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Abstract: *This study introduces a new approach in graph coloring known as harmonious dynamic coloring which is useful in optimization of social network analysis, telecommunications, airspace, resource allocation and related engineering problems. The minimum quantifying colors to produce harmonious dynamic coloring is the Harmonious dynamic chromatic number. The goal of the study is to find the Harmonious dynamic chromatic number $\chi_{hd}(G)$ of shell graph, uniform shell bow graph, subdivided uniform shell graph, subdivided uniform shell bow graph. Furthermore, we illustrate Central graph C(G) of Shell graph and the uniform shell bow graph.*

Keywords: *Harmonious dynamic coloring, optimization, shell graph, uniform shell bow graph, subdivided shell graph, subdivided uniform shell bow graph, Central graph.*

1. Introduction

Harmonious dynamic coloring, a novel graph coloring hybrid approach by connecting the ideas of harmonious coloring [1] and dynamic coloring [8]. Deb and Limaye [2], who first defined the Shell graph. In the year 2014, the subdivided shell graph and the subdivided uniform shell bow graph were introduced [4]. The Central graph [3, 8, 9] of G is attained by partitioning exactly once in every edge of G also connecting all vertices of G that are not adjacent. It is denoted by $C(G)$. A shell graph [5, 9] is a cycle C_n, with $(n-3)$ chords sharing the common end point called the apex; otherwise, shell graph is the joint of a complete graph K_1 and path P_n, the path with n vertices. In this paper, it is symbolized as $S(n, n-3)$.

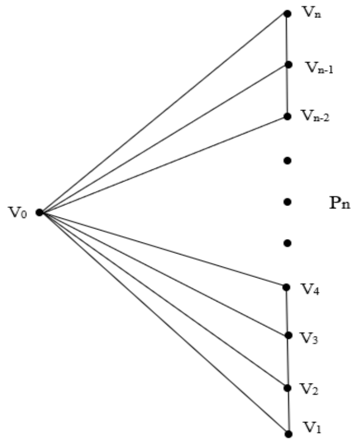


Figure 1. Shell graph

A uniform shell bow graph [4, 5, 6] is defined as double shell graph where every shell maintains the same order. In this paper, we denote the uniform shell bow graph as $\mathcal{SB}$ *(n, n-3)*. Hence, a double shell consists of two separate shells with a common apex. It is represented as v_0.

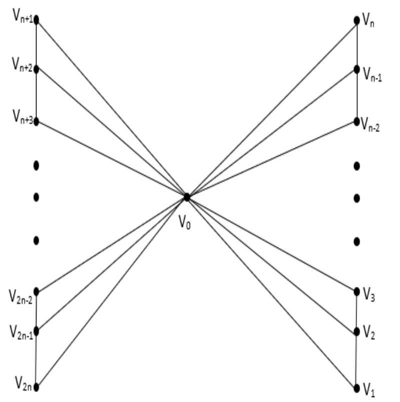


Figure 2. Uniform shell bow graph

The shell graph $G = P_n \cup K_1$ which only sections edges in the path P_n is defined as subdivided shell graph [4, 5]. A subdivided shell graph has the new path P_{2n-1}. In this paper, the subdivided shell graph is represented as $\mathcal{SSh}$ *(n, n-3)*.

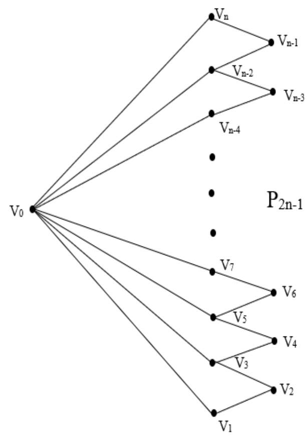


Figure 3. subdivided shell graph

The two subdivided shell graphs of the same order connected with one vertex are known as subdivided uniform shell bow graph [6]. In this paper, the subdivided uniform shell bow graph is denoted as $\mathcal{SSB}$ *(n, n-3)*.

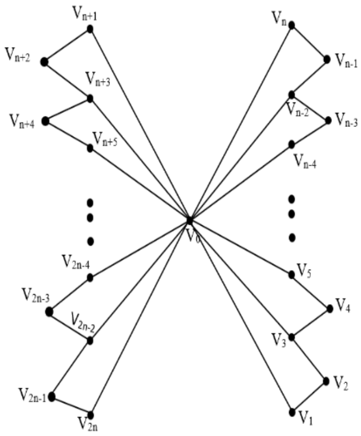


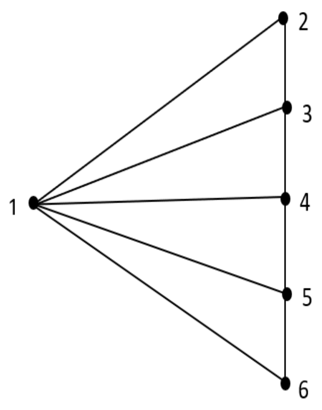
Figure 4. subdivided uniform shell bow graph

2. Main results

This section describes the shell graph, uniform shell bow graph, subdivided shell graph, subdivided uniform shell bow graph in Harmonious dynamic coloring and also shows Harmonious dynamic coloring of the Central graph of shell graph, uniform shell bow graph.

Theorem 2.1. If $S(n, n\text{-}3)$ is a shell graph, then Harmonious dynamic chromatic number on the shell graph is χ_{hd} $(S\,(n, n\text{-}3)) = n$.

Proof. Let $S\,(n, n\text{-}3)$ be the shell graph with n vertices and $(2n\text{-}3)$ edges. Apex of shell graph $S(n, n\text{-}3)$ is denoted as v_0. Assign the vertices of the shell graph are $v_1, v_2, v_3, v_4, \dots v_{n-1}$. For $n \geq 4$, the maximum degree of shell graph $\Delta(S\,(n, n\text{-}3))$ is $n\text{-}1$, minimum degree of shell graph $\delta(S\,(n, n\text{-}3))$ is 2. Here, single apex vertex with degree $(n\text{-}1)$ and *two* vertices possess a degree of 2 whereas $(n\text{-}3)$ vertices have a degree 3. By definition Harmonious dynamic coloring, two adjacent vertices have the distinct color. Also, the color pair should not repeat additionally, each vertex has at least two degrees and is adjacent to vertices of at least two distinct colors. Furthermore, adjacent vertices receive colors, however earlier preferred colors cannot be reused, because outcome coloring will not give a harmonious dynamic coloring. The above-colored vertices are harmonious with the lowest number of colors.



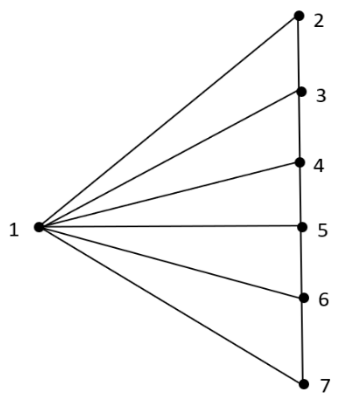


Figure 5. a. **Figure 5. b.**

Figure 5. a. Harmonious dynamic coloring of shell graph χ_{hd} *(S (6, 3))* =*6* where n=6 even

Figure 5. b. χ_{hd} ***(S (7, 4))* =7 where n=7 odd.**

Hence χ_{hd} *(S (n, n-3))* = *n.*
i.e., Harmonious dynamic chromatic number of shell graph is *n*.

Theorem 2.2. If *S (n, n-3)* is a shell graph, then Harmonious dynamic coloring on Central graph of shell graph χ_{hd} *(C (S (n, n-3)))* is *n+3.*

Proof. Let *S (n, n-3)* be shell graph with *n* vertices and *(2n-3)* edges. Apex of shell graph *S (n, n-3)* denoted as v_1. Assign the vertices of shell graph are $v_2, v_3, v_4, \dots v_n$. By central graph definition, every edge of graph is partitioned by a vertex only once, and also joins the non-adjacent vertices. Clearly, it is proper coloring. Since every vertex and its neighbors obtain distinct colors. Additionally, colors are also not the same as the adjacent; every pair of vertices occurs only once. In Harmonious dynamic coloring, calculating the colors of vertices is the lowest number of colors.

Case *(i)*. The color covers *n+3*. If assigning *n+2* colors, then colors will be the same, which are contradiction to the Harmonious dynamic coloring definition.

Case *(ii)*. Assuming neighbors of same color, the color pair repetitions conflicts with Harmonious dynamic coloring definition. Therefore, the lowest quantify of colors necessary for Harmonious dynamic coloring for central graph of shell graph with *n* vertices and (*2n-3)* edges is *n*+3.

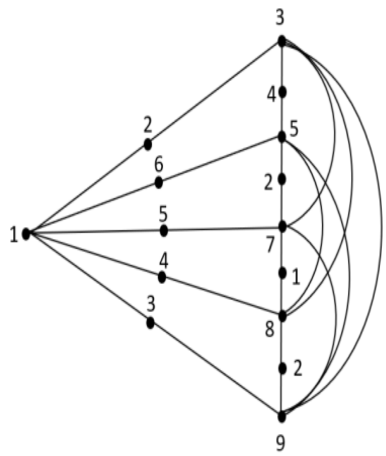


Figure 6. a.

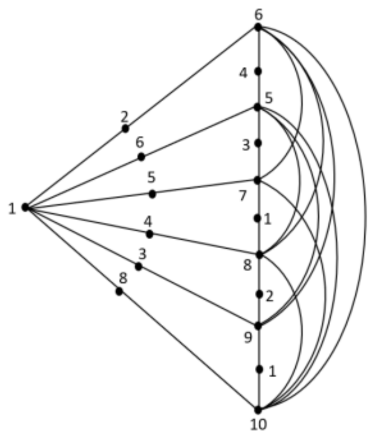


Figure 6. b.

Figure 6. a. Harmonious dynamic coloring of Central graph χ_{hd} ***(C (S (6, 3)))* is *n+3* if *n* is even.**

Figure 6. b. Harmonious dynamic coloring of Central graph χ_{hd} ***(C (S (7, 4)))* is *n+3* if *n* is odd.**

Theorem 2.3. Let *SSh (n, n-3))* be a subdivided shell graph, then Harmonious dynamic chromatic number on the subdivided shell graph is *n*, also χ_{hd} *(SSh (n, n-3))* ≥ *Δ*(*SSh (n, n-3))*.

Proof. Let *SSh (n, n-3)* be subdivided shell graph which has (2*n-2)* vertices and (3*n-5)* edges. Apex of subdivided shell graph *SSh (n, n-3))* is denoted as v_1. Assign the vertices of the subdivided shell graph are $v_2, v_3, v_4, \dots v_{2n-2}$. For *n≥4*, maximum degree of subdivided shell graph *Δ(SSh (n, n-3))* is *n-1*, minimum degree of subdivided shell graph *δ(SSh (n, n-3))* is 2. Here, only a single vertex with a degree (*n-1)* and *n* vertices possess a degree of *two*, also *(n-3)* vertices have degree *three*. By definition Harmonious dynamic coloring, two adjacent vertices have the distinct color. Also, the color pair should not repeat additionally, each vertex has at least two degrees and is adjacent to vertices of at least two distinct colors. Furthermore, the colors cannot be previously used in adjacent vertices because the resultant colors will be improper or not harmonious

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dynamic coloring. Thus, the colored vertices are harmonious with the least number of colors.

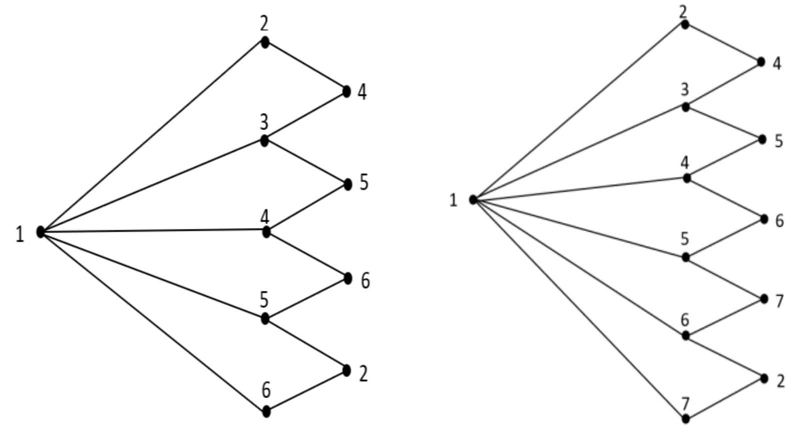

Figure 7. a. **Figure 7. b.**

Figure 7. a. Harmonious dynamic coloring of subdivided shell graph χ_{hd} *(SSh (6, 3)) =6,*
Figure 7. b. Harmonious dynamic coloring of subdivided shell graph χ_{hd} *(SSh (7, 4)) =7.*

Hence χ_{hd} *(SSh (n, n-3))* = *n.*

Theorem 2.4. Let $\mathcal{SB}$ *(n, n-3)* be the uniform shell bow graph of the Harmonious dynamic chromatic number of is 2*n-1, n* ≥ *4.*

Proof: Let $\mathcal{SB}$ *(n, n-3)* be uniform shell bow graph with *(2n-1)* vertices and *(4n-6)* edges. Apex of uniform shell bow graph $\mathcal{SB}$ *(n, n-3)* is denoted as v_1. Assign the vertices be $v_2, v_3, v_4, \dots v_{2n-1}$. Here, *(2n-4)* vertices take degree *3,* and only one apex vertex has degree *(2n-2)* also *4* vertices have degree 2. By definition Harmonious dynamic coloring, two adjacent vertices have the distinct color. Also, the color pair should not repeat additionally, each vertex has at least two degrees and is adjacent to vertices of at least two distinct colors. For *n≥4,* maximum degree of uniform shell bow graph $\Delta(\mathcal{SB}$ *(n, n-3))* is 2*n*. minimum degree of uniform shell bow graph $\delta(\mathcal{SB}$ *(n, n-3))* is 2. We allocate colors to vertices in uniform shell bow graph for identifying Harmonious dynamic coloring. If we assign color to vertex as same color pairs then it's a contradiction to our definition.

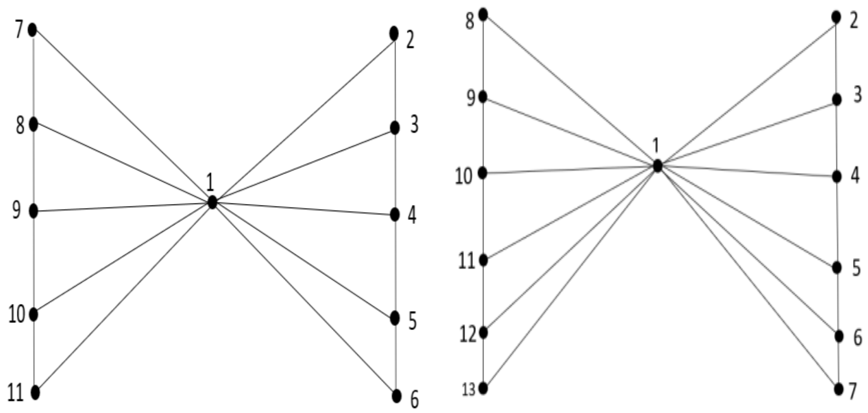

Figure 8. a. **Figure 8. b.**

Figure 8. a. Harmonious dynamic coloring of uniform shell bow graph χ_{hd} *(SB (6, 3)) =11,*
Figure 8. b. Harmonious dynamic coloring of uniform shell bow graph χ_{hd} *(SB (7, 4)) =13.*

Here the vertices will have the degrees 2, 3 and *n+4*.
Maximum degree of uniform shell bow graph $\Delta(\mathcal{SB}$ *(6, 3))* is 10
Minimum degree of uniform shell bow graph $\Delta(\mathcal{SB}$ *(6, 3))* is 2

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Maximum degree of uniform shell bow graph $\delta(\mathcal{SB}\ (7, 4))$ is 12
Minimum degree of uniform shell bow graph $\delta(\mathcal{SB}\ (7, 4))$ is 2
Hence, the Harmonious dynamic chromatic number of uniform shell bow graph is *2n-1* for *n≥4*.

Theorem 2.5. Let $\mathcal{SSB}\ (n, n\text{-}3)$ be the subdivided uniform shell bow graph Harmonious dynamic chromatic number is $\chi_{hd}(\mathcal{SSB}\ (n, n\text{-}3)) = 2n\text{-}1, n \geq 4$.

Proof: Let $\mathcal{SSB}\ (n, n\text{-}3)$ be subdivided uniform shell bow graph having (*2n-1)* vertices and (*4n-6)* edges. Apex of subdivided uniform shell bow graph $\mathcal{SSB}\ (n, n\text{-}3)$ is denoted as v_1. Assign the vertices be $v_2, v_3, v_4, \dots v_{2n-1}$. By definition Harmonious dynamic coloring, two adjacent vertices have the distinct color. Also, the color pair should not repeat additionally, each vertex has at least two degrees and is adjacent to vertices of at least two distinct colors.
Maximum degree of the subdivided uniform shell bow graph $\Delta(\mathcal{SSB}\ (n, n\text{-}3))$ is *2n-2* for *n≥4*. Minimum degree of the subdivided uniform shell bow graph $\delta(\mathcal{SSB}\ (n, n\text{-}3))$ is 2 for *n≥4*. The color covers 2*n-1*. If assume that fewer than *2n-1* colors are assigned, then colors will be the same, which is a contradiction to the Harmonious dynamic coloring definition.

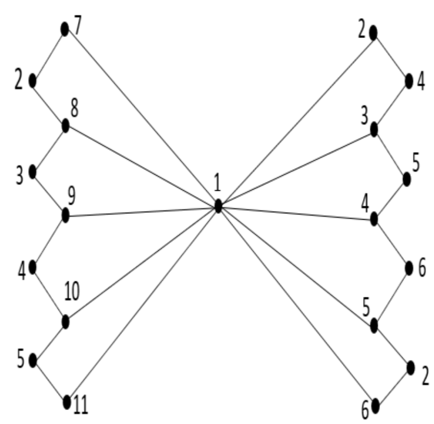

Figure 9. a.

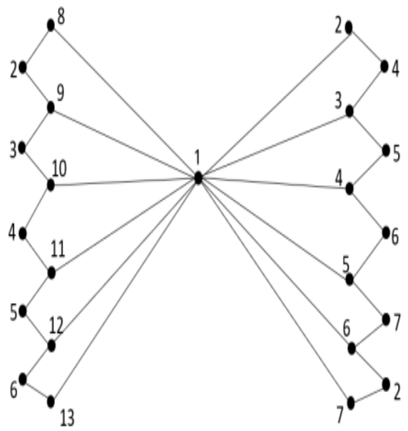

Figure 9. b.

Figure 9. a. Harmonious dynamic coloring of subdivided uniform shell bow graph χ_{hd} *(SSB (6, 3)) =11,*
Figure 9. b. Harmonious dynamic coloring of subdivided uniform shell bow graph χ_{hd} *(SSB (7, 4)) =13.*

Hence χ_{hd} $(\mathcal{SSB}\ (n, n\text{-}3))=2n\text{-}1, n \geq 4$.

Theorem 2.6. If $\mathcal{SB}\ (n, n\text{-}3)$ be a uniform shell bow graph, then Harmonious dynamic chromatic number of central graph of uniform shell bow graph is $\chi_{hd}(C(\mathcal{SB}\ (n, n\text{-}3)) = 2n+1$.

Proof: Let us consider the uniform shell bow graph $\mathcal{SB}\ (n, n\text{-}3)$ with (*2n-1)* vertices and (*4n-6)* edges. Apex of uniform Shell bow graph $\mathcal{SB}\ (n, n\text{-}3)$ is denoted as v_1. Assign the vertices of the uniform shell bow graph are $v_2, v_3, v_4, \dots v_{2n-1}$. By definition of Central graph, partitioning exactly once in every edge of *G* also connecting all vertices of *G* that are not adjacent. It is denoted by *C(G)*.
Case *(i)*. The color covers 2*n+1*. If assigning *2n* colors, then colors will be the same, which is a contradiction to the Harmonious dynamic coloring definition.
Case *(ii)*. Assuming neighbors of similar color, then color pair repetitions, which conflicts with Harmonious dynamic coloring definition.

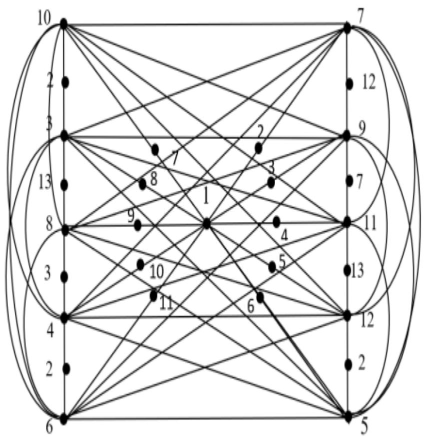

Figure 10. Harmonious dynamic coloring of central graph in uniform shell bow graph χ_{hd}*(C (SB (6, 3)) =13.*

Hence χ_{hd} *(C (SB (n, n-3)) =2n+1.*

3. Analysis

Table 1. Harmonious Dynamic Chromatic Number of Families of Shell Graph

S.NO.	Graphs	Harmonious Dynamic chromatic number
1	Shell Graph	*n*
2	Uniform Shell Bow Graph	*n+3*
3	Subdivided Shell Graph	*n*
4	Subdivided Uniform Shell Bow Graph	*2n-1*
5	Central Graph of Shell Graph	*n+3*
6	Central Graph of Uniform Shell Bow Graph	*2n+1*

4. Conclusion

We have described the Harmonious dynamic chromatic number of shell graph, uniform shell bow Graph, Subdivided Shell Graph, subdivided uniform shell bow graph also results the Harmonious dynamic coloring of Central graph of shell graph, uniform shell bow graph. Harmonious Dynamic Coloring has been applied to improve conflict free resource allocations, optimizing energy consumptions in wireless sensor networks and also reduce communication frequencies. In future, research directions could involve determining the Harmonious Dynamic Chromatic number of other families of Graphs.

Acknowledgments

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