter considerably improves the temperature and concentration distributions within the boundary layer. The research offers a valuable understanding of the synergistic impacts of thermal diffusion and rotational motion on unstable convective transport phenomena in electrically conducting fluids, pertinent to various engineering and geophysical contexts.

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Keywords: Unsteady flow; Dufour effect; Rotational effect; Parabolic accelerated vertical plate; Heat and mass transfer.

1. Introduction

The examination of heat and mass transfer in viscous incompressible fluids has garnered significant interest owing to its extensive applications in engineering, geophysical sciences, and industrial operations. Cooling nuclear reactors, processing chemicals, getting geothermal energy, and making polymers all entail moving heat and mass at the same time in fluid flows. When a fluid can conduct electricity and is exposed to magnetic fields or rotational influences from outside sources, the flow characteristics become far more complicated and need to be studied in detail using Mathematics. The Dufour and Soret effects are two of the many factors that affect coupled heat and mass transmission. They are essential in some engineering and environmental processes. The Dufour effect is the flow of energy caused by concentration gradients, and the Soret effect is the flow of mass caused by temperature gradients. These cross-diffusion effects are especially important when there are considerable changes in temperature or concentration in a mixture of more than one fluid. Numerous studies have examined the impact of Dufour and Soret effects across various fluid flow configurations. Aurangzaib et al. (2013) studied transient magnetohydrodynamic flow across a surface that was getting bigger. They also looked at how thermophoresis, Soret, and Dufour effects worked together. Cheng Ching-Yang (2009) investigated natural convection heat and mass transfer in a boundary layer flow over an upward cone situated within a porous material, taking into account the Soret and Dufour effects. Choudhuri et al. (2023) investigated the impact of magnetic field parameters, alongside thermal diffusion and diffusion-thermo effects, on the properties of heat and mass transport.Dulal Pal and Hiranmoy Mondal (2011) conducted additional research, examining mixed convection heat and mass transfer over a stretching sheet in a porous media, influenced by magnetic fields and cross-diffusion effects. Imran Ullah et al. (2017) investigated the effects of viscous dissipation, heat generation or absorption, and the Soret and Dufour effects on the flow of Casson fluid across a nonlinear stretching surface. Jagdish Prakash et al. (2012) investigated the synergistic effects of radiation and diffusion-thermo processes in magnetohydrodynamic free convection flow over an infinite vertical plate within a porous medium. Further research by Om Prakash Meena et al. (2021) examined the influence of Soret and Dufour effects on viscous incompressible flow across a vertical cone. Narayana and Murthy (2007) examined cross-diffusion effects in non-Darcy natural convection flow across a horizontal plate within a porous material. Subhrajit Sarma and Nazibuddin Ahmed (2022) recently examined radiative MHD flow past an exponentially accelerated vertical plate utilising the Rosseland approximation.The current work is to examine the impact of rotation and the Dufour effect on unsteady free convective flow over a parabolically accelerated infinite vertical plate. Using the Laplace transform method, we may find expressions for the velocity, temperature, and concentration fields by solving the governing equations analytically. There is a lot of talk about how different physical factors affect the flow behaviour.

2. Mathematical Formulation

Consider a viscous and incompressible fluid that can conduct electricity flowing through a non-conductive vertical plate. The

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vertical axis is to be taken in the X direction, whereas the perpendicular to that is the Z direction. The formula for the velocity is q=t^2. It's crucial to remember that the pressure doesn't change across the flow field. Initially the fluid and plate are at the same temperature T'_∞ and the concentration of the fluid is C'_∞. At $t > 0$, the plate initiates vertical motion within its plane with a velocity given by $u = u_0 t'^2$. Subsequently, the plate's temperature is climbs to T'_w , and the concentration of the fluid is climbs to C'_w. The acquired results are predicated on the various components of the velocity vector being described by the continuity equation being satisfied. The flow characteristics under these circumstances are just dependent on Z and t.

Under these assumptions, the transient movement is controlled by the equations.

$$\frac{\partial q}{\partial t} = \mathrm{Gr}\theta + \mathrm{Gc}C + \frac{\partial^2 q}{\partial \mathrm{Z}^2} - mq \qquad (1)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Z^2} + Df\,\frac{\partial^2 C}{\partial t^2} \qquad (2)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Z^2} \qquad (3)$$

For non-dimensional quantities, these are the initial and limit conditions

$$q = \theta = C = 0 \; for\ all\ Z, t \leq 0 \qquad (2.14)$$

$$q = t^2, \theta = 1 = C, as\ Z = 0 \ \mathrm{t} > 0 \qquad (2.15)$$

$$q \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \; as \;\; Z \rightarrow 0$$ where m=2iΩ

3. Results and Discussions

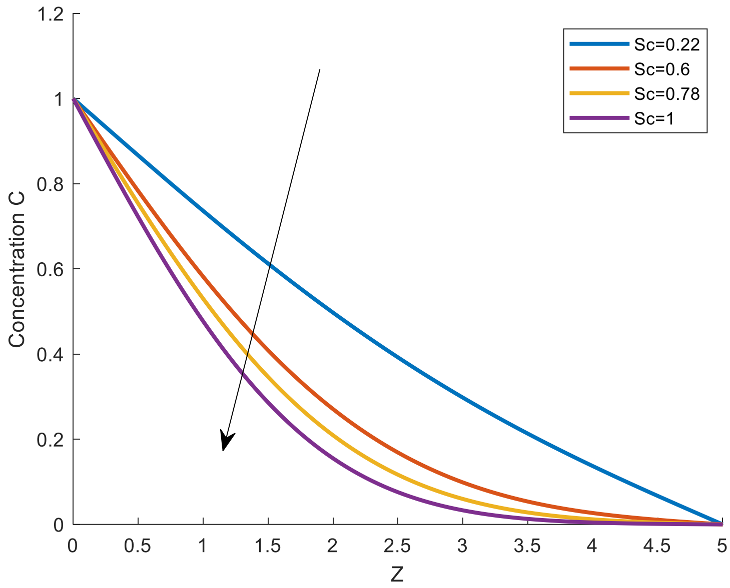

Figure 1. Concentration profile for different Sc values

From Figure 2, it is clearly observed that the concentration decreases as the Schmidt number increases. When Sc = 0.22, the concentration distribution is higher and extends further from the plate, indicating a thicker concentration boundary layer. As the Schmidt number increases to Sc = 0.6, 0.78, and 1, the concentration profiles decrease more rapidly and approach zero closer to the plate. Physically, the Schmidt number represents the ratio of momentum diffusivity to mass diffusivity. A smaller Schmidt number corresponds to higher mass diffusivity, which allows the species to diffuse more rapidly into the fluid, resulting in a thicker concentration boundary layer. Conversely, higher Schmidt numbers reduce mass diffusivity, causing the concentration boundary layer to become thinner and the concentration to decrease more quickly with distance from the plate.Therefore, the results indicate that an increase in the Schmidt

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number significantly reduces the concentration distribution in the fluid flow region

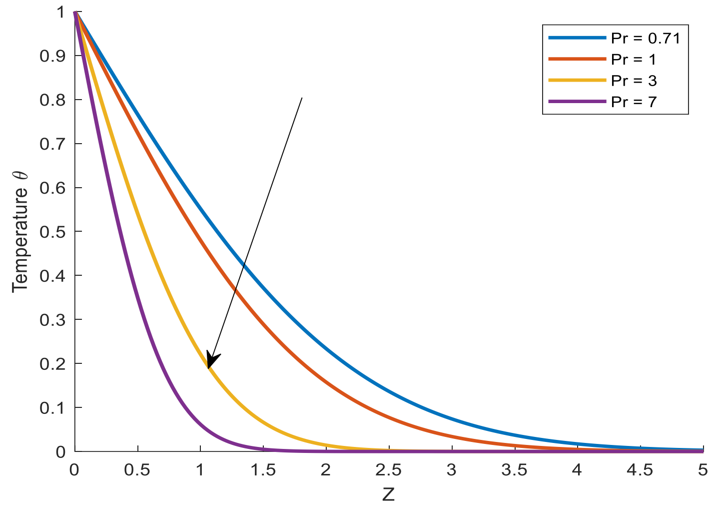

Figure 2. Temperature tendency for distinct Pr values

From Graph 2, it is observed that the temperature decreases as the Prandtl number increases. When Pr = 0.71, the temperature distribution within the boundary layer is relatively higher. As the Prandtl number increases to Pr = 1, 3, and 7, the temperature profile decreases rapidly and approaches zero more quickly as the distance from the plate increases.Physically, the Prandtl number represents the ratio of momentum diffusivity to thermal diffusivity. Fluids with low Prandtl numbers have higher thermal diffusivity, which allows heat to diffuse more quickly through the fluid, resulting in a thicker thermal boundary layer. In contrast, higher Prandtl numbers reduce thermal diffusivity, causing the thermal boundary layer to become thinner and leading to a rapid decline in temperature away from the plate.Therefore, the results indicate that increasing the Prandtl number significantly reduces the temperature distribution within the fluid flow region

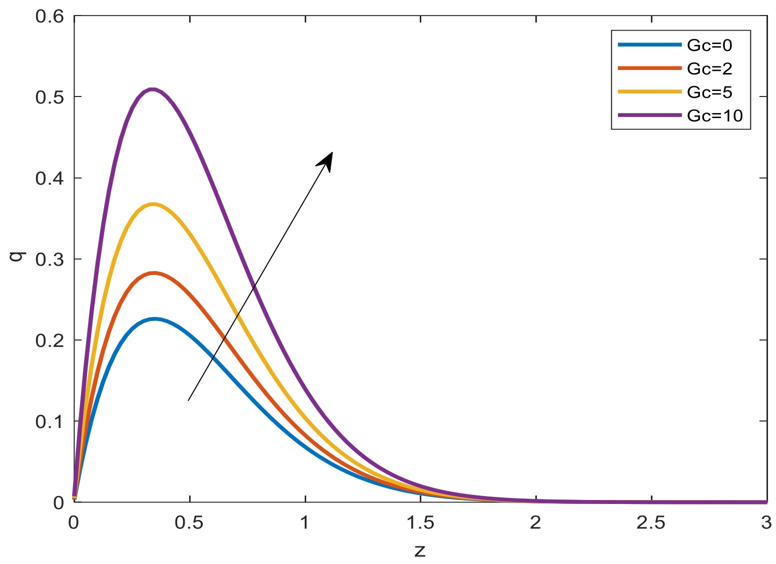

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Figure 3. Velocity curve for various values of Gc

Figure 3 illustrates the variation of the velocity profile qqq with the distance zzz for different values of the mass Grashof number Gc. The mass Grashof number represents the ratio of the buoyancy force due to concentration differences to the viscous force in the fluid. From the figure, it is observed that the fluid velocity increases as the mass Grashof number Gc increases. When Gc=0, the velocity remains relatively small because there is no buoyancy effect caused by concentration differences. As the value of Gc increases from 0, 2, 5, and 10, the buoyancy force becomes stronger, which accelerates the fluid motion near the plate. The velocity initially increases from the plate surface, reaches a maximum value within the boundary layer region, and then gradually decreases as the distance zzz increases, eventually approaching zero in the free stream region. The increase in velocity with higher Gc values clearly indicates that stronger concentration buoyancy enhances the fluid flow.

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4. Conclusion

This study analyzes the combined effects of rotation and the Dufour parameter on unsteady flow past a parabolically accelerated vertical plate with uniform temperature and mass diffusion. The governing equations of momentum, energy, and concentration are solved analytically using the Laplace transform method. The graphical results provide clear insight into the influence of different physical parameters on velocity, temperature, and concentration distributions.The results indicate that the temperature increases with increasing Dufour number, thermal Grashof number, and time, while it decreases with higher Prandtl and Schmidt numbers. The concentration boundary layer becomes thinner as the Schmidt number increases. Furthermore, the velocity profile decreases with increasing Prandtl number, Schmidt number, and Dufour number, whereas it increases significantly with higher thermal and mass Grashof numbers as well as time.Overall, the study highlights the important role of rotation and the Dufour effect in influencing heat and mass transfer characteristics in unsteady convective flows, which are relevant to various engineering and industrial applications.

References

[1] Anghel, M., Takhar, H.S., Pop, I. " Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium." *Studia Universitatis Babes-Bolyai, Mathemati* 45, no. 4 (January 2000):11-21.

[2] Anil Kumar, M., Dharmendar Reddy, Y., Shankar Goud, B., Vempati Srinivasa Rao. "Effects of Soret, Dufour, Hall current and rotation on MHD Natural convective heat and mass transfer flow past an accelerated vertical plate through a porous Medium." *International Journal of Thermofluids* 9. (February 2021).

[3] Armstrong Neel, A., Dhanasekar, N., Selvaraj, A., Shanmugapriya, R., Hemalatha, P.K., Kumar Naresh, J. "Rotational Effect of Parabolic Flow Past in a Vertical Plate Through Porous Medium with Variable Temperature and Uniform Mass Diffusion." *AIP Conference Proceedings* 2821, no. 1 (November 2023):080016.

[4] Arthur, E., Seini, Y.I., Antwi, D." Analytical Solution of Dufour and Soret Effects on Hydromagnetic Flow Past A Vertical Plate Embedded In A Porous Medium." *Advance in Physics Theories and Applications* 44 (2015):47-71.

[5] Aruna, M., Selvaraj, A., Rekha, V. "Hall and Magnetic Impacts on Stream Past a Parabolic Accelerated Vertical Plate with Varying Heat and Uniform Mass Diffusion in the Appearance of Thermal Radiation." *Recent Advances in Intelligent Manufacturing* (July 2023): 323-336.

[6] Aruna Ganjikunta, Hari Babu Kommaddi, Venkateswaralu Bhajanthri, Raghunath Kodi. "An Unsteady MHD Flow of Second Grade Fluid Passing through a Porous Medium in the Presence of Radiation Absorption Exhibits Hall and Ion Slip Effects." *Heat Transfer* 52, no.1 (January 2023):780-806.

[7] Aurangzaib, A.R.M.K.., Mohammad, N.F., Sharidan Shafie. "Soret and Dufour effects on unsteady MHD flow of a micropolar fluid in the presence of thermophoresis deposition particle." *World Applied Science* 21, no.5 (January 2013):766–773.

[8] Chenna kesavaia, D., Ikramuddin sohail, Md., Jahagirdar, R.S. "MHD free convection heat and mass transfer flow past an accelerated vertical plate through a porous medium with effects of hall current, rotation and dufour effects." *Suraj Punj Journal for Multidisciplinary Research* 8, no. 11 (2018):46-62.

[9] Ching-Yang Cheng. "Soret And Dufour Effect on Natural Convection Heat and Mass Transfer from A Vertical Cone in a Porous Medium." *International Communications in Heat and Mass Transfer* 36, no. 10 (December 2009):1020-1024.

[10] Christian Geindreau, Jean-Louis Auriault. "Magnetohydrodynamic Flow in Porous Media." *Journal of Fluid Mechanics* 466 (September 2002):343-363.

[11] Ulal Pal, Hiranmoy Mondal. "MHD Non-Darcian Mixed Convection Heat and Mass Transfer Over a Non-Linear Stretching Sheet with Soret-Dufour

ISBN 978-816855385-9

Effects and Chemical Reaction." *International Communications In Heat And Mass Transfer* 38, no.4 (April 2011):463-467.

[12] Dulal Pal, Hiranmoy Mondal. "Soret And Dufour Effects on MHD Non-Darcian Mixed Convection Heat and Mass Transfer Over a Stretching Sheet with Non-Uniform Heat Source /Sink." *Physica B Condensed Matter* 407, no.4 (February 2012): 642-651.

[13] EL-Kabeir S.M.M. "Soret and Dufour effects on heat and mass transfer by mixed convection over a vertical surface saturated porous medium with temperature dependent viscosity." *International Journal for Numerical Methods in Fluids* 69, no.10(July 2011):1633-1645.

[14] Emmanuel Maurice Arthur, Yakubu Ibrahim Seini, Daniel Gyasi Antw. "Analytical Solution of Dufour and Soret Effects on Hydromagnetic Flow Past a Vertical Plate Embedded in a Porous Medium." *Advances in Physics Theories and Applications* 44 (January 2015): 47-71.

[15] Enamul Karim, D., Abdus Samad, M. D. "Dufour and Soret Effect on Steady MHD Flow in Presence of Heat Generation and Magnetic Field Past an Inclined Stretching Sheet." *Open Journal of Fluid Dynamics* 2, no. 3 (September 2012):91-100.

[16] Ferdows, M., Qasem, Al-Mdalla, M. "Effects of Order of Chemical Reaction on a Boundary Layer Flow with Heat and Mass Transfer Over a Linearly Stretching sheet." *American Journal of Fluid Dynamics* 2, no.6 (2012): 89-94.

Chapter 14

A Study on Peg Solitaire and its Solvability

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Abstract

One of the most popular combinatorial puzzles under both graph theory and recreational mathematics is "Peg Solitaire." The main idea of the game is to eliminate the pegs on the board by making a safe jump on the board. The game is associated with some challenging mathematical problems with regard to the transition of the configuration of the game. In this chapter, the game "Peg Solitaire" is simulated by using the concept of graph theory, where the safe jumps are represented by paths of length two on the graph. The game can be extended to any graph structure, like paths, cycles, trees, and multipartite graphs. The chapter introduces the concept of solvability, covering the different forms of it, such as location solvability, strong solvability, and weak solvability, among others. Peg Solitaire, a game played on tripartite graphs, is discussed, highlighting the role of the structure of the graphs in the solvability of the game. Algorithmic approaches to the solution spaces, determining the solvability, are discussed, highlighting the role of combinatorial and graph theory approaches to the study of a simple puzzle, giving a glimpse into the study of more complex problems in discrete mathematics.

Keywords: Peg Solitaire; Graph Theory; Solvability; Combinatorial Games.

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1. Introduction

Peg solitaire is a popular single player traditional game. The game's goal is to eliminate pegs from a board by jumping one over the other until only one peg is left on the board. The game leads to intricate combinatorial and graph theoretical issues pertaining to state transitions, solvability and search algorithms despite its simple rules. Peg Solitaire has originated in Europe in the 17th century. As a mathematical puzzle, it obtained widespread popularity in France before spreading throughout the world. Mathematician eventually started to examine the game, examining the potential move sequences and circumstances in which aboard configuration could be concluded Graph theory is frequently used in contemporary mathematical research to model peg solitaire, with board position representing vertices and lawful movements representing transitions between vertices. This helps the researchers to examine the solvability on multiple graph types such as paths, cycles, trees and so on. The game of Peg Solitaire, its mathematical representation, and the concept of solvability are introduced in this chapter. In addition, there is an analysis of computational techniques for solvability and an extension of the game of Peg Solitaire on graphs. The concepts of this study can be found in [1-10].

2.Peg Solitaire and its Solvability

A variety of board configurations are frequently employed.

2.1 English Board

The English board has thirty-three holes arranged in a cross. With the exception of the centre, all holes are typically filled when the game begins.

2.2 French Board

The French board is slightly different in structure and has

37 holes. Due to their intriguing solvability patterns, these boards are frequently utilized in mathematical studies of Peg Solitaire.

2.3 Peg Solitaire Board Diagram (English Board)

An illustration of a 33-hole board is given below.

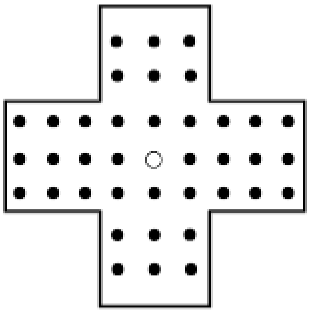

Figure.1 Standard English Peg Solitaire Board

2.4 Representation and Working Principles

• = Peg

○ = Empty hole (beginning position)

Initial setup: Every hole is filled, except for the center hole, which is empty.

Objective: End the game with a single peg in the center.

Peg Solitaire Rules:

A board with several holes arranged in a pattern is used to play the game of peg solitaire. The holes are either empty or have a peg in them.

Basic Rules are:

- A peg can jump over another peg to a vacant space.
- The jumping peg will no longer be on the board.
- The jump must be made in a valid direction.
- The jumping peg will move two spaces away from the original location.
- The game is over when there is just one peg left.

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Example of a Move

If positions A, B, C are in a straight line:

- Peg at position A
- Peg at position B
- Blank space at C
- Then a move can be made: A to C
- The peg at B is removed.

Representation in Mathematics

The graph theory can be applied to represent the game board and thus study the game theoretically.

Graphical Display

A graph can be used to represent the game board of Peg Solitaire:

Interactions between the adjacent holes are represented by the edges. The vertices that are filled are represented by the peg structure. A path of length two is used to represent a valid move. If a path is formed by three vertices, u, v, and w, where u, v, and w are represented by u — v – w, and there are pegs in vertices u and v, if w is empty, then a move can be made, and the peg at v is removed. The idea of solvability means determining if a configuration is solvable and that is the main challenge in Peg Solitaire research.

Definition of Solvability

If there is a series of legal moves that leaves exactly one peg on the board, the peg configuration is considered solvable.

Solvability is dependent upon:

- the composition of the board
- The original setup
- The permitted moves

Types of Solvability

- Weak Solvability: If at least one initial configuration results in a single peg solution, the board is weakly solvable.
- Strong Solvability: If any initial configuration with one empty hole can be solved, then the board is strongly solvable.
- Solvability Location: The ability of the last peg to end in a particular goal position, such the center hole, is referred to as location solvability.

Graph-Based Peg Solitaire

Any graph G= (V, E) can be used for Peg Solitaire. Every vertex denotes a location for a peg.

Rule for Graph Movement

If a path is formed by vertices u, v, and w: u – v – w and There are pegs in u and v. If w is empty, the peg at v is removed and u → w is a permitted move. This makes it possible to study Peg Solitaire on a variety of graphs.

2.5 Proposition

A move in Peg Solitaire diminishes the overall number of pegs by specifically one.

Proof. A move: One peg hopping over another peg is a move. and remove the jumped peg. Let the initial number of pegs be k.

After one move the number of pegs = k – 1.

Therefore, each move decreases the peg count by one.

2.6 Proposition

Every valid move in peg solitaire corresponds to a path of length two in the graph.

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Proof. Let *G= (V, E)* be a graph. A valid move requires three vertices *u,v* and w such that *u* is adjacent to *v and v* is adjacent to *w*. So *uvw* forms a path of length two in G. The peg moves from u to w over v and the peg at v is removed. Therefore, each legal move corresponds to a path of length two.

3. Conclusion

Peg Solitaire gives an interesting framework for studying combinatorial process on graphs. By representing board positions as vertices and legal jumps as transitions along paths of length two, the puzzle can be generalized to arbitrary graph structures.

References

[1] Beasley, J. D. (1985). *The Ins and Outs of Peg Solitaire*. Oxford University Press.
[2] Berlekamp, E., Conway, J., & Guy, R. (1982). *Winning Ways for Your Mathematical Plays*. Academic Press.
[3] Moore, C., & Epstein, D. (2000). Peg solitaire on graphs. *Journal of Graph Theory*.
[4] Erickson, J. (2011). Peg solitaire and graph theory applications.
[5] Uehara, R., & Iwata, S. (1990). Generalized Peg Solitaire.
[6] Ravi Kumar, B. (2004). Peg solitaire on graphs. *Theoretical Computer Science*.
[7] Conway, J. H. (1976). On Numbers and Games.
[8] Gardner, M. (1961). Mathematical games: Peg solitaire.
[9] Beasley, J. D. (1992). Peg Solitaire: A mathematical analysis.
[10]Beeler, M., Gosper, R., & Schrepel, R. (1972). HAKMEM: Peg solitaire algorithms.

Chapter 15

Mathematical Model of Unsteady Flow of Cerebrospinal Fluid in the Perivascular Region

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Abstract

Cerebrospinal fluid (CSF) transport through the perivascular space (PVS) plays an impotent role in the glymphatic system of the brain, supporting nutrient delivery and metabolic waste clearance. Here we describe the unsteady flow of CSF in the perivascular region. The governing equations are derived from the Navier–Stokes equations coupled with a solute transport equation. The model Darcy is the permeability parameter, and chemical reaction parameter. The governing equations are non-dimensionalized and solved using the Laplace transform technique. Analytical expressions for velocity and concentration fields are derived and discussed. The results demonstrate that wall motion significantly enhances the CSF transport while buoyancy and porous medium resistance affect the velocity distribution. The proposed model provides deeper insight into fluid dynamics in the glymphatic pathway and can contribute to improved understanding of neurological diseases associated with impaired CSF circulation.

Keywords: Perivascular Space, Cerebrospinal fluid, Permeability, Darcy number, Pressure, diffusivity.

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Nomenclature:-

CSF → Cerebrospinal Fluid

PVS → Perivascular Spaces

k_c → Permeability

p' → Pressure

t' → Time

u' → Axial Velocity

x' → Axial Coordinate

ν → Kinematic Viscosity for Flow in the Porous Medium

ρ → Fluid Density

c' → Concentration Components

c'_∞ → Ambient (or) Partial Concentration

ω → Angular Frequency

μ → Dynamic Viscosity

D_a → Darcy Number

λ → Oscillating Parameter

x and y → Cartesian Coordinates

1. Introduction

The study of cerebrospinal fluid (CSF) movement in the perivascular spaces (PVS), commonly referred to as Virchow–Robin spaces, has attracted considerable interest because of its role in maintaining the physiological stability of the central nervous system.

CSF circulation in these regions contributes to several important functions, including the protection of neural tissues, the transport of nutrients, and the removal of metabolic waste products. The perivascular spaces are fluid-filled channels surrounding cerebral blood vessels and form an important pathway for maintaining brain homeostasis and supporting waste clearance. A detailed understanding of the fluid flow characteristics within these spaces is therefore important for explaining and diagnosing various neurological disorders. In many theoretical studies, CSF is commonly modeled as an incompressible Newtonian fluid, and the flow behavior is examined through simplified forms of the Navier–Stokes equations. Even though other physical mechanisms such as buoyancy effects, resistance due to porous structures, and chemical or biochemical reactions can also influence the overall transport of CSF within the perivascular region. In order to remove solutes from the brain, advection and diffusion are essential, and tissue characteristics and artery pulsations affect the flow of cerebrospinal fluid (CSF) [1]. Advection is replaced by shear-augmented dispersion in oscillatory CSF flow in channels that simulate the spinal subarachnoid space and perivascular spaces (PVSs) [2]. Although CSF flow direction is driven by arterial movements, the exact mechanism is unclear, with astrocyte end feet possibly controlling flow between the PVSs and the extracellular space [3, 16]. Perivascular spaces around leptomeningeal arteries create low-resistance routes, promoting CSF flow, with velocity amplitudes of 60-260 μm/s observed due to artery wall expansion [4, 5].

It is observed that the amplitude and phase of perivascular pumping, a major source of CSF flow, deviate from theoretical predictions [8]. Metabolic waste, such as misfolded tau and amyloid

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beta proteins, accumulates in CSF and serves as a clinical marker for Alzheimer's disease [9]. Numerical analyses in axisymmetric channels with pulsating barriers show that reduced models accurately predict pressure and flux, capturing pulsatile flow characteristics [10, 11]. The peristaltic deformation of arteries and brain tissue elasticity affect CSF streaming, with peristaltic waves creating flow highly dependent on tissue permeability and rigidity [12]. Systolic pulsations modeled as traveling waves ensure fluid movement in the direction of arterial wave travel, even with modestly negative pressure gradients [13]. Simplified diffusion-weighted image analysis (DWI-ALPS) assesses interstitial fluid status across age groups, supporting the evaluation of neurodegenerative conditions [14]. We provide a comprehensive characterization of the fluid flow parameters. Numerous biomechanical and computational studies have demonstrated that arterial pulsations, vessel-wall oscillations, and geometric properties of the PVS significantly influence CSF transport dynamics.

2. Mathematical Formulation

Mathematically modeling the circulation of cerebrospinal fluid (CSF) inside perivascular spaces (PVS), some basic assumptions are made to simplify the research. The fluid is Newtonian and incompressible since its density and viscosity don't change while it flows. The unsteady flow of cerebrospinal fluid in the perivascular space surrounding a cerebral artery. The flow is assumed to occur in a two-dimensional channel where x' represents the axial direction and y' represents the radial direction. The arterial wall oscillates with angular frequency ω. The CSF is assumed to be incompressible and Newtonian. The approach is predicated on laminar flow, which is characteristic of the small-scale and relatively low-velocity nature of CSF transport in PVS. The use of steady-state conditions implies that fluid properties remain constant across time. To further simplify the problem, the PVS is projected to have an axisymmetric geometry around the major blood vessel, and gravitational effects are regarded as negligible. At the PVS walls, when the fluid velocity equals the boundary surface, the model applies a no-slip boundary condition. Furthermore, the perivascular space and surrounding tissues are regarded as homogeneous and isotropic, and a continuous pressure gradient is postulated across the PVS to aid analytical solutions. Together, these presumptions simplify the fluid dynamics in PVS's mathematical formulation and solution. In a Cartesian coordinate system, we study the unsteady laminar flow of a viscous, incompressible fluid (CSF) with a uniform cross section h. The governing equations for such flow can be obtained by combining the continuity and species equation with the Navier-Stokes equations for incompressible fluids.

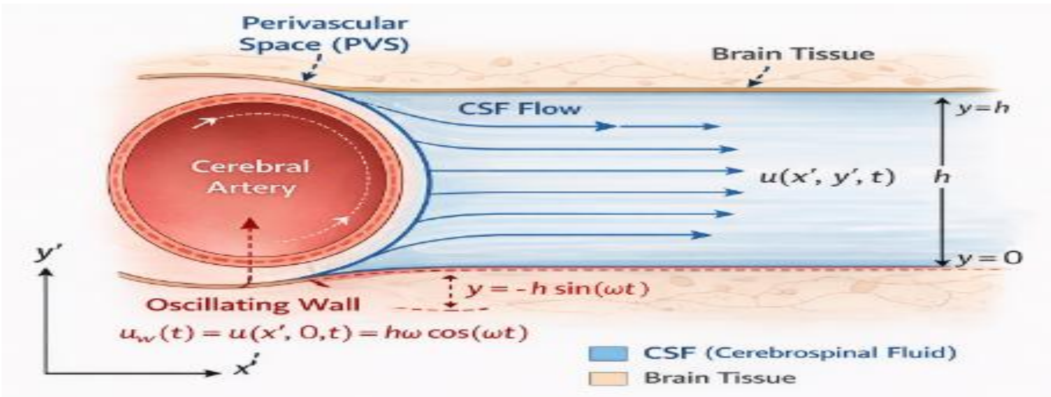

Figure.1 Physical Configuration

Continuity Equation: -

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{1}$$

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Naiver Stokes (or) Momentum Equation: -

$$\rho\left(\frac{\partial u'}{\partial t'}+v_0\frac{\partial u'}{\partial y'}\right)=-\frac{\partial p'}{\partial x'}+\mu\frac{\partial^2 u'}{\partial y'^2}-\frac{\mu}{k}u' \tag{2}$$

Concentration (or) Species Equations: -

$$\frac{\partial c'}{\partial t'}=D\frac{\partial^2 c'}{\partial y'^2}-k_c c' \tag{3}$$

The problem can be reduced to a 2D analysis where the velocity component v can be ignored, and the equations are further simplified, for the case of a uniform cross-section h and taking into account an axisymmetric flow. Boundary conditions for this problem also include suitable conditions at the channel's inlet and exit, as well as no-slip conditions at the channel walls. Time-dependent variables are added to the Navier-Stokes equations to account for the flow's unsteadiness. The impact of pulsatile artery movements and the existence of perivascular spaces (PVSs) add complexity to the flow of cerebrospinal fluid (CSF). Terms that depict the pulsatile nature of the flow and the interaction between the CSF and the surrounding brain tissue are used to model these effects [1, 2].To solve these equations under realistic boundary conditions and geometries, numerical approaches like finite difference or finite element methods are frequently used [10, 11]. Pressure and velocity profiles, which are essential for comprehending the dynamics of CSF flow and its function in eliminating metabolic waste from the brain, can be predicted from the data [9, 12, 13].The initial conditions, Wall motion and Far field condition are prescribed as follows:

Initial conditions $\quad when\ t'=0,\ y'=0, u'=0\ and\ c'=0\ \forall x$

Wall motion $\quad when\ t'\geq 0, \mathrm{y}'=\mathrm{h}, u'=h\omega cos\omega t'$

Far field condition $\quad u\rightarrow 0, c'\rightarrow 0\ as\ y\rightarrow\infty$

All parameters are defined in the nomenclature. It is useful to introduce dimensionless quantities which are defined as follows:

$$u = \frac{u'}{h\omega}, x = \frac{x'}{h}, y = \frac{y'}{h}, c = \frac{c'}{c_0}, p = \frac{p'}{\rho\omega v}, t = \omega t',$$

The transverse pressure gradient decays exponentially with time in CSF flow in the regime. Implementing dimensionless quantities in Eqs. (1) – (3) yields the following system of dimensionless coupled partial differential equations:

The equation (1), (2) and (3) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$\alpha^2 \frac{\partial u}{\partial t} = e^{-\lambda t} + \frac{\partial^2 u}{\partial y^2} - {D_a}^2 u \tag{5}$$

$$\frac{\partial c}{\partial t} = \frac{1}{\beta^2}\frac{\partial^2 c}{\partial y^2} - k'c \tag{6}$$

In Eq. (5), $\frac{\partial p}{\partial x} = -e^{-\lambda t}$ expresses the exponentially decaying transverse pressure gradient. Furthermore, the dimensionless numbers in Eqs. (4) – (6) take the definitions:

$$\nu = \frac{\mu}{\rho}$$

$$-\frac{\nu}{k} \rightarrow Darcy\,Term\,(Porous\,Medium)$$

$$g\beta\rho(c' - c'_\infty) \rightarrow Buoyancy\,Force$$

$$\alpha^2 = \frac{h^2\omega}{v}\ (Womersely\,Number)$$

$$D_a = \frac{k}{h^2}(Darcy\,Number)$$

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$$\frac{\partial p}{\partial x} = -e^{-\lambda t}, (Transverse\ pressure\ gradient)$$

$$\beta^2 = \frac{h^2\omega}{D}\ (oscillatory\ diffusion\ parameter)$$

$$c = \frac{c'}{c_0},\ k' = \frac{k_c}{\omega}$$

Now Boundary Conditions becomes: -

$as\ t = 0,\ y = 0\ then\ u = 0; as\ t \geq 0, y = 1\ then\ \ u = cost$

$when\ y \to \infty,\ then\ u \to 0\ and\ c \to 0,\ when\ y \to 0\ then\ u \to 1$ and c $\to 1$

Taking Laplace Transform on both side of equation (6) we get

$$L\left(\frac{\partial c}{\partial t}\right) = \frac{1}{\beta^2} L\left(\frac{\partial^2 c}{\partial y^2}\right) - k'L(c)$$

$$SL(c) - c(0) = \frac{1}{\beta^2} D^2 L(c) - k'L(c)$$

$$D^2 L(c) + \beta^2\ SL(c) + \beta^2 k' L(c) = 0$$

$$\left(D^2 + \beta^2(s + k')\right) L(c) = 0$$

$$L(c) = A_1 \exp\left(\beta\sqrt{s + k'}y\right) + A_2 \exp\left(-\beta\sqrt{s + k'}y\right) \tag{7}$$

When $t \geq 0, c \to 0\ as\ y \to \infty, L(c) = 0\ \Rightarrow A_1 = 0$

$$L(c) = A_2 \exp\left(-\beta\sqrt{s + k'}y\right) \tag{8}$$

When $t = 0, y = 0, c = 1, L(c) = \frac{1}{s}\ \Rightarrow A_2 = \frac{1}{s}$

$$L(c) = \frac{1}{S} \exp\left(-\beta y\sqrt{s + k'}\right) \tag{9}$$

Taking Inverse Laplace Transform on both side we get

$$c(y,t) = \frac{1}{2}\left[\exp\left(-\sqrt{k'}\beta y\right) erfc\left(\frac{\beta y}{2\sqrt{t}} - \sqrt{k't}\right) + \exp\left(\sqrt{k'}\beta y\right) erfc\left(\frac{\beta y}{2\sqrt{t}} + \sqrt{k't}\right)\right] \tag{10}$$

Taking Laplace Transform on both side of equation (5) we get

$$L(u) = \left\{\frac{S}{S^2+1} - \frac{1}{D_a^2 - \alpha^2\lambda}\left[\frac{1}{S+\lambda} - \frac{1}{S+\frac{D_a^2}{\alpha^2}}\right]\right\}\exp\left(-\sqrt{\alpha^2 S + D_a^2}y\right) + \frac{1}{D_a^2 - \alpha^2\lambda}\left[\frac{1}{S+\lambda} - \frac{1}{S+\frac{D_a^2}{\alpha^2}}\right] \tag{11}$$

Taking Inverse Laplace Transform on both side we get

$$\begin{aligned} u = {} & L^{-1}\left[\frac{S}{S^2+1}\exp\left(-\sqrt{\alpha^2 S + D_a^2}y\right)\right] \\ & - \frac{1}{D_a^2 - \alpha^2\lambda}L^{-1}\left[\frac{1}{S+\lambda}\exp\left(-\sqrt{\alpha^2 S + D_a^2}y\right)\right] \\ & + \frac{1}{D_a^2 - \alpha^2\lambda}L^{-1}\left[\frac{1}{S+\frac{D_a^2}{\alpha^2}}\exp\left(-\sqrt{\alpha^2 S + D_a^2}y\right)\right] \\ & + \frac{1}{D_a^2 - \alpha^2\lambda}L^{-1}\left[\frac{1}{S+\lambda}\right] - \frac{1}{D_a^2 - \alpha^2\lambda}L^{-1}\left[\frac{1}{S+\frac{D_a^2}{\alpha^2}}\right] \end{aligned} \tag{12}$$

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$$u(y,t) = \left\{\frac{e^{-it}}{4}\left[e^{-\sqrt{n}y} erfc\left(\eta\sqrt{\alpha^2} - \sqrt{(m-i)t}\right) + e^{\sqrt{n}y} erfc\left(\eta\sqrt{\alpha^2} + \sqrt{(m-i)t}\right)\right] + \frac{e^{it}}{4}\left[e^{-\sqrt{n}y} erfc\left(\eta\sqrt{\alpha^2} - \sqrt{(m+i)t}\right) + e^{\sqrt{n}y} erfc\left(\eta\sqrt{\alpha^2} + \sqrt{(m+i)t}\right)\right]\right\} - \frac{e^{-\lambda t}}{2a}\left[e^{-\sqrt{a}y} erfc\left(\eta\sqrt{\alpha^2} - \sqrt{(m-\lambda)t}\right) + e^{\sqrt{a}y} erfc\left(\eta\sqrt{\alpha^2} + \sqrt{(m-\lambda)t}\right)\right] \tag{13}$$

Where $a = D_a^2 - \lambda\alpha^2, m = \frac{D_a^2}{\alpha^2}, n = D_a^2 + \alpha^2 S, \eta = \frac{y}{2\sqrt{t}}, D_a^2 = \frac{1}{D_a}$

3. Results and Discussions

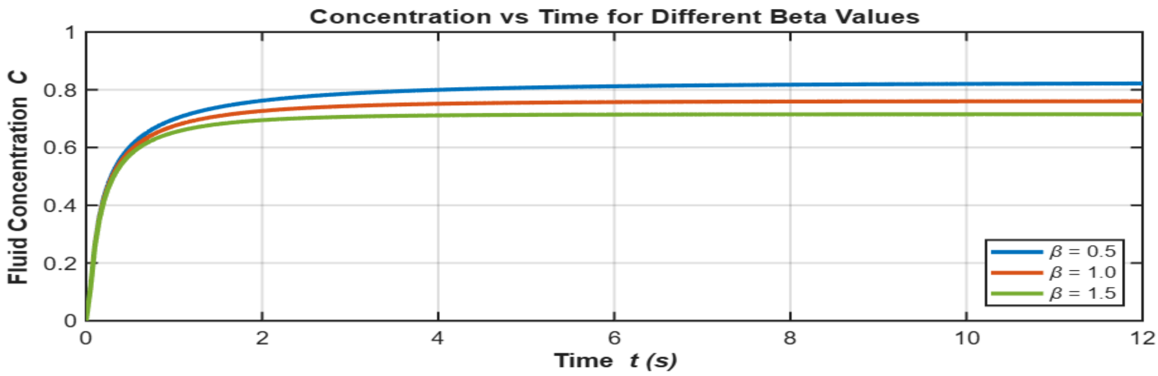

Figure.2 Concentration of the fluid for different values of coefficient of solute parameter

The Figure 2 illustrates that the time dependent variation of cerebrospinal fluid (CSF) concentration within the perivascular space (PVS) is analyzed with different values of the parameter β, which represents the combined influence of solutal diffusivity and convective transport. As β increases, the concentration rises more quickly, indicating enhanced solute movement due to stronger buoyancy-driven flow. Lower β values result in slower diffusiondominated transport. The plot provides insight into how changes in flow parameters affect CSF dynamics, supporting the analytical findings derived using the Laplace transform method.

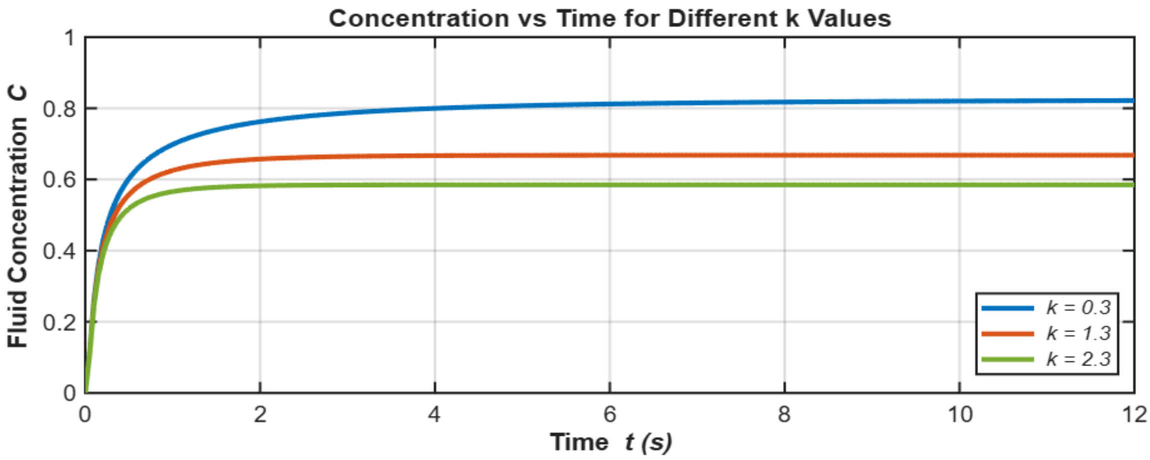

Figure.3 Concentration of the fluid for different values of porous medium

The Figure 3 illustrates that the concentration of cerebrospinal fluid (CSF) in the perivascular space changes over time for different permeability values (k = 0.3, 1.3, 2.3). The findings show that as permeability increases, solute transport improves, leading to a faster and higher buildup of concentration. Since changes in permeability in these pathways are often linked to problems with clearing metabolic waste, this research helps us better understand the fluid dynamics behind neurological conditions like Alzheimer's disease. The different values of k reflect structural or pathological changes in the perivascular space that can impact how CSF flows and how solutes accumulate in the brain.

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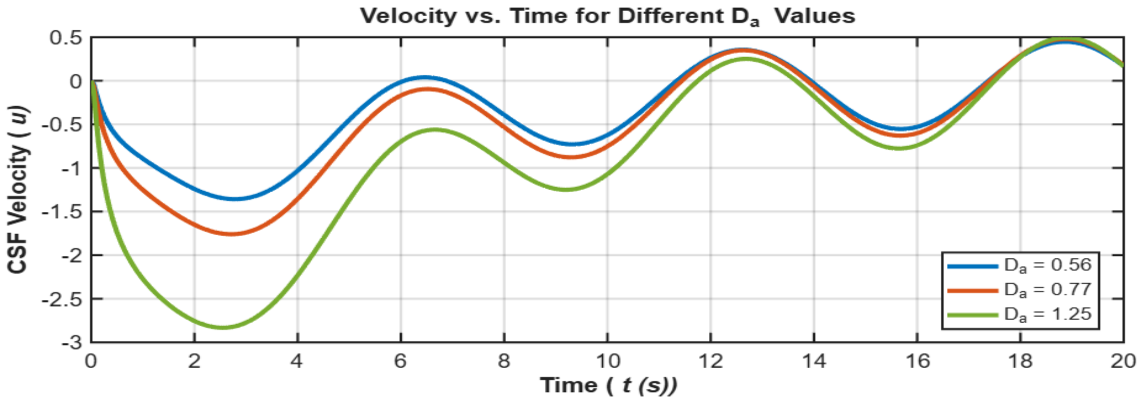

Figure.4 Velocity of the CSF for different values of Darcy number

The Figure 4 illustrates the velocity of cerebrospinal fluid (CSF) in the perivascular space changes over time for different Darcy numbers (Da = 0.56, 0.77, 1.25). The Darcy number measures how permeable the medium is, which affects how easily the fluid can flow through the space. The graph shows that higher Darcy numbers lead to faster CSF flow, meaning more efficient fluid movement. On the other hand, lower Darcy values cause the velocity to drop significantly, showing increased resistance to flow. Since reduced permeability in the perivascular space is often linked to problems clearing CSF is a common issue in neurological conditions like Alzheimer's and small vessel disease these findings are important for understanding the underlying physiology.

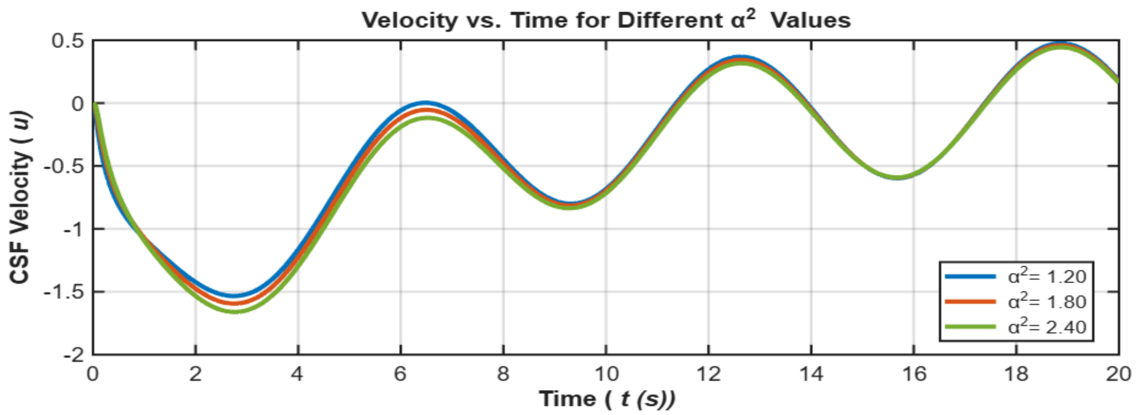

Figure. 5 Velocity of the CSF for different values of Womersley Number

The CSF velocity varies more noticeably over time as the diffusion coefficient α^2 increases, as shown in Figure 5. Higher α^2 gives more variations because of the influence of diffusion increasing, whereas lower α^2 produces a smoother and more stable velocity profile. Modeling and studying fluid dynamics in systems where diffusion is an important requires an understanding of this relationship.

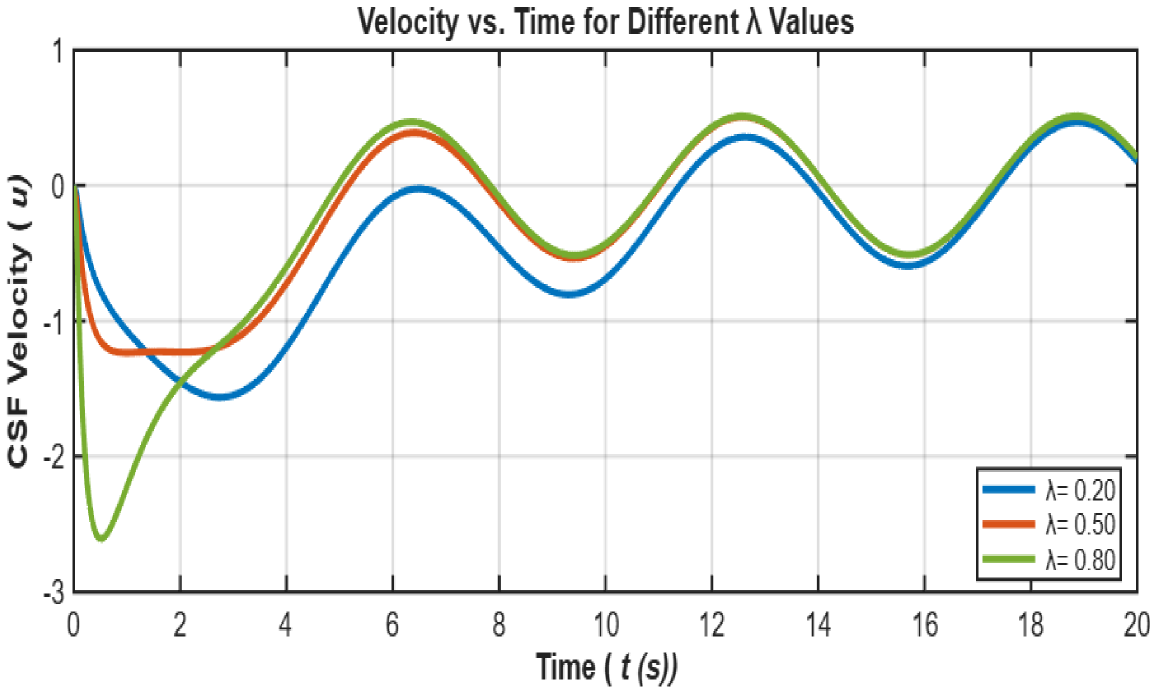

Figure.6 Velocity of the CSF for different values of damping coefficient

Figure 6 illustrates how the velocity of cerebrospinal fluid (CSF) changes over the time in the range of relaxation parameter values. The CSF velocity clearly gives λ rises from 0.2 to 0.8. This pattern suggests that increased values of λ fluid resistance in the system, resulting in damped or delayed flow responses.

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Table 1: Findings & References

Fig. No.	Parameter	Effect on CSF Transport	Physiological / Neurological Significance	Supporting References
Fig. 2	β (diffusion + convection)	Higher β → faster concentration rise; Lower β → slow diffusion transport	Solute concentration gradients contribute to the movement of cerebrospinal fluid, playing an important role in the clearance processes within the perivascular spaces (PVS).	[1–4]
Fig. 3	Permeability (k = 0.3, 1.3, 2.3)	Higher k → enhanced solute transport and accumulation	Variations in permeability are often associated with reduced clearance efficiency, which has been linked to neurological conditions such as Alzheimer's disease and certain vascular disorders.	[4]
Fig. 4	Darcy number (Da = 0.56,0.77, 1.25)	Higher Da → higher CSF velocity; lower Da → reduced velocity	Lower values of the Darcy parameter (Da) indicate reduced permeability of the medium, which may hinder the effectiveness of glymphatic clearance mechanisms.	[1], [7], [8]
Fig. 5	Diffusion coefficient (α^2)	Higher α^2 → increased velocity variations; lower α^2 → smoother velocity	These parameters significantly influence the patterns of solute transport and removal within the glymphatic pathway.	[2], [5], [7]
Fig. 6	Relaxation parameter (λ)	Higher λ → decreased CSF velocity (damped response)	They also represent the structural and viscoelastic resistance offered by the perivascular region to the flow of cerebrospinal fluid.	[15]

4. Conclusion

This study presents a mathematical investigation of the unsteady flow of cerebrospinal fluid (CSF) in the perivascular region. The results highlight the significant role of arterial wall motion in enhancing CSF transport within the perivascular space. It is also observed that parameters such as buoyancy effects, porous medium resistance represented through the Darcy permeability parameter, and chemical reaction processes influence the velocity distribution and solute concentration in the flow region. These factors collectively affect the transport and clearance mechanisms of solutes within the glymphatic pathway. The mathematical model useful insight into the dynamics of CSF flow in the perivascular region and may contribute to a better understanding of neurological disorders associated with impaired CSF circulation.

References

[1] Thomas, John H.. (2019). Fluid dynamics of cerebrospinal fluid flow in perivascular spaces. Journal of The Royal Society Interface, 16(159), 20190572–. doi:10.1098/rsif.2019.0572.

[2] Keith Sharp, M.; Carare, Roxana O.; Martin, Bryn A. . (2019). Dispersion in porous media in oscillatory flow between flat plates: applications to intrathecal, periarterial and paraarterial solute transport in the central nervous system. Fluids and Barriers of the CNS, 16(1), 13–. doi:10.1186/s12987-019-0132-y.

[3] Gan, Yiming, et al. "Perivascular pumping of cerebrospinal fluid in the brain with a valve mechanism." Journal of the Royal Society Interface 20.206 (2023): 20230288.

[4] Bedussi B, Almasian M, de Vos J, VanBavel E, Bakker EN. Paravascular spaces at the brain surface: Low resistance pathways for cerebrospinal fluid flow. Journal of Cerebral Blood Flow & Metabolism. 2018;38(4):719-726. doi:10.1177/0271678X17737984.

[5] Daversin-Catty, Cécile, et al. "The mechanisms behind perivascular fluid flow." Plos one 15.12 (2020): e0244442.

[6] Yokoyama N, Takeishi N, Wada S. Cerebrospinal fluid flow driven by arterial pulsations in axisymmetric perivascular spaces: Analogy with Taylor's swimming sheet. *J Theor Biol.* 2021;523:110709. doi:10.1016/j.jtbi.2021.110709.

[7] Daversin-Catty C, Gjerde IG and Rognes ME (2022) Geometrically Reduced Modelling of Pulsatile Flow in Perivascular Networks. *Front. Phys.* 10:882260. doi: 10.3389/fphy.2022.882260.

ISBN 978-816855385-9

[8] Romanò, F., Suresh, V., Galie, P.A. *et al.* Peristaltic flow in the glymphatic system. *Sci Rep* 10, 21065 (2020).
[9] Lynne E. Bilston , David F. Fletcher , Andrew R. Brodbelt & Marcus A. Stoodley (2003) Arterial Pulsation-driven Cerebrospinal Fluid Flow in the Perivascular Space: A Computational Model, Computer Methods in Biomechanics and Biomedical Engineering, 6:4, 235-241, DOI: 10.1080/10255840310001606116.
[10] Taoka T, Ito R, Nakamichi R, et al. Diffusion-weighted image analysis along the perivascular space (DWI-ALPS) for evaluating interstitial fluid status: age dependence in normal subjects. *Jpn J Radiol.* 2022;40(9):894-902. doi:10.1007/s11604-022-01275-0.
[11] S. Kavitha; Ayothi Selvaraj; Senthamilselvi Sathiamoorthy; P. Rajesh. A Parabolic Flow With MHD, the Dufour and Rotational Effects of Uniform Temperature and Mass Diffusion through an Accelerating Vertical Plate in the Presence of Chemical Reaction. J. Adv. Res. Fluid Mech. Therm. Sc. 2023, 110, 192-205.
[12] Balasundaram, Hemalatha, et al. "Hydrocephalic cerebrospinal fluid flowing rotationally with pulsatile boundaries: a mathematical simulation of the thermodynamical approach." Theoretical and Applied Mechanics Letters 13.1 (2023): 100418.
[13] Mestre, H. et al. “Flow of cerebrospinal fluid is driven by arterial pulsations and is reduced in neurodegeneration.” Nature Communications 9, 4878 (2018).
[14] Iliff, J.J. et al. “A paravascular pathway facilitates CSF flow through the brain and clearance of interstitial solutes, including amyloid β.” Science Translational Medicine 4.147 (2012): 147ra111.

Chapter 16

Mathematical Model of Blood Plasma Flow through Pulmonary Capillaries

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Abstract

The transport of blood plasma through pulmonary capillaries plays a crucial role in the exchange of oxygen and carbon dioxide within the human respiratory system. Understanding the dynamics of plasma flow in the presence of external forces is important for studying physiological processes and biomedical applications. In the present study, a mathematical model is developed to investigate the unsteady laminar flow of blood plasma in the lung mechanism under the influence of an applied magnetic field. Blood plasma is treated as a viscous, incompressible and electrically conducting fluid flowing through a permeable medium representing the pulmonary tissue. The governing equations of continuity, momentum, energy and mass transfer are formulated using the principles of magnetohydrodynamics. These equations are converted into dimensionless form using suitable non-dimensional parameters such as the Grashof number, Hartmann number, Prandtl number and Schmidt number. An analytical solution is obtained using the perturbation method. The influence of various physical parameters on plasma velocity, temperature and oxygen concentration distributions is analyzed graphically. The results indicate that magnetic field strength, buoyancy effects and permeability of lung

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tissue significantly influence plasma transport in pulmonary circulation. The present model provides a theoretical framework for understanding blood plasma flow and gas transport in the human lung.

Keywords: Blood Plasma Flow Pulmonary Circulation, Mathematical Modelling, Oxygen Transport, Perturbation technique.

1. Introduction

The transport of blood plasma through pulmonary capillaries plays a crucial role in the exchange of oxygen and carbon dioxide within the human respiratory system. Understanding the dynamics of plasma flow in the presence of external forces is important for studying physiological processes and biomedical applications [2,18,20] and the erythrocyte in flowing blood assumes a variety of shapes and drop, cup and parachute shapes have been described and photographed Skalak and Branemark [17].

In the present study, a mathematical model is developed to investigate the unsteady laminar flow of blood plasma in the lung mechanism under the influence of an applied magnetic field. Blood plasma is treated as a viscous, incompressible and electrically conducting fluid flowing through a permeable medium representing the pulmonary tissue [15] . The governing equations of continuity, momentum, energy and mass transfer are formulated using the principles of magnetohydrodynamics. These equations are converted into dimensionless form using suitable non-dimensional parameters such as the Grashof number, Hartmann number, Prandtl number and Schmidt number [13,15,21]An analytical solution is obtained using the perturbation method [22]. The influence of various physical parameters on plasma velocity, temperature and oxygen concentration distributions is analyzed graphically. The results indicate that magnetic field strength, buoyancy effects and permeability of lung tissue significantly influence plasma transport in pulmonary circulation. The present model provides a theoretical framework for understanding blood plasma flow and gas transport in the human lung.

2. Mathematical Formulation

In the present model the flow of blood plasma in the pulmonary capillary region is considered. We have taken into account an unsteady two-dimensional laminar flow of a viscous, incompressible fluid of uniform cross section h, fluid is lying above and below bounded by porous layers, and the physical situation can be written in Cartesian form under the assumptions based on study areas. The initial time-dependent slip boundary condition at the porous medium interface at $t \leq 0$, assumes that the region is at the same temperature T and concentration C. When $t > 0$, the temperature and concentration of the area instantly rise to T_0 and C_0, respectively. These values oscillate with time and are thus kept constant. Make the x axis parallel to the fluid flow and the y axis perpendicular. The problem's schematic representation is displayed below. The appropriate boundary conditions are used to derive the governing equation and the ensuing assumptions are then generated. In the y direction, the blood plasma flow is subjected to a consistent magnetic field, designated B_0.

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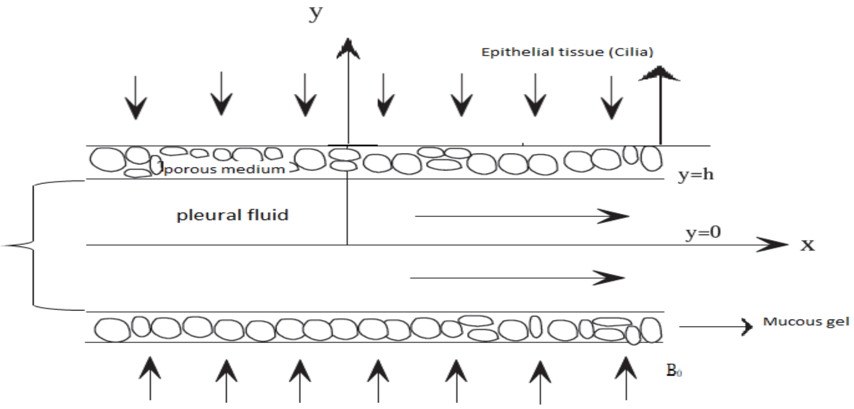

Figure. A *Physical configuration*

The balances of the mass, linear momentum, energy, and concentration species serve as the foundation for the governing equations.

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Navier-stokes Equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + g\beta(T - T_0) + g\beta^*(C - C_0) - \frac{\nu}{\sqrt{k_1}}u - \frac{u\sigma{\beta_0}^2}{\rho} \qquad (2)$$

According to the presumptions, the following are the proper boundary conditions for fields of velocity, temperature, and concentration involving moving fluids:

At $t = 0, \quad u = 0$ **(3)**

At $y = 0, \quad u = 0, T$ **(4)**

At $y = h, \frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{k_1}}u$, **(5)** By substituting the following non-dimensional quantities, the given problem becomes non-dimensional.

$$\boldsymbol{u^*} = \frac{u}{v_0}\ \boldsymbol{v^*} = \frac{v}{v_0}, \boldsymbol{x^*} = \frac{xv_0}{\nu}, \boldsymbol{y^*} = \frac{yv_0}{\nu},$$

$$\boldsymbol{\theta^*} = \frac{T-T_0}{T_a-T_0}, \boldsymbol{\varphi^*} = \frac{C-C_0}{C_a-C_0}, \ \boldsymbol{t^*} = \frac{tv_0^2}{\nu}, \ \boldsymbol{p^*} = \frac{p}{\rho v_0^2} \tag{6}$$

where temperature and concentration are represented by the dimensionless variables θ *and* φ respectively.

Neglecting the (*) symbol results in the main governing Equations (1) and (2) no longer implying $\boldsymbol{v} = \boldsymbol{v_0}$, where $\boldsymbol{v_0}$ is a real positive constant and is also taken into account as the characteristic velocity.

$$\frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial y^2}\right) + Gr\theta + Gc\varphi - \sigma^{*2}u - H_a{}^2 u \tag{8}$$

Using non-dimensional values, the beginning and boundary condition becomes,

$$\left.\begin{aligned} &\text{At, } t = 0, \quad u = \varphi = \theta = 0, \\ &\text{At, } y = 0, \quad u = 0, \theta = T_{x2}, \frac{\partial \varphi}{\partial y} = 0 \\ &\text{At, } y = H, \quad \frac{\partial u}{\partial y} = \alpha\sigma u, \frac{\partial \theta}{\partial y} = \alpha\sigma(\theta + T_{x3}), \varphi = C_{x2} \end{aligned}\right\} \tag{9}$$

Where, $Gr = \frac{g\beta\nu(T_W-T_0)}{v_0{}^3}$ [Grashof number] ,

$Gc = \frac{g\beta^*\nu(C_W-C_0)}{v_0{}^3}$ [Solutal Grashof number],

$\sigma = \frac{\nu}{v_0\sqrt{k_1}}$, [Porosity parameter], $H_a = \frac{\nu\sigma\beta_0{}^2}{\rho v_0{}^2}$ [Hartmann number].

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The perturbation method can be used to obtain the analytical solution for the set of dimensionless equations from (7) and (8), along with the set of boundary conditions (9). This procedure can be carried out by depicting the perturbation parameter ($\epsilon << 1$) as being very, very small and then following it with flow velocity u, temperature θ and concentration φ .

$$U(x, y, t) = u_0\,(y) + \epsilon e^{(\lambda t+nx)} u_1(y) + o(\epsilon^2) \tag{10}$$

$$P(x, y, t) = p_0(x) + \epsilon e^{(\lambda t+nx)} p_1(y) + o(\epsilon^2) \tag{11}$$

By substituting the aforementioned equation into the basic equations (7) and (8), we can move further with our problem while ignoring higher order terms, i.e., $o(\epsilon^2)$. The resulting set of equations are as follows u_0 .

$$\left(\frac{\partial^2 u_0}{\partial y^2}\right) - C_t \frac{\partial u_0}{\partial y} - ({\sigma^*}^2 + {H_a}^2)\, u_0 = \frac{\partial p_0}{\partial x} - Gr\,\theta_0 - Gc\,\varphi_0 \tag{12}$$

$$\left.\begin{array}{ll} \text{At } y = 0, & u_0 = 0 \\ \text{At } y = H, & \dfrac{\partial u_0}{\partial y} = \alpha\sigma\, u_0 \end{array}\right\} \tag{13}$$

Boundary conditions (13) can be used to obtain zeroth-order velocity (u_0),

$$u_0 = A_9 e^{m_9 y} + A_{10} e^{e^{m_{10} y}} - \frac{g_1}{N} - (\,R_1 e^{m_1 y} + R_2\, e^{m_2 y} + C_1\, e^{m_5 y} + C_2\, e^{m_6 y}) \tag{14}$$

$$\left(\frac{\partial^2 u_1}{\partial y^2}\right) - C_t \frac{\partial u_1}{\partial y} - (\sigma^{*2} + H_a{}^2 + \lambda)\, u_1 = np_1 - Gr\,\theta_1 - Gc\,\varphi_1 \qquad (15)$$

$$\left.\begin{array}{l} \text{At } y = 0,\ u_1 = 0, \\ \text{At } y = H,\ \frac{\partial u_1}{\partial y} = \alpha\sigma\, u_1 \end{array}\right\} \qquad (16)$$

Boundary conditions (16) can be used to simultaneously derive first order velocity u_1.

$$u_1 = A_{11} e^{m_{11} y} + A_{12} e^{e^{m_{12} y} y} - \frac{g_1}{N} - (R_3\, e^{m_3 y} + R_4\, e^{m_4 y} + C_3\, e^{m_7 y} + C_4\, e^{m_8 y}) \qquad (17)$$

3. Results and Discussion

The goal of the current study is to assess how well numerical computations of the analytical results for the following fluid particles velocity distributions can characterize the flow of the fluid at various values of physical parameters. Computations are performed using MATHEMATICA 12.0 and the results are presented graphically to understand the behavior of plasma flow in the lung mechanism. To evaluate the graph throughout the algorithm, we use equivalent values:($t = 2, \mu = 3 \times 10^{-6}; \rho = 992.2 \times 10^{-15},\ g = 9.81 \times 10^{6}$,

$$C_t = 0.001, \lambda = 0.005, \epsilon = 0.002, n = 0.001, x = 0.01,$$

$$H = 0.5, Gr = 0.5, Gc = 0.5, \epsilon = 0.03)$$

Until the other parameter values are maintained in their fixed state. The Grashof number, Hartmann number, porosity parameter are among the parameters we are computing.

Figure.1 depicts how different magnetic fields affect the velocity distribution. Lorentz force is the way of referring to such a force. The graph below shows that the fluid's velocity rises as the HARTMANN

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parameter increases. As a result, the velocity distribution of plasma flow within the capillary region is modified. This is important in biomedical applications where magnetic field are used for diagnostic purpose. We noticed that the fluid's velocity rises as the porosity permeable parameter rises in Figure.2. As the permeability increases the resistance to plasma flow decreases which results in the increasing velocity. The Grashof number represents the buoyancy force caused by temperature differences between blood plasma and surrounding lung tissues is shown in Figure.3 and Figure.4. It is evident that a buoyant flow develops during fluid flow inside the lung as a result of the gravitational force's acceleration.

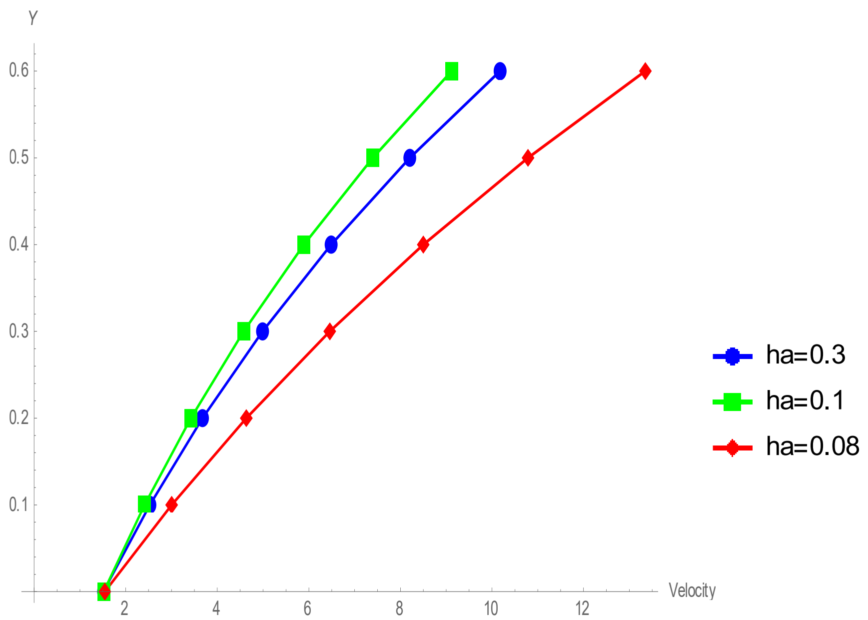

Figure.1 *Velocity field for various values of Ha*

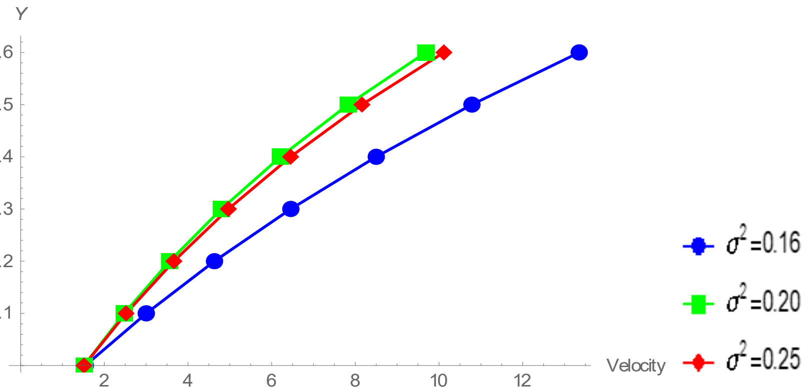

Figure.2 *Velocity field for various values of σ**

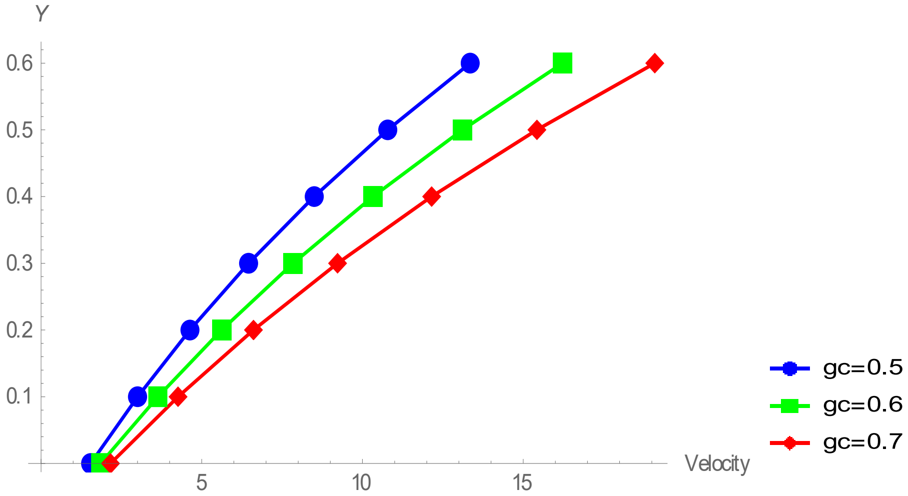

Figure.3 *Velocity field for various values of Gr*

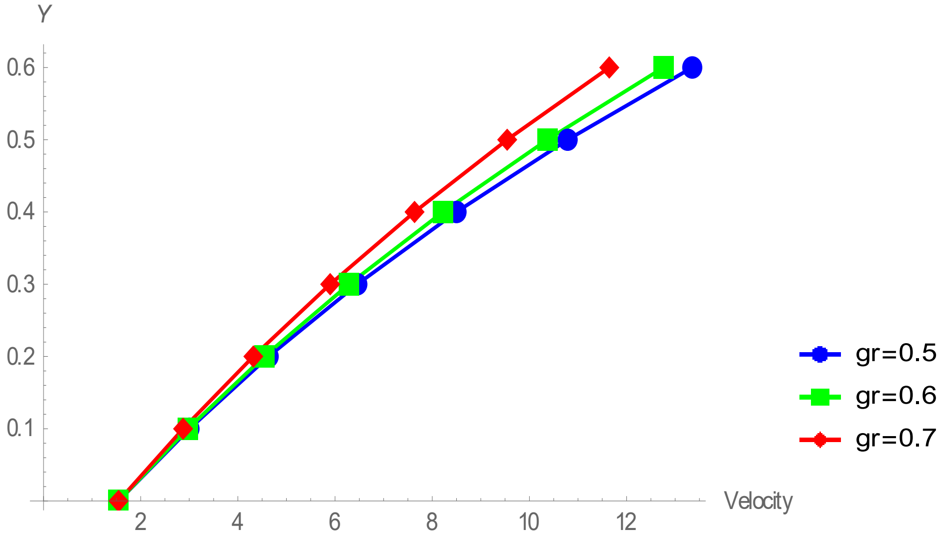

Figure.4 *Velocity field for various values of Gc*

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4.CONCLUSION

A mathematical model has been developed to analyze the flow of blood plasma flow in the pulmonary capillary region. In the use of the perturbation technique, the dimensionless governing equations above are analytically resolved. The effects of various physical parameters on blood plasm a flow velocity is studied. An increase in mass Grashof and Solutal Grashof number raises the plasma velocity due to stronger buoyancy effects caused by temperature and concentration differences. The effects of this study may contribute to further research in biomedical fluid dynamics and help in understanding the role of magnetic fields in medical applications related to blood flow and respiratory processes.

References

[1] *Burrowes KS, Hunter PJ, Tawhai MH. Anatomically-based finite element models of the human pulmonary arterial and venous trees including supernumerary vessels. J Appl Physiol. 2005; 99:731–8. doi: 10.1152/japplphysiol.01033.2004.*

[2] *Eldesoky, "Mathematical Analysis of Unsteady MHD Blood Flow through Parallel Plate Channel with Heat Source," World Journal of Mechanics, Vol. 2 No. 3, 2012, pp. 131-137. doi: 10.4236/wjm.2012.23015.*

[3] *Fletcher GF, Balady GJ, Amsterdam EA, Chaitman B, Eckel R, Fleg J, et al. Exercise standards for testing and training: A statement for healthcare professionals from the American Heart Association. Circulation. 2001; 104:1694–740. Doi: 10.1161/hc3901.095960.*

[4] *Gan R, Yen R. Vascular impedance analysis in dog lung with detailed morphometric and elasticity data. J Appl Physiol. 1994; 77:706–17. doi: 10.1152/jappl.1994.77.2.706.*

[5] *Grotberg JB, Romanò F, Grotberg JC (2025) Flow mechanisms of the air-blood barrier. PLOS Computational Biology 21(4): e1012917. https://doi.org/10.1371/ journal.pcbi. 1012 917.*

[6] *Godbey PS, Graham JA, Presson RG, Jr, Wagner WW, Jr, Lloyd TC., Jr Effect of capillary pressure and lung distension on capillary*

recruitment. J Appl Physiol. 1995; 79:1142–7. Doi: 10.1152/jappl.1995.79.4.1142.

[7] *Huang W, Tian Y, Gao J, Yen RT. Comparison of theory and experiment in pulsatile flow in cat lung. Ann Biomed Eng. 1998; 26:812–20. Doi: 10.1114/1.107. 20. Krenz GS, Dawson CA. Flow and pressure distributions in vascular networks consisting of distensible vessels. Am J Physiol. 2003; 284:H2192–203. Doi: 10.1152/ajpheart.00762.2002.*

[8] *Hunter PJ, Smith NP, Fernandez J, Tawhai MH. Integration from proteins to organs: The IUPS Physiome Project. Mech Ageing Dev. 2005;126:187–92. doi: 10.1016/j.mad.2004.09.025.*

[9] *Hunter P, Nielsen P. A strategy for integrative computational physiology. Physiology. 2005; 20:316–25. doi: 10.1152/physiol.00022.2005*

[10] *Hunter P, Coveney PV, de Bono B, Diaz V, Fenner J, Frangi AF, et al. A vision and strategy for the virtual physiological human in 2010 and beyond. Philos Transact A Math Phys Eng Sci. 2010; 368:2595–614. Doi: 10.1098/rsta.2010.0048.*

[11] *Horsfield K. Morphometry of the small pulmonary arteries in man. Circ Res. 1978; 42:593–7. Doi: 10.1161/01.res.42.5.593.*

[12] *Horsfield K, Gordon WI. Morphometry of pulmonary veins in man. Lung. 1981; 159:211–8. Doi: 10.1007/BF02713917.*

[13] *Huang W, Yen RT, McLaurine M, Bledsoe G. Morphometry of the human pulmonary vasculature. J Appl Physiol. 1996; 81:2123–33. Doi: 10.1152/jappl.1996.81.5.2123.*

[14] *Hussein, Ahmed & Ashorynejad, H. & Sivanandam, Sivasankaran & Kolsi, Lioua & Shikholeslami, M. & Kayode, Isaac. (2016). Modeling of MHD natural convection in a square enclosure having an adiabatic square shaped body using Lattice Boltzmann Method. Alexandria Engineering Journal. 55. 203–214. 10.1016/j.aej.2015.12.005.*

[15] *Kahshan, Muhammad & Lu, Dianchen & Abu-Hamdeh, Nidal & Golmohammadzadeh, Ali & Farooq, A. & Rahimi-Gorji, Mohammad. (2020). Darcy-Brinkman flow of a viscous fluid through a porous duct: Application in blood filtration process. Journal of the Taiwan Institute of Chemical Engineers. 117. 10.1016/j.jtice.2020.11.033.*

[16] *Karwe, Mukund & Deo, I.. (2003). Grashof number. Encyclopedia of Agricultural, Food, and Biological Engineering. 1. 454-456.*

[17] *Mélot C, Delcroix M, Closset J, Vanderhoeft P, Lejeune P, Leeman M, et al. Starling resistor vs.distensible vessel models for embolic*

ISBN 978-816855385-9

pulmonary hypertension. Am J Physiol. 1995; 268:H817–27. Doi: 10.1152/ajpheart.1995.268.2.H817.

[18] *Maseri A, Caldini P, Harward P, Joshi RC, Permutt S, Zierler KL. Determinants of pulmonary vascular volume: Recruitment versus distensibility. Circ Res. 1972; 31:218–28. doi: 10.1161/01.res.31.2.218.*

[19] *Oka S, Nakai M. Effects of non-Newtonian blood and metabolic states of the blood and vessel wall on the optimum design of single vessels and the vascular bifurcation. Biorheology. 1989;26(5):921-34. PMID: 2620089.*

[20] *Paiva M, Engel L. Pulmonary interdependence of gas transport. J Appl Physiol. 1979; 47:296–305. Doi: 10.1152/jappl.1979.47.2.296.*

[21] *Sugihara-Seki, Masako & Fu, Bingmei. (2005). Blood flow and permeability in microvessels. Fluid Dynamics Research. 37. 82-132. 10.1016/j.fluiddyn.2004.03.006.*

[22] *Shon, Sudeok & Lee, Seung-Jae. (2015). A Study on the Analytical Solution using Homotopy Perturbation Method of Shell-like Shallow Trusses. Journal of the Architectural Institute of Korea Structure & Construction. 31. 27-34. 10.5659/JAIK_SC.2015.31.9.27.*

[23] *Tawhai MH, Hunter PJ, Tschirren J, Reinhardt JM, McLennan G, Hoffman EA. CT-based geometry analysis and finite element models of the human and ovine bronchial tree. J Appl Physiol. 2004; 97:2310–21. doi: 10.1152/japplphysiol.00520.2004.*

[24] *Verbanck S, Paiva M. Model simulations of gas mixing and ventilation distribution in the human lung. J Appl Physiol. 1990; 69:2269–79. Doi:10.1152/jappl.1990.69.6.2269.*

[25] *Warren NJ, Crampin EJ, Tawhai MH. The role of airway epithelium in replenishment of evaporated airway surface liquid from the human conducting airways. Ann Biomed Eng. 2010; 38:3535–49. doi: 10.1007/s10439-010-0111-6.]*

[26] *West JB. Regional differences in gas exchange in the lung of erect man. J Appl Physiol. 1962; 17:893–8. doi: 10.1152/jappl.1962.17.6.893.*

[27] *West JB, Dollery CT, Naimark A. Distribution of blood flow in isolated lung: Relation to vascular and alveolar pressures. J Appl Physiol. 1964; 19:713–24. doi: 10.1152/jappl.1964.19.4.713.*

[28] *Zhou Q, Gao J, Huang W, Yen M. Vascular impedance analysis in human pulmonary circulation. Biomed Sci Instrum. 2006; 42:470–5.*

Chapter 17

A Study on Fuzzy Graphs

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Abstract

Fuzzy graph theory is an extension of classical graph theory designed to model systems that involve uncertainty, vagueness, and partial relationships. In many real-world problems, relationships between elements are not strictly binary; instead, they exist with varying degrees of strength. Fuzzy graph theory integrates the principles of fuzzy set theory with graph theory to represent such uncertain structures. In this paper, the fundamental concepts of fuzzy graphs are introduced, including fuzzy vertices, fuzzy edges, and membership functions. The methodology used to construct fuzzy graphs and analyze their properties is also discussed. Applications of fuzzy graphs in social networks, communication systems, decision making, and transportation modeling are highlighted. The study demonstrates that fuzzy graph models provide a flexible and effective framework for representing complex systems where classical graphs fail to capture uncertainty.

Keywords: Graph Theory; Fuzzy Graph Theory; Graph Models.

1. Introduction

Graph theory is an important area of mathematics used to represent relationships among objects. A graph typically consists of

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a set of vertices and edges connecting these vertices. Classical graph theory assumes that connections either exist or do not exist. In many practical situations, however, relationships are not strictly defined. Instead, they may exist with different levels of strength, reliability, or probability. Fuzzy set theory was introduced to address uncertainty and vagueness in mathematical models. By allowing elements to belong to a set with a degree of membership between 0 and 1, fuzzy set theory provides a powerful tool for modeling real-world problems. When the principles of fuzzy set theory are applied to graphs, the resulting structure is called a fuzzy graph. Fuzzy graphs allow both vertices and edges to have membership values that indicate their degree of existence or strength of connection. This feature makes fuzzy graphs particularly useful in representing systems such as communication networks, social relationships, transportation systems, and biological networks.

The development of fuzzy graph theory has enabled researchers to study network systems under uncertainty. Various extensions and models of fuzzy graphs have been developed to improve analysis and decision making in complex systems. This chapter provides an overview of fuzzy graph theory, including its basic concepts, methodology, and applications. The objective is to present a clear understanding of how fuzzy graphs can be constructed and applied to solve real-world problems involving uncertain relationships.

2. Construction of Fuzzy Graphs

A fuzzy graph is typically represented as $G = (V, \sigma, \mu)$, where V is a set of vertices, σ is a membership function that assigns each vertex a value between 0 and 1, and μ is a membership function that assigns a value between 0 and 1 to each edge between vertices. These values represent the degree of participation of vertices and the strength of connections between them. The membership value of an edge must not exceed the minimum membership value of its endpoints. This ensures consistency within the graph structure.

The process of constructing a fuzzy graph generally involves the following steps: Identification of vertices representing entities in the system. Assignment of membership values to each vertex based on their relevance or importance. Identification of relationships between vertices. Assignment of membership values to edges representing the strength of relationships.

3. Types of Fuzzy Graphs

Researchers have developed several variations of fuzzy graphs to represent different types of uncertain relationships.

Strong fuzzy graphs: In these graphs, the membership value of each edge is equal to the minimum membership value of its end vertices.

Complete fuzzy graphs: Every pair of vertices is connected with an edge having a certain membership value.

Regular fuzzy graphs: Each vertex has the same degree of membership with respect to edge connections.

Fuzzy trees and fuzzy cycles: These structures extend classical tree and cycle concepts into fuzzy environments.

4.Analytical Techniques

Several methods are used to analyze fuzzy graphs. These include determining fuzzy paths, fuzzy connectivity, fuzzy degrees, and fuzzy distances. A fuzzy path refers to a sequence of vertices connected through edges with non-zero membership values.

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Connectivity analysis helps determine how strongly different parts of the graph are linked. Graph measures such as centrality, density, and clustering can also be extended to fuzzy graphs to evaluate network characteristics. These measures help researchers analyze the importance of nodes, identify clusters, and study network reliability.

To apply fuzzy graphs in practical systems, data is first collected regarding relationships among elements. These relationships are then converted into membership values using expert judgment, statistical analysis, or computational models. Once the fuzzy graph is constructed, mathematical techniques are used to analyze connectivity patterns and identify significant structures within the network.

4. Conclusion

Fuzzy graph theory provides a powerful extension of classical graph theory by incorporating the concept of partial membership. This allows researchers to model complex systems that involve uncertainty and imprecise relationships. By assigning membership values to vertices and edges, fuzzy graphs can represent varying levels of connectivity and importance within networks. The methods discussed in this paper demonstrate how fuzzy graphs can be constructed and analyzed effectively. Their flexibility makes them suitable for applications in communication networks, decision-making systems, transportation planning, and social network analysis.

As research continues, fuzzy graph theory is expected to expand further with the development of new models and analytical techniques. These advancements will enhance the ability of researchers and engineers to analyze uncertain systems and make better decisions in complex environments.

References

[1] Rosenfeld, A. (1975). Fuzzy graphs. Fuzzy Sets and Their Applications.
[2] Zadeh, L. A. (1965). Fuzzy sets. Information and Control.
[3] Mordeson, J. N., & Nair, P. S. (2000). Fuzzy Graphs and Fuzzy Hypergraphs.
[4] Bhutani, K. R., & Rosenfeld, A. (2003). Strong arcs in fuzzy graphs.
[5] Samanta, S., & Pal, M. (2013). Fuzzy graph theory applications.
[6] Gani, A. N., & Radha, K. (2010). Properties of fuzzy graphs.
[7] Sunitha, M. S., & Vijayakumar, A. (2002). Complement of fuzzy graphs.
[8] Akram, M. (2011). Bipolar fuzzy graphs and their applications.

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Chapter 18

Dialect Words of Vellore District

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Abstract

Language reflects the geographical setting, social life, and cultural practices of the community that uses it. Regional dialects emerge as variations of a language shaped by local environments, occupations, traditions, and interactions among people. The present study examines the dialect words used by the people of Vellore District and analyses how these words function in everyday communication. The research categorizes dialect expressions into three major groups: occupation-related, nature-related, and culture-related vocabulary. Through selected examples collected from local speech usage, the study highlights how dialect words represent the lifestyle, ecological surroundings, and cultural identity of the region. The findings reveal that dialect expressions not only serve as linguistic tools for communication but also preserve the cultural heritage and social memory of the community. By documenting and analysing these regional expressions, the study contributes to the understanding of dialectal diversity in Tamil and emphasizes the importance of preserving local linguistic traditions in contemporary society.

Keywords: Dialect words, Regional language variation, Tamil dialectology, Cultural vocabulary, Nature-related vocabulary.

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1. Introduction

Language is closely related to the geographical location and living environment of the people who use it. The form and usage of a language often vary depending on the regional setting, cultural background, and social context of a particular area. As a result, the words used in everyday speech undergo changes in pronunciation, meaning, and usage from one district to another. In each region, people adapt language according to their local culture and communicative needs. These variations give rise to regional dialects and vocabulary that reflect the unique linguistic identity of a community. In this context, the present study aims to examine how such dialect words are used in the everyday speech of people living in Vellore District and to analyse the forms in which these expressions continue to function within the local linguistic tradition.

2. Dialect Words

Every language spoken by people tends to develop several regional variations over time. These variations emerge due to differences in geographical location, social interaction, and cultural practices. Such linguistic variations are commonly referred to as regional dialects. Dialects represent distinctive forms of language that are shaped by the speech habits of particular communities.Based on this understanding, the present study examines how people in Vellore District employ dialect words in their everyday communication. It attempts to identify the usage patterns and linguistic characteristics of these regional expressions within the local speech tradition.

3. Origin of the Name "Vellore District"

The name Vellore is believed to have originated from the presence of abundant Velam trees (Prosopis juliflora / Seemai Karuvelam) that once surrounded the region. Hence, the land covered with these trees came to be known as Vellore. Among the 38 districts of Tamil Nadu, Vellore District occupies an important administrative and historical position.In 1989, the former North Arcot District was reorganized into two separate districts. These were named North Arcot Ambedkar District and Tiruvannamalai Sambuvarayar District. Later, in 1996, the North Arcot Ambedkar District was officially renamed as Vellore District. Subsequently, in 2019, the district underwent further administrative reorganization, resulting in the formation of three separate districts: Vellore District, Ranipet District, and Tirupattur District.

4.Taluks of Vellore District

Vellore District consists of several administrative taluks, which play an important role in local governance and administration. The major taluks of the district include:

Vellore Taluk
Gudiyatham Taluk
K. V. Kuppam Taluk
Katpadi Taluk
Pernambut Taluk
Anaicut Taluk

4.1 Words Spoken by People in the Border Areas of Vellore District

The people of Vellore District primarily use Tamil as their spoken language in everyday communication. However, since the district shares a border with the state of Andhra Pradesh, linguistic interaction has influenced the speech patterns of people living in the border regions. As a result, some residents of these areas also speak

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Telugu, and elements of the Telugu language can be observed in their local speech.

4.2 Categories of Dialect Words Used in Vellore District

The dialect words spoken in Vellore District can be broadly classified into the following categories:

1. Occupation-related dialect words – terms associated with various professions and livelihood activities.

2. Nature-related dialect words – vocabulary related to natural elements such as land, weather, plants, and animals.

3. Culture-related dialect words – expressions reflecting customs, traditions, rituals, and social practices of the local community.

4.3 Occupation-Related Dialect Words

Occupation-related dialect words refer to the specialized vocabulary used by people engaged in a particular profession. Such words are commonly employed within workplaces and among workers belonging to the same occupational group. These expressions often denote tools, techniques, materials, and work procedures associated with specific professions. Furthermore, different regions and occupations develop their own unique sets of terminology based on local practices and cultural contexts. Based on this understanding, some of the occupation-related dialect words used by people in Vellore District are listed below. Examples of Dialect Words:

Dialect Word- Meaning (English)

கருக்கா-Empty paddy grains without rice inside

கூடகட்ட- A wooden block used for sharpening a sickle

அறுப்புகுட்ட -A goat raised specifically for meat

புடி-A unit or measure of weight

காது -A type of cooking pan (Vadachatti)

மைகோதீ- A metal comb used to remove tangles in hair

வாலுருவீ-Cattle trader

கச்சால்-A tool used for catching fish

பறி-Fishing activity or fishing method

கட்ட கறி-Fish meat

சிமிட்டா-A thorn remover or tweezer-like tool

கரண-Seed sugarcane cut into pieces for planting

கோது-Sugarcane residue or fibre

பாகு -Jaggery syrup

பப்பிள்ள-A type of fried snack (Ottavadai)

கொழலுகாரன்-A person who plays the flute

பொட்டி-Harmonium (musical instrument)

தொரப்பணம்-A digging tool used for planting trees

வாச்சி-Tool used for lifting wood

கொட்டாபுலி-Wood cutter

மனகத்த அரிசி-Red rice

1 புட்டி-Equivalent to 40 Marakkal(traditional grain measure)

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கூர்-Approximate quantity without precise measurement

நவுத்தடி-Agricultural tool used with cattle plough

கொடிஜல்-Tool used for leveling soil

These dialect expressions demonstrate how local occupational practices influence vocabulary. Such words not only represent linguistic diversity but also preserve the traditional knowledge and livelihood practices of the people in the region.

4.4 Nature-Related Dialect Words

Nature-related dialect words refer to vocabulary that emerges from the interaction between human life and the natural environment. The words used by people often change according to the geographical setting, climate, and ecological conditions in which they live. As a result, regional communities develop unique expressions that describe natural objects, plants, animals, landscapes, and environmental features.

Based on this perspective, some of the nature-related dialect words used by people in Vellore District are listed below. Examples of Nature-Related Dialect Words:

Dialect Word-Meaning (English)

திரங்கல்-Pepper

சினை-Fat

குண்டலம்-Ear ornament (earring)

வாகீ-Rabbit

தூர்-Mud

கானகோழி-Water hen

கந்தம்-Lotus

சுறுக்கு-Sharpness

ஈசானம்-Northern direction

கூவான்-Rooster

தச்சாரம்-Camphor

வலயம்-Boundary

குரண்ட-Hook

கரிசல்-Black soil

மல்லு-Fight

நாகு -Snail

உடத்தை-Squirrel

முத்த-Fermented porridge (Kali)

களிகோல்-Tool used for preparing ragi porridge

பனங்கா-Palmyra fruit (Nungu)

மோத்து-Small shoot of sugarcane

சொறுக்கு -Sugarcane

தாக்கு-Pit or hollow

குலவி கல்-Grinding stone (Ammikkal)

மச்சி வீடு -House with an upper floor (terraced house)

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கரகம்-Rain

தாராதம்-Cloud

ஆலி -Oil flow or oil stream

சுள்ளி-Dry stick

கலக்கா-Groundnut

கொசவன் குட்டை-Water body or pond near the village

These dialect expressions illustrate how environmental conditions influence language use. Such words not only describe natural elements but also reflect the close relationship between the local community and their surrounding ecological landscape.

4.5 Culture-Related Dialect Words

Culture-related dialect words represent distinctive linguistic forms that reflect the geographical setting, lifestyle, traditions, and belief systems of a community. Such words are closely tied to the social structure and cultural practices of the people who use them. They often transmit cultural values, customs, and social identities across generations. Based on this perspective, several culture-related dialect expressions used by the people of Vellore District in their everyday life are presented below.

Examples of Culture-Related Dialect Words:

Dialect Word-Meaning (English)

வாராகி-Earth / land

எழிலி-Cloud

தக்கல்-A virtuous or admirable woman

வர்ணன்-King

சுரம் -Life / vital force

பக்கறை-Confusion

பீட்டா-Grandson

சுதி-Music / melody

ஒட்டான்-Enemy

கருங்காலி-Informer / traitor

நாமக்கார்-Vaishnavite devotee

பரிகாரி-Physician / traditional healer

பள்ளி-Vanniyar community

குடியானத்தெரு-Street inhabited by upper-caste groups

காசங்குடி -Migrant settlers

காயக்குடி-Indigenous inhabitants

கோல்காரன்-Village announcer who conveys public information

மருகி-Daughter-in-law

மருகன்-Son-in-law

ஜாமம்-Traditional method of time calculation

சகாயம்-Help / assistance

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These expressions highlight how language functions as a cultural marker. The dialect vocabulary reflects social relationships, kinship structures, community organization, and traditional practices that shape everyday life in the region.

5. Conclusion

Language adapts to the geographical environment and socio-cultural context in which people live. Each community modifies linguistic forms according to its lifestyle, occupation, and cultural practices. In this context, the dialect words used by the people of Vellore District illustrate how regional language reflects the lived experiences of the community. This study has examined selected dialect expressions used in the district and categorized them into occupational, nature-related, and cultural vocabulary. Through these examples, it becomes evident that dialect words continue to play a vital role in preserving regional identity and cultural heritage.

References

[1] M. S. Boopathi. Vattara Vazhakku. Chennai: New Century Book House, 2003.
[2] Senthamizh Anthanar R. Ilangkumaranar. Vattara Vazhakku Sol Akarathi (Dictionary of Dialect Words). Chennai: Manivasagar Pathippagam, 1998.
[3] M. Gunasekaran. Vellore Varalattru Sirappu (Historical Significance of Vellore). Vellore: Vani Publications, 2005.
[4] Dr. P. Venkatesan. Katpadi – K.V. Kuppam Thadayangal. Vellore: Tamil Research Publication / Local Research Publication, 2012.

Chapter 19

Scientific Messages in the Novels of Nanjil Nadan

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Abstract

Tamil literature has always reflected the knowledge systems embedded in everyday life. The works of Nanjil Nadan, one of the significant contemporary Tamil writers, provide rich insights into the scientific knowledge present in rural culture and traditional lifestyles. His novels portray a variety of scientific elements related to agriculture, ecology, engineering, food science, microbiology, economics, and mathematics. Through realistic narratives rooted in the socio-cultural life of Kanyakumari district, the author documents indigenous knowledge systems that have evolved through generations. This chapter analyses the scientific messages embedded in Nanjil Nadan's novels and highlights how traditional practices reflect principles of modern science. The study demonstrates that science is not confined to laboratories but is deeply integrated into the daily life of farmers, homemakers, and traders. By documenting such experiential knowledge, Nanjil Nadan contributes significantly to the intellectual richness of Tamil literature and promotes a sustainable way of life closely connected with nature.

Keywords: Nanjil Nadan, Scientific Knowledge in Literature, Indigenous Knowledge Systems, Tamil Novels, Food Science, Ecology.

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1. Introduction

Nanjil Nadan is widely recognized for his natural narrative style, sharp observation of human behaviour, and deep attachment to the land and rural life. His novels capture the social and cultural life of the Kanyakumari district, particularly the agricultural traditions and environmental practices of the region. One of the distinctive features of his writings is the integration of scientific ideas within literary narratives. His novels contain insights related to agricultural science, soil science, food science, economics, engineering, and ecology. Rather than relying heavily on imagination, he bases his narratives on realistic experiences and the lived knowledge of rural communities. This chapter aims to analyse the scientific messages embedded in the novels of Nanjil Nadan and to explore how traditional practices reflect principles of modern science.

2. Science in Literary Narratives

The integration of scientific ideas in literature plays an important role in disseminating knowledge to future generations. Scientific concepts embedded within narratives not only enhance readers' curiosity but also contribute to their intellectual development. Scientific narratives often explore the potential impact of technological discoveries and innovations on society. Such literary works are commonly referred to as science fiction (Tamil Virtual University, 2026).

In the novel Thalaikeezh Vigithangal, the author portrays the contradictions of human life through the character Sivathanu, illustrating how progress in one dimension of life may result in setbacks in another. Similarly, the novel Ettuthikkum Mathayaanai explains various agricultural practices such as natural fertilizers, seed characteristics, and irrigation systems.

Another novel, Sathuranga Kudhirai, deals with economic decision-making, consumer behaviour, and market logic. Through these narratives, the author subtly introduces scientific and analytical thinking.

2.1 Water Irrigation Systems

The irrigation systems of Kanyakumari district represent an excellent example of traditional engineering knowledge. Nanjil Nadan vividly describes the ponds, lakes, and canal systems used for agriculture in the region.Rainwater flowing from the Western Ghats is naturally channelled into streams and ponds. When one pond becomes full, water flows into another through specially designed sluice systems (madagu). These interconnected water bodies form an efficient irrigation network that ensures optimal distribution of water to agricultural fields.Farmers also possess experiential knowledge about the moisture-retention capacity of various soils such as red soil, black soil, and alluvial soil. Such knowledge is reflected in the novels Thalaikeezh Vigithangal and Ettuthikkum Mathayaanai.

2.2 Green Manure and Agricultural Ecology

Traditional agricultural practices emphasised the use of green manure long before the introduction of chemical fertilizers. These organic materials increased microbial activity in the soil and improved fertility. Nanjil Nadan also documents the biodiversity of the Western Ghats by listing numerous plants, trees, and creepers found in the region. His novels provide valuable information about coconut diseases, paddy varieties, and traditional seed selection methods. Farmers predicted rainfall using natural indicators such as wind direction and cloud formation. These practices demonstrate the depth of indigenous environmental knowledge.

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2.3 Soil Science

The novels highlight that fertile soil contains nutrients such as nitrogen and phosphorus, which are essential for plant growth. Traditional agricultural practices such as kidaipoduthal (penning livestock in fields) enriched the soil with organic matter. Goat urine and manure decomposed naturally and improved soil fertility.

The author also explains the design of the plough, which uses the pulling force of cattle efficiently to break the soil at an optimal angle. These descriptions demonstrate a strong understanding of soil systems and agricultural engineering.

2.4 Ecological Adaptation

Migration and environmental adaptation are important themes in Nanjil Nadan's works. In one of his narratives, the character Shanmugam migrates to Mumbai in search of employment and struggles to adapt to the new environment.

The author vividly describes the biological and psychological stress experienced by individuals when exposed to unfamiliar climatic conditions. Such narratives illustrate the concept of ecological displacement adaptation. Similarly, the novel Vidhavai highlights the physical stress caused by drastic environmental changes.

2.5 Environmental Awareness and Deforestation

Nanjil Nadan repeatedly emphasises the importance of environmental conservation. His novels warn about ecological damage caused by deforestation and the neglect of traditional water bodies. By documenting the flora and fauna of the Western Ghats, the author portrays the region's rich biodiversity. He also explores the relationship between ecological changes and social transformations.

2.6 Mathematical Knowledge in Daily Life

Mathematical thinking appears frequently in the author's narratives through references to measurements, distance, and time calculations. Traditional measurement units such as marakkal, padi, and kottai are mentioned alongside modern measurements. The novel Thalaikeezh Vigithangal also highlights the economic implications of dowry, illustrating how social practices can reflect market values.

2.7 Economic Principles in Fiction

Nanjil Nadan's novels contain insights into economic principles such as demand, supply, and market fluctuations. In Sathuranga Kudhirai, the protagonist Narayanan demonstrates statistical reasoning by predicting market trends. The author also critiques modern consumer culture, particularly the commercial strategies used in restaurants to attract customers.

2.8 Health and Food Science

Food culture in Nanjil Nadu is treated as an important aspect of life. The author explains the medicinal properties of herbs and spices used in traditional cuisine. The novel Maamisa Padaippu discusses the scientific aspects of cooking, including temperature control and the timing of adding spices. Nanjil Nadan also explains fish preservation techniques using salt and sun-drying methods.

2.9 Microbiology in Traditional Practices

The author highlights the role of microorganisms in food preparation and health. Fermented rice porridge (kanji) is described as a nutritious food rich in vitamin B12. He also discusses the risks

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associated with contaminated water in rural areas and the spread of waterborne diseases. The preservation of fish using salt is explained through the principle of osmosis, which prevents bacterial growth.

The irrigation systems of Nanjil Nadu represent sophisticated indigenous engineering practices. Sluice structures regulate water pressure and prevent erosion through layered stone arrangements. Traditional houses were also designed scientifically. Features such as thinnai (raised platforms) and mutram (central courtyards) ensured ventilation, natural lighting, and temperature regulation.

3. Conclusion

Nanjil Nadan's novels demonstrate that scientific knowledge exists not only in laboratories but also in everyday life. Farmers, homemakers, and traders possess valuable experiential knowledge that reflects fundamental scientific principles. By documenting traditional practices and ecological wisdom, the author promotes a sustainable lifestyle closely connected with nature. His works therefore enrich Tamil literature by integrating cultural heritage with scientific understanding.

References

[1] Kannan, T. (2010). Karpanavum ini amaiyum – Nanjil Nadanudan oru nerkaanāl. Pathagai.
[2] Nanjil Nadan. (1977). Thalaikeezh Vigithangal.
[3] Nanjil Nadan. (2005). Sathuranga Kudhirai.
[4] Nanjil Nadan. (2008). Midhavai.
[5] Nanjil Nadan. (2009). Thalaikeezh Vigithangal (Reprint edition).
[6] Nanjil Nadan. (2010). Mamisap Padaippu.
[7] Nanjil Nadan. (2026). Ettuthikkum Mathayaanai, Tamil Virtual University, https://www.tamilvu.org.

Chapter 20

Scientific Thoughts in Thirumandiram

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Abstract

Siddha literature holds a distinctive place in the history of Tamil literature. The Siddhars proposed a holistic way of life that integrates the physical, mental, and spiritual dimensions of human existence. Their ideas are not limited to spiritual philosophy alone; they also contain scientific insights related to human anatomy, breathing mechanisms, nutrition, pharmacology, herbal medicine, and other fields. The text Thirumanthiram includes various scientific perspectives such as embryology, dietary principles, herbal medicine, yogic science, and psychology. This study examines and explains the scientific concepts found in Thirumanthiram with appropriate literary evidence and analysis.

Keywords: Thirumanthiram, Siddha Literature, Scientific Thought, Yogic Science, Siddha Medicine.

1. Introduction

Science is a field of knowledge that helps in understanding the functioning of the world, explaining natural phenomena, and improving human life. In the modern era, scientific advancements have brought remarkable progress in various aspects of human living. However, the foundational ideas of scientific thinking can be traced back to ancient times and are reflected in many classical

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literary works and Siddhar texts. Tamil Siddha literature presents a unique synthesis of ideas related to spirituality, medicine, yoga, biology, psychology, and natural sciences. Among these works, Thirumanthiram occupies a prominent place. Composed by the Siddhar Tirumoolar, this text explains concepts related to the human body, life force, mind, diet, breathing practices, and herbal medicine through poetic expressions. This study examines the scientific ideas found in Thirumanthiram by analyzing relevant literary evidence and comparing them with concepts found in modern science. Through this analysis, the study highlights that the Siddhar tradition serves as an important historical source for early scientific thought.

2. Embryology

In modern biology, the process through which human life begins is referred to as fertilization. Thirumanthiram contains verses that indirectly describe the formation of the embryo. In these verses, Tirumoolar explains the process of conception through symbolic expressions, describing how the male reproductive element enters the female womb and results in the creation of life. According to modern biological science, human life begins when the male and female chromosomes combine during fertilization to form a zygote. Although the Siddhars did not use modern scientific terminology such as "chromosomes," their writings reveal an understanding of the basic process involved in the origin of human life. Tirumoolar states in the following verse:

"**Seeking the sacred womb, it entered within;**

the two forces rushed and fell together as one." (Thirumanthiram, -154)

In this verse, the word “Thiru” symbolically refers to the woman. The lines describe the event in which the male reproductive element enters the female womb and leads to the formation of a new life. In contemporary biological terms, this process is known as fertilization. Thus, the verse suggests that Tirumoolar possessed an insightful understanding of the process of conception, which reflects an early form of scientific thought regarding embryological development.

3.Health Maintenance

Although the Siddhars generally regarded the human body as impermanent and often emphasized asceticism, Tirumoolar strongly stressed the importance of preserving and caring for the body. He states in the following verse:

“**If the body decays, life too will decline;**

firm realization of true wisdom will not be attained.

Knowing the means to nurture the body,I nourished the body and thus strengthened life.” (Thirumanthiram,724)

This verse highlights the significance of maintaining physical health. It suggests that when the body is properly protected and sustained, life continues in a stable manner. A healthy body also becomes the foundation for attaining spiritual realization. This idea closely corresponds with modern medical concepts such as health maintenance and preventive medicine, which emphasize protecting the body and maintaining well-being in order to prevent disease and support long-term health.

3.1 Psychology and Inner Consciousness

The Thirumandiram strongly emphasizes the power of the human mind. It presents the idea that a person's thoughts play a crucial role

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in shaping their life. In other words, life tends to reflect the nature of one's thoughts.According to this perspective, the psychological process of thought → speech → action determines the quality and direction of human life. Positive thoughts naturally lead to positive words and constructive actions, which ultimately contribute to a meaningful and virtuous life. A similar ethical principle is expressed in the Thirukkural. As stated in Kural 34: "**Manaththukkan māsilan āthal anaiththaran**." This verse explains that true virtue lies in maintaining a mind free from impurities such as harmful or negative thoughts.In addition, Thirumandiram also refers to the physiological and neurological aspects of the human body. Thirumoolar describes the subtle energy channels known as Ida, Pingala, and Sushumna, which play an important role in yogic practices and Siddha medical traditions. These channels are believed to regulate the flow of vital energy within the body. Furthermore, the text highlights the significance of breathing patterns, the circulation of life force, and the internal functioning of the body as essential elements of human health and spiritual development. These ideas can be compared with modern scientific discussions on the nervous system and bodily regulatory processes. The role of the mind is crucial in determining the quality of human life. Positive thoughts lead to positive actions and ultimately contribute to a better and more meaningful life. The Thirumandiram highlights control of the mind as one of the central goals of yogic practice. According to the teachings of the Siddhars, negative mental states such as stress, fear, and excessive desire can adversely affect physical health. These insights suggest that the condition of the mind directly influences the well-being of the body. Interestingly, this view aligns with the perspectives of modern

Psychology, which recognizes the close relationship between mental states and physical health.

3.2 Nutrition

Food plays a vital role in maintaining human health, both in terms of quantity and quality. In the Thirumandiram, Thirumoolar explains that moderate and regulated eating helps preserve the balance of the body. Excessive consumption as well as insufficient intake of food can both lead to various diseases. This idea is also emphasized in modern Nutrition Science. Contemporary nutrition studies highlight the importance of a balanced diet, which provides the nutrients necessary for the body's proper growth, functioning, and overall health. Thirumoolar similarly points out that the nature and quantity of food directly influence human health.

The following verse from Thirumandiram explains the consequences of improper eating habits:

"**Mēviya vannattāl vilaṅkiyatu iccaḍam**

Pāviye koṉṟu paḻiyuṟu mannantān

Āṉmitam tappilaiyuṟṟu nōyāgum

Kāvilivai yellām kaṇḍu koḷ annamē." (Thirumanthiram,1820–1825)

This verse indicates that excessive eating can lead to several physical ailments and that one must understand the importance of moderation in food. A similar principle is expressed in the Thirukkural: "**Mikkīnum kuṟaiyinum nōy**."(Kural – meaning: both excess and deficiency lead to disease). Thus, both classical Tamil wisdom and modern medical science stress the importance of balanced and moderate eating habits as essential for maintaining good health.

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3.3 Neurology

In the Thirumandiram, Thirumoolar also refers to the neurological structure and subtle energy pathways of the human body.

“**Mūnru nathi yuḷē muthir naṭakkum mūkkiṟ**

Senṟum thiriyum irupakka nātham

Ninrathor thūṇḍil neḍiya uḍampir

Ponrāmē vāzhum punal nāḍi thānē.” (Thirumanthiram, - 566)

This verse describes the three important channels (nāḍis) within the human body. These are traditionally identified as Ida, Pingala, and Sushumna. In yogic philosophy, these channels are believed to regulate the movement of vital energy and breath within the body.The Ida and Pingala nāḍis are said to function on the left and right sides of the body, while the Sushumna runs along the central axis. These pathways are closely associated with breathing patterns and the flow of life energy. In yogic practices, controlling the breath and balancing these nāḍis is considered essential for physical and mental well-being. Such ideas can be compared with modern discussions in Neuroscience, which explore the functioning of the nervous system and its influence on bodily regulation.

3.4 Medicine

Siddha Medicine is a traditional healing system based largely on natural herbs and medicinal plants. In Thirumandiram, several references are made to herbal remedies and their therapeutic uses. The Siddhars described the medicinal properties of herbs and their application in treating various diseases. This traditional knowledge of herbal medicine has continued to influence many forms of alternative and complementary medical practices today. For example, Thirumoolar suggests a herbal remedy for eye ailments in the following verse:

“**Induppu thippili iyal pītharōgiṇi**

Nandiyāvaṭṭa chāṟṟil nayanthē yaraithida

Anthakan kaṇṇukku arundhathi thōnṛidum

Nandhikku nāthan nayanthuraitthittadē.”(Thirumanthiram, 2049)

According to this verse, a medicinal preparation can be made using herbs such as Induppu, Thippili, Peetharogini, and Nandiyavattai. These ingredients are ground together and used as a treatment for eye-related diseases. Similar herbal remedies are still practiced in certain rural and traditional medical settings.

3.5 Yoga Science

In the Thirumandiram, Thirumoolar emphasizes that yogic practices contribute significantly to the improvement of both physical and mental health. Practices such as āsana (postures), prāṇāyāma (breath regulation), and meditation help maintain balance within the body and mind. These practices are considered essential for achieving harmony between the body’s physiological functions and mental stability. Modern medical and scientific studies also acknowledge that yogic exercises reduce stress, enhance mental clarity, and improve overall physical well-being. Thus, the yogic principles described in Thirumandiram can be viewed as closely aligned with the perspectives of contemporary Yoga Therapy and health sciences.

4. Discussion

The scientific ideas presented in Thirumandiram relate to several aspects of human life, including the body, food, mind, medicine, and

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spiritual discipline. These insights can be considered valuable elements of traditional knowledge systems. When compared with modern scientific understanding, many of these concepts show notable similarities. Such parallels suggest that the Siddhars possessed a deep observational understanding of nature and the human body. Their insights into health, diet, mental discipline, and medicine reflect an early form of systematic thinking that resembles scientific inquiry.

5. Conclusion

Although Thirumandiram is primarily regarded as a spiritual text, it also contains numerous scientific ideas. The work addresses various fields such as embryology, physiology, nutrition, yogic science, and herbal medicine. The concepts proposed by Thirumoolar serve as meaningful guidelines for maintaining human health and a balanced lifestyle. Therefore, Siddhar literature can be viewed as an important source of traditional scientific knowledge. These works demonstrate that the Tamil Siddhars not only contributed to spiritual philosophy but also offered profound insights into scientific thinking related to human life and health.

References

[1] Tirumular. (2002). Tirumandiram (Tamil edition). Chennai: Saiva Siddhanta Publishing House.
[2] Zvelebil, K. (1995). Tamil Literature. Leiden: Brill Academic Publishers.
[3] Kailasapathy, K. (1987). Tamil Heroic Poetry. Oxford University Press.
[4] Subramanian, N. (1998). Siddha Medicine and Science. Chennai: Tamil University.
[5] Thiruvalluvar. (2014). Thirukkural. Chennai: International Institute of Tamil Studies.

Chapter 21

Workship Practices of the Ancient Tamizhar in Sangam Literature

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Abstract

Sangam Literature represents one of the earliest and most significant literary traditions of the Tamil language. These works provide valuable insights into the lifestyle, social organization, religious beliefs, and ritual practices of the ancient Tamil people. The early Tamizhar lived in close harmony with nature and considered natural elements such as mountains, trees, seas, and rivers as sacred manifestations worthy of worship. In addition, they followed the custom of commemorating fallen heroes by erecting hero stones (Nadukal) and performing ritual worship in their honour. Through the study of Sangam texts, it is possible to understand the diverse forms of worship practiced by the ancient Tamizhar, including devotion to natural forces, village deities, and major gods. This study examines the various worship traditions reflected in Sangam literature with relevant literary evidence and attempts to highlight the spiritual outlook and religious consciousness of the early Tamil society.

Keywords: Sangam Literature, Religion, Workship, Ancient Tamizhar.

1. Introduction

Sangam Literature represents the earliest and most ancient literary tradition of the Tamil language. These texts are generally dated

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between 300 BCE and 300 CE and provide significant insights into the cultural patterns, social organization, and everyday life of the ancient Tamil people. Sangam literature is broadly classified into two major categories: Akam (interior) and Puram (exterior). Akam literature primarily deals with themes of love, emotions, and personal relationships within the private sphere of human life. In contrast, Puram literature focuses on public life, highlighting heroic deeds, warfare, political activities, generosity of kings, and various aspects of social life. The Concepts can be found in [1-6].

2. Life and Culture of the Ancient Tamizhar

The assembly of Tamil scholars who gathered to study, discuss, and promote the Tamil language was traditionally known as the Sangam. This scholarly academy played an important role in nurturing literary creativity and intellectual exchange among poets. The significance of such gatherings is reflected in the verse:

"Tamil kezhuk koodal than kōl vēndhē" (Purananuru,58)

This verse highlights the importance of assemblies where learned scholars convened to deliberate on Tamil language and literature. The people who lived before and during the Sangam period are generally referred to as the ancient Tamizhar (Tholkudiyinar / Tholkudiyar). Their lifestyle was deeply rooted in nature. Natural environments such as mountains, forests, agricultural lands, and seas shaped their livelihood, culture, and social practices. A well-known verse that expresses the universal humanistic outlook of the ancient Tamizhar

is: **"Yaadhum Oore, Yaavarum Kelir"** (Purananuru,192)

This line conveys the idea that all places are one's own and all people are kin. It reflects the inclusive worldview and humanitarian values of the ancient Tamil society, demonstrating their belief in universal harmony and unity among humankind.

3.Religious Thought

The term religion (Samayam) refers to an organized system or set of principles formulated for the worship of a divine power. In classical usage, the word samayam denotes an established doctrine or disciplined path associated with spiritual practice. According to K. S. Pillai, the concept of samayam corresponds to the idea of a structured system similar to the English term "system," implying an organized framework through which religious practices and beliefs are carried out (Pillai, Tamilar Samayam, All-India Tamil Conference Welcome Address, p. 25). The word samayam is also related to the root samai, which signifies the act of preparing or refining. In this sense, religion functions as a process that cultivates and refines human character. It shapes individuals in a manner that promotes both personal well-being and harmonious social living. Thus, religion plays a vital role in guiding human beings toward spiritual maturity and divine grace (S. Thirunavukkarasu, Nalvar Nanmanimalai: Or Aayvu).Scholars have further explained that religion emerges as a complete system when emotional elements, ritual practices, and intellectual understanding function together in harmony (Kalaikkalanjiyam, Vol. 4, p. 452).During the Sangam period, religion functioned as an important social institution that regulated human life and moral conduct. It guided individuals toward virtues such as righteousness, discipline, and compassion. The existence of religious institutions during this period is indicated in the following verse: "**Pagatteruththin pala saalaith thavappalli**" (Pattinappaalai,52–53)

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This verse suggests the presence of ascetic centers and religious establishments, indicating that spiritual practices and religious learning were integral components of early Tamil society.

4.Belief in God

The people of the Sangam age possessed a deep belief in divine powers. Observing the forces of nature often evoked awe and reverence, which gradually developed into forms of worship. The ancient Tamizhar believed that divine energy was present within natural elements and phenomena. Early grammatical and cultural references indicate that certain ritual practices were associated with divine worship. In this context, references such as Kodi Nilai, Kanthazhi, and Valli, mentioned in Tolkappiyam, are interpreted as symbolic forms of early worship traditions connected with divine belief.Sun worship has also existed among people from ancient times. Natural forces were regarded as manifestations of divine power and were revered accordingly. A verse from Natrinai illustrates the worship of the sun:

“**Munnīr mīmisai pala thozhath thōndri**

Ēmuṟa viḷangiya sudarinum” (Natrinai,283)

This verse indicates that the sun, shining above the vast seas and receiving the reverence of many, was considered a divine entity worthy of worship.The belief that the universe was created by a divine power is also reflected in Sangam poetry. A verse in Natrinai expresses the idea that the world itself was created by God:

“**Aithē kamma! ivvulagu padaithōnē**!” (Natrinai,240)

Similarly, another poetic line suggests that devotion to the divine would bring prosperity and blessings such as rainfall:

"**Theyvam thozhāḷ kozhunan**

thozhudhezhvāḷ

peyyenap eyyum mazhai." (Natrinai,240)

These lines reveal the strong faith that people placed in divine intervention and grace. Natural forces that caused fear or wonder were often personified as deities. In Sangam literature, certain supernatural powers are referred to as "Sūr."

"**Sūr uru manjnaiyin nadunga**" (Akananuru,98)

Furthermore, the deity believed to determine or measure the span of human life was referred to as "Kootruvan," the god associated with death.

"**Kootruvan pōla kodiyaḍhu pirivu**" (Purananuru,42)

Thus, Sangam literature clearly reflects that the ancient Tamizhar maintained a profound belief in divine powers, perceiving them both in natural phenomena and in the cosmic order governing human life.

5.The Five Landscapes (Ainthinai) and Deity Worship

The concept of the five ecological landscapes (Ainthinai) described in Tolkappiyam reflects the close relationship between nature and the lifestyle of the ancient Tamizhar. Early Tamil society revered nature and perceived divine presence within natural environments. Accordingly, the land was classified into five distinct ecological regions, each associated with a particular deity and mode of life. The inhabitants of these landscapes worshipped different deities based on their natural surroundings. The mountainous Kurinji region was associated with the worship of Murugan; the pastoral Mullai region with Tirumal (Vishnu); the fertile agricultural Marutham region with

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Indra; the coastal Neithal region with Varuna; and the arid Palai region with Korravai, the goddess of victory and war. This classification clearly illustrates how religious beliefs in ancient Tamil society were closely intertwined with ecological conditions and geographical landscapes.

5.1 Nature Worship

The Tamizhar of the Sangam period regarded nature itself as sacred and worthy of reverence. Natural elements such as mountains, trees, rivers, and seas were considered manifestations of divine power and were worshipped accordingly. This form of nature worship reveals the deep ecological awareness and environmental sensitivity of the ancient Tamil people. Their reverence for natural forces demonstrates a cultural outlook in which human life was viewed as inseparable from the natural world.

5.2 Tree Worship

The ancient Tamizhar considered certain trees to be sacred and worshipped them as divine manifestations. References to such practices can be found in Natrinai (303) and Purananuru (260). These literary sources indicate that particular trees were revered and regarded as the abodes of divine power. Tree worship therefore formed an integral component of the broader tradition of nature worship among the early Tamil people.

5.3 Nadukal (Hero Stone) Worship

Another important religious practice of the Sangam period was the worship of hero stones (Nadukal). Stones were erected in memory of warriors who died heroically in battle. These memorial stones were not merely symbolic markers but objects of ritual reverence. The practice reflects the cultural values of the ancient Tamizhar, who held bravery and sacrifice in high regard. Through the worship of hero stones, the community honored the valor of fallen warriors and preserved their memory for future generations.

5.4 Worship of Minor Deities

In addition to the worship of major gods, the Sangam people also believed in the existence of minor or local deities who functioned as protective guardians of villages and communities. These deities were often associated with specific natural locations such as trees, hills, and forests. People believed that these sacred spaces were inhabited by divine forces that protected the community from harm and ensured prosperity.

5.5 Worship of Major Deities

Sangam literature also contains references to the worship of major deities such as Murugan, Siva, Tirumal (Vishnu), and Brahma. These references demonstrate the presence of established religious traditions and the reverence accorded to prominent divine figures.

Murugan

“Porulum ponnnum pōgamum alla ninpāl

arulum anbhum aṟanum” (Paripadal,5)

This verse highlights Murugan as a deity associated with grace, love, and righteousness rather than material wealth.

Tirumal (Vishnu)

“Māyōn mēya kāduṟai ulagamum” (Kalithogai,140)

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This line refers to Tirumal, who is associated with pastoral landscapes and divine protection.

Siva

"Mukkaṇ selvan nagar valam seyarkē" (Akananuru,181)

Here Siva is referred to as the three-eyed lord, indicating the presence of Saivite beliefs during the Sangam age.

Brahma

"Nānmuga oruvar payantha" (Perumpanarruppadai,42–43)

This reference mentions Brahma, the four-faced creator, showing that cosmological concepts were also present in early Tamil religious thought.

5.6 Methods of Worship

During the Sangam period, people practiced various forms of worship as part of their religious life. These practices included ritual offerings (puja), presenting flowers to the deity, lighting lamps, offering fermented beverages such as kal as part of ritual observances, and the worship of hero stones (nadukal). Such practices were closely connected with both social customs and spiritual beliefs. Through these ritual acts, people expressed devotion to divine powers and sought protection, prosperity, and well-being for their communities.

6.Spiritual Thought of the Ancient Tamizhar

The spiritual outlook of the ancient Tamizhar was deeply connected with nature. They perceived human life as an integral part of the natural world. Their beliefs and religious practices reveal a worldview in which nature, humanity, and the divine were interrelated. This perspective demonstrates that the early Tamil people maintained a harmonious relationship with nature while also recognizing the presence of sacred power within it.

7. Conclusion

The worship traditions of the ancient Tamizhar were fundamentally rooted in nature-centered cultural practices. Forms of worship such as nature worship, hero-stone worship, the veneration of minor local deities, and the devotion to major gods together reveal the spiritual consciousness of early Tamil society. Sangam literature serves as an important historical and cultural source that clearly illustrates the religious beliefs, cultural heritage, and spiritual worldview of the ancient Tamil people.

References

[1] VU. VE. Swaminatha Iyer. Purananuru. Chennai, 1894.
[2] VU. VE. Swaminatha Iyer. Natrinai. Thanjavur, 1985.
[3] VU. VE. Swaminatha Iyer. Pattinappaalai. Chennai, 1889.
[4] VU. VE. Swaminatha Iyer. Kalithogai. Chennai, 1887.
[5] VU. VE. Swaminatha Iyer. Pathitrupathu. Chennai, 1957.
[6] K. A. Nilakanta Sastri. Studies in Sangam Literature. 1956.

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9 788168 553859

Chapter 22

A Novel Encryption Framework Using Self-Invertible Matrices and Complete Graph Adjacency Matrix Structures

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Abstract

Encryption techniques form the foundation for securing communication and protecting information in the modern digital era. With the rapid growth of internet usage and network-based interactions, the need for robust encryption methods has become increasingly critical. Transmitting sensitive information over unsecured or vulnerable networks exposes it to risks such as cyberattacks, data breaches, and unauthorized access. To address these challenges, cryptographic techniques have been widely developed and applied. Classical methods such as the Caesar Cipher, Atbash Cipher, Hill Cipher, along with various symmetric encryption schemes, have significantly contributed to secure communication. In this paper, we propose an encryption approach that integrates a self-invertible matrix, the adjacency matrix, and the structure of a complete graph with a Hamiltonian circuit to generate complex ciphertext from given message units. A key feature of this method is the use of a self-invertible matrix as the encryption key, which inherently guarantees the existence of its inverse without

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requiring explicit computation. This property simplifies the decryption process and considerably reduces computational complexity while maintaining the strength of the encryption scheme.

Keywords: Adjacency Matrix, Complete graph, Graph Encryption, Hamiltonian circuit, Self-Invertible Matrix.

1. Introduction

Cryptography is a mathematical discipline designed to protect communications, data, and digital content from unauthorized access, playing a crucial role in securing sensitive information [10]. In this process, the original message, known as plaintext, is transformed into an encrypted form called ciphertext[10]. Although both plaintext and ciphertext may retain alphabetic structures, they do not necessarily use the same set of symbols; in many cases, additional characters such as numbers, punctuation marks, and special symbols are incorporated to enhance security and reduce vulnerability to attacks.

A	B	C	D	E	F	G	H	I	J	···	W	X	Y	Z		?	!
1	2	3	4	5	6	7	8	9	10	···	23	24	25	26	27	28	29

To facilitate efficient encoding of message units, this study employs a structured encoding table. Concepts from graph theory have found extensive applications in mathematics, particularly in cryptography, where numerous encryption schemes are based on graph-theoretic principles. For instance, some approaches utilize complete graphs [13] and Hamiltonian paths with lower triangular matrices as encryption keys, while others adopt upper triangular matrices in combination with complete graphs and their corresponding minimum spanning trees [14]. Alternative methods have also explored graph labeling techniques alongside triangular matrices for encoding and decoding processes [2].

Most of these symmetric encryption techniques rely on a shared key exchanged between the sender and receiver, often in the form of upper or lower triangular matrices transmitted over communication channels. The security of such systems depends heavily on maintaining the secrecy of this key [9]; if it is intercepted or discovered, the encryption becomes vulnerable to compromise. Moreover, securely sharing a key matrix over unreliable networks presents significant challenges. To address these issues and enhance security, this work proposes a novel encryption scheme that employs the adjacency matrix of a graph together with a self-invertible matrix as the keys for both encryption and decryption.

This work introduces a method for encrypting and decrypting plaintext message units by integrating fundamental concepts from graph theory with a self-invertible matrix [1, 5, 6, 7, 8] as the key. The proposed approach is designed to enhance security while offering a novel and efficient encryption framework. A key advantage of using a self-invertible matrix is that decryption can be performed without explicitly computing the inverse of the key matrix, thereby reducing computational effort.

In this scheme, message units are represented as vertices of a graph, and the sender constructs the adjacency matrix of a complete graph based on a Hamiltonian circuit. A self-invertible key matrix, generated by both communicating parties, is then multiplied with this adjacency matrix to produce the encrypted output, which is transmitted over an insecure channel. Upon receiving the

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ciphertext, the recipient retrieves the original message by applying the reverse procedure. The remainder of the paper is organized as follows: Section 2 presents the basic concepts of graph theory, Section 3 describes the construction of self-invertible matrices, Section 4 outlines the proposed method, Section 5 illustrates its implementation, and Section 6 concludes with results and suggestions for future research.

2. Basic definitions

In this section, our objective is to outline fundamental graph definitions.

Undirected/Directed Graphs: In an undirected graph, the set uv will be an edge connecting nodes u and v, therefore it arc nodes u and v, which is equivalent to connecting nodes v and u, which are both identical. The graphs are not in any particular order, and neither is there any direction to them. For a directed graph, it's crucial to consider the graphs' directions and order.

Path: A path without repeating vertices is called a route. A path from u to v exists for every u and v in a graph called V if the graph is connected.

Hamiltonian Path: A path through a linked graph G that visits all vertices precisely once is known as a "Hamiltonian path.

Complete Graph: Each pair of vertices are adjacentin a graph, the graph is said to be complete.

Adjacency Matrix: Adjacency matrix refers to a matrix that is mostly dependent on the vertices of a certain network. $A = \{1;\ v_i = v_j\ 0;\ v_i \neq v_j$

3. The generation process of self-invertible key matrices

If $P = P^{-1}$, or $P \cdot P^{-1} = P^{-1} \cdot P = I$ then such a matrix P is called as a self-invertible matrix, It was crafted through the utilization of the following:

Consider any arbitrary $\frac{n}{2} \times \frac{n}{2}$ matrix P_{22} (where n being selected based on the order of adjacency matrix). With P_{22} as a starting point, we can able to identify the other $\frac{n}{2} \times \frac{n}{2}$ matrices as follows,

$$P_{11} + P_{22} = I \Rightarrow P_{12} = I - P_{11} \Rightarrow P_{21} = I + P_{11}$$

After computing P_{11}, $P_{12,}$ P_{21}, P_{22} the self-invertible matrix J was created by

$$P = \left[P_{11} \ \cdots \ P_{12} \ \vdots \ \ddots \ \vdots \ P_{21} \ \cdots \ P_{22}\right] = \left[P_{11} \, P_{12} \ \cdots \ P_{21} \, P_{22} \ \cdots \ \cdots \ \cdots \ \cdots \ \vdots \ \vdots \ \vdots \ \cdots$$

4. The proposed cryptosystem

This section provides more details on the suggested approach, which makes use of the adjacency matrix, the complete graph of the undirected graph's Hamiltonian circuit, and the key matrix for encryption as a self-invertible matrix.

4.1 Methodology for the Suggested Encryption Approach

The phases needed in the encryption process are as follows:

Level 1: To create a Hamiltonian circuit, join the successive letters in the given plaintext message units.

Level 2: Utilizing Table 1, convert the given message bits into the appropriate numerical values. Level 3: Utilizing the created Hamiltonian circuit, create a complete graph.

Level 4: Prepend an additional character, such as 'A', before the first letter of the given plaintext to indicate the start of the original message.

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Level 5: To find the weights of each edge, compute the numerical distance between neighboring vertices.

Level 6: Find the connected full graph's adjacency matrix.

Level 7: Create the self-invertible matrix using the given data and steps described in Section 3.

Level 8: Encrypts the initial plaintext message by multiplying the newly created key matrix by the existing adjacency matrix. Through an unprotected channel, the end user can obtain these encrypted matrix of the plaintext message, the adjacency matrx's order, and the matrix which aids to built the key matrix.

4.2 Methodology for the Suggested Decryption Approach:

The decryption process involves the following levels:

Level 1: The end user determines the sequence of matrices, the matrix that utilized to create the self-invertible key matrix and the encoded matrix of the original message, after obtaining the encrypted data from the sender.

Level 2: The end user also creates the self-invertible matrix by following the guidelines given in Section 3.

Level 3: The above created self-invertible matrix is multiplied with the encrypted matrix.

Level 4: After that, the receiver adds modulo 29 to the resultant matrix in order to calculate the matrix of adjacency.

Level 5: Entails the recipient reconstructing the entirety of the graph, encompassing both its nodes and weights, through retracing the graph.

Level 6: By summing the weights, the original message can be deduced. It's noted that vertex v_1 corresponds to 'N', with a numerical equivalent of 14, and vertex v_2 equals the value of v_1 plus the weight e_1, and so forth.

5. Illustration of implementation:

Assume that the Transmitter wishes to transmit the plaint text message "NOTICE" to the receiver. Using the method described in the aforementioned section and the key matrix described in Section 3.

5.1 User A (The sender):

The encrypting process involves the subsequent techniques:

At first, the sender (User A) converts the given plaintext message "NOTICE" into the vertices of a graph and forms a Hamiltonian circuit. Sequential letters of the given message are connected to create the vertices

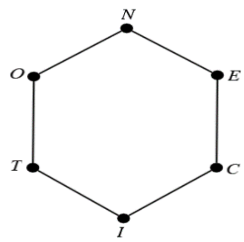

Figure 1. The Hamiltonian circuit of original plaintext message

Using the furnished encoded table we obtain,

N → 14, O → 15, T → 20, I → 9, C →3, E → 5

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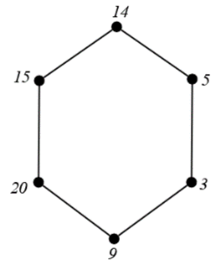

Figure 2. Encoded Hamiltonian circuit

The initial letter of the original plain text message unit to indicate the starting point of the original plaintext message. Now to make this Hamiltonian circuit into a complete graph K_6 by connecting unpaired vertices of the Hamiltonian circuit.

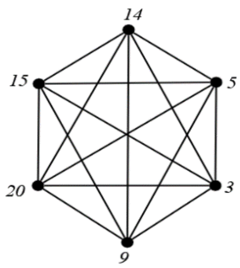

Figure 4. The Complete graph K_6

The numerical separation between the successive vertices is calculated, and modulo 29 is then applied to compute the weights of the graph's edges. (e_1 = Code O – Code N, e_2 = Code T − Code O …

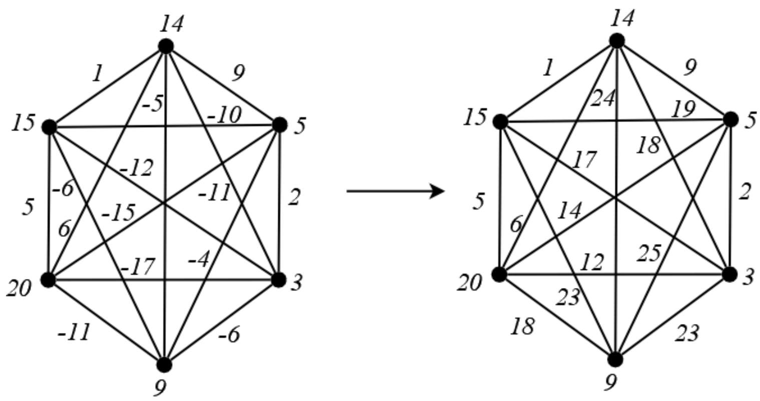

Figure 5. Complete graph K_6 with weights

After computation, A was assigned to the adjacency matrix for the entire graph.

$$A = \begin{bmatrix} 0 & 1 & 6 & 24 & 18 & 9 \\ 1 & 0 & 5 & 23 & 17 & 19 \\ 6 & 5 & 0 & 18 & 12 & 14 \\ 24 & 23 & 18 & 0 & 23 & 25 \\ 18 & 17 & 12 & 23 & 0 & 2 \\ 9 & 19 & 14 & 25 & 2 & 0 \end{bmatrix}$$

Now, we generate the self-invertible matrix 'G' using $\frac{n}{2} \times \frac{n}{2}$ matrix G_{22}

Let $G_{22} = \begin{bmatrix} 1 & 4 & 5 \\ 6 & 3 & 9 \\ 7 & 8 & 9 \end{bmatrix}$ then $G_{11} = \begin{bmatrix} 28 & 25 & 24 \\ 23 & 26 & 20 \\ 22 & 21 & 20 \end{bmatrix}$

$G_{12} = I - G_{11} = \begin{bmatrix} 2 & 4 & 5 \\ 6 & 4 & 9 \\ 7 & 8 & 10 \end{bmatrix}$ and $G_{21} = I + G_{11} = \begin{bmatrix} 0 & 25 & 24 \\ 23 & 27 & 20 \\ 22 & 21 & 21 \end{bmatrix}$

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} 28 & 25 & 24 & 2 & 4 & 5 \\ 23 & 26 & 20 & 6 & 4 & 9 \\ 22 & 21 & 20 & 7 & 8 & 10 \\ 0 & 25 & 24 & 1 & 4 & 5 \\ 23 & 27 & 20 & 6 & 3 & 9 \\ 22 & 21 & 21 & 7 & 8 & 9 \end{bmatrix}$$

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$$C = A.G = \begin{bmatrix} 0 & 1 & 6 & 24 & 18 & 9 \\ 1 & 0 & 5 & 23 & 17 & 19 \\ 6 & 5 & 0 & 18 & 12 & 14 \\ 24 & 23 & 18 & 0 & 23 & 25 \\ 18 & 17 & 12 & 23 & 0 & 2 \\ 9 & 19 & 14 & 25 & 2 & 0 \end{bmatrix} \begin{bmatrix} 28 & 25 & 24 & 2 & 4 & 5 \\ 23 & 26 & 20 & 6 & 4 & 9 \\ 22 & 21 & 20 & 7 & 8 & 10 \\ 0 & 25 & 24 & 1 & 4 & 5 \\ 23 & 27 & 20 & 6 & 3 & 9 \\ 22 & 21 & 21 & 7 & 8 & 9 \end{bmatrix}$$

$$C = \begin{bmatrix} 767 & 1427 & 1265 & 243 & 274 & 432 \\ 947 & 1563 & 1415 & 295 & 339 & 494 \\ 867 & 1348 & 1210 & 230 & 264 & 399 \\ 2676 & 2722 & 2381 & 625 & 601 & 939 \\ 1203 & 1761 & 1606 & 259 & 344 & 496 \\ 1043 & 1692 & 1516 & 267 & 330 & 499 \end{bmatrix}$$

5.2. User B (The receiver)

The encrypted matrix has the flexibility to be converted into either a row wise matrix or a column wise matrix format and transmitted to another user using various communication channels, specifying the matrix's order, the matrix which plays a crucial role in the computation of the self-invertible matrix.

$$C = \begin{bmatrix} 767 & 1427 & 1265 & 243 & 274 & 432 \\ 947 & 1563 & 1415 & 295 & 339 & 494 \\ 867 & 1348 & 1210 & 230 & 264 & 399 \\ 2676 & 2722 & 2381 & 625 & 601 & 939 \\ 1203 & 1761 & 1606 & 259 & 344 & 496 \\ 1043 & 1692 & 1516 & 267 & 330 & 499 \end{bmatrix}$$

The self-invertible matrix is also being produced by the receiver in accordance with the method described in Section 3.

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} 28 & 25 & 24 & 2 & 4 & 5 \\ 23 & 26 & 20 & 6 & 4 & 9 \\ 22 & 21 & 20 & 7 & 8 & 10 \\ 0 & 25 & 24 & 1 & 4 & 5 \\ 23 & 27 & 20 & 6 & 3 & 9 \\ 22 & 21 & 21 & 7 & 8 & 9 \end{bmatrix}$$

$$C.G = \begin{bmatrix} 767 & 1427 & 1265 & 243 & 274 & 432 \\ 947 & 1563 & 1415 & 295 & 339 & 494 \\ 867 & 1348 & 1210 & 230 & 264 & 399 \\ 2676 & 2722 & 2381 & 625 & 601 & 939 \\ 1203 & 1761 & 1606 & 259 & 344 & 496 \\ 1043 & 1692 & 1516 & 267 & 330 & 499 \end{bmatrix} \begin{bmatrix} 28 & 25 & 24 & 2 & 4 & 5 \\ 23 & 26 & 20 & 6 & 4 & 9 \\ 22 & 21 & 20 & 7 & 8 & 10 \\ 0 & 25 & 24 & 1 & 4 & 5 \\ 23 & 27 & 20 & 6 & 3 & 9 \\ 22 & 21 & 21 & 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 97933 & 105387 & 92632 & 23862 & 24146 & 36897 \\ 112260 & 120930 & 106522 & 326964 & 27509 & 41924 \\ 96750 & 103390 & 91147 & 22899 & 23444 & 35684 \\ 224397 & 239244 & 213023 & 49155 & 52455 & 78673 \\ 128343 & 135766 & 119724 & 30009 & 30740 & 46779 \\ 120040 & 127967 & 112679 & 28590 & 29118 & 44399 \end{bmatrix}$$

By applying addition modulo 29, the result is:

97933(mod 29) = 0, 105387(mod 29) = 1, 92632 (mod 29) = 6, 23863 (mod 29) =24 …

$$C.G = \begin{bmatrix} 0 & 1 & 6 & 24 & 18 & 9 \\ 1 & 0 & 5 & 23 & 17 & 19 \\ 6 & 5 & 0 & 18 & 12 & 14 \\ 24 & 23 & 18 & 0 & 23 & 25 \\ 18 & 17 & 12 & 23 & 0 & 2 \\ 9 & 19 & 14 & 25 & 2 & 0 \end{bmatrix} = A$$

A complete graph corresponding to the aforementioned adjacency matrix was created.

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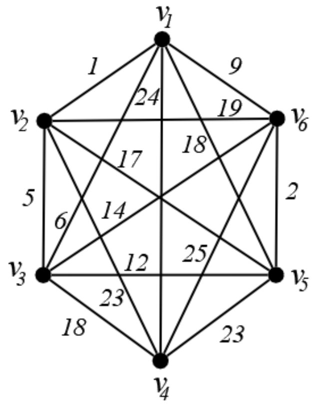

Figure 6. Complete graph K_6 of decoded adjacency

The nodes of the aforementioned graph mentioned was created by combining the value of each vertex with its associated edge. Take $v_1 = 14$, so $v_2 = 14 + 1 = 15,\ v_3 = 15 + 5 = 20,$

$$v_4 = 20 + 18 = 38 = 9,\ v_5 = 9 + 23 = 32 = 3,\ v_6 = 3 + 2 = 5.$$

Therefore vertices are 14, 15, 20, 9, 3, 5.

Hence the original message is N → 14, O → 15, T → 20, I → 9, C →3, E → 5. i.e., Notice.

6. Conclusion

In conclusion, ensuring the security of information has become increasingly critical in the modern era. While existing techniques such as the Hill cipher and graphical methods provide a foundation for encryption, the proposed cryptosystem offers a more efficient and secure alternative. By incorporating the concepts of Hamiltonian circuits, adjacency matrices of complete graphs, and even-ordered self-invertible matrices, the method enhances resistance against unauthorized access while simplifying both encryption and decryption processes. The use of a reduced $\frac{n}{2} \times \frac{n}{2}$

matrix to generate the key matrix eliminates the need for inverse computation, thereby improving computational efficiency. Built upon fundamental graph theory and cryptographic principles, this approach ensures both practicality and robustness. Future work may focus on extending the method to accommodate self-invertible matrices of any order and exploring more advanced graph-theoretic structures to further strengthen the system.

References

[1] Acharya, B., Rath, G.S., Patra, S.K., Panigrahy, S.K., A Novel methods of generating self-invertible matrix for Hill Cipher Algorithm, International Journal of Security(2007), pp.14-21.

[2] Amudha P, Jayapriya J, Gowri J, An algorithmic approach for encryption using graph Labeling, Journal of physcis,1770(1): 012072, (2021), pp. 375-384.

[3] Arumugam S, Ramachandran S, Invitation to Graph theory, Scitech Publications, (2015).

[4] Diffie, W., Hellman, M., New directions in Cryptography, IEEE Trans. Inf. Theory 22 (6), (1976), pp.644-654.

[5] Mohan. P, Rajendran. K, Rajesh. A, An Encryption Technique using a complete graph with a Self-invertible matrix, Journal of Algebraic statistics, Volume 13. No 3, (2022), https://publishoa.com/index.php/journal/article/view/816, pp.1821-1826.

[6] Mohan P, Rajendran K, Rajesh A. A Hamiltonian Path-Based Enciphering Technique with the use of a Self-Invertible Key Matrix, Indian Journal of Science and Technology, 15(44) (2022), pp.2351-2355.

[7] Mohan P, Rajendran K, Rajesh A. An encryption techniques using the adjacency matrices of certain graphs with a self-invertible key matrix, E3 Web of conferences, Vol 376, 01108(2023).

[8] Mohan P, Rajendran K, Rajesh A. Enhancing computational performance of minimal spanning tree of certain graphs based

ISBN 978-816855385-9

enciphering technique using self-invertible key matrix, Journal of Aeronautical Meterials, Vol.43(01) (2023), pp.359-371.

[9] Nandhini R, Maheswari V and Balaji V, A Graph Theory Approach on Cryptography, Journal of Computational Mathematics,2(1),(2018), pp.97-104.

[10] Neal Koblitz, A course in Number Theory and Cryptography, second edition, Springer.

[11] Saniah Sulaiman Zurina Mohd Hanpi, Extensive analysis on Images Encryption using Hybrid Elliptic Curve Cryptosystem and Hill cipher, Journal of Computer Science, 17(3), (2021), pp.221-320.

[12] Uma Dixit, Cryptography a Graph theory approach, International journal of Advance Research in Science and Engineering, 6(01), (2017), pp.218-221.

[13] Weal Mahmoud AI Etaiwi, Encryption algorithm using Graph theory, Journal of Scientific Research and Reports, 3(19), (2014), pp. 2519-2527.

[14] Yamuna M, Meenal Gogia, Ashish Sikka, Md. Jazib Hayat Khan, Encryption Using Graph Theory and Linear Algebra, International Journal of Computer Application, ISSN:2250-1797,Issue 2 Vol 5(2012),pp.102-107.

[15] Ziad E. Dawahdeh, Shahrul N. Yaakob, Rozmie Razif bin Othman, A new image encryption technique combining Elliptic Curve Cryptosystem with Hill Cipher, Journal of King Saud University – Computer and Information Sciences,30(3),(2018), pp.349-355.

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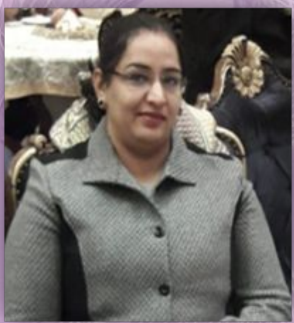

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