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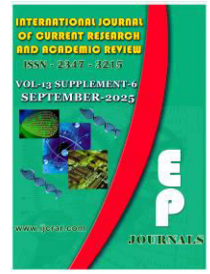

Full Length Article **IJCRAR/FL/73**

Optimization of Inventory Model Using Hexagonal Fuzzy Number

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Abstract

In this paper total inventory cost was minimized under Hexagonal fuzzy numbers .The parameters such as carrying cost and ordering cost where given in Hexagonal Fuzzy Numbers. The fuzzy numbers are defuzzified into crisp number by applying Magnitude ranking technique and Pascal's ranking technique .Numerical example were given to illustrate the comparison of total inventory cost obtained using fuzzy environment and classical inventory model under different defuzzification techniques . Sensitivity analysis for various demands were also discussed for various defuzzification techniques.

Keywords: Fuzzy Inventory Models, Hexagonal fuzzy numbers, Magnitude Ranking Technique, Pascal's ranking technique

Introduction

Operations Research is an important tool for the decision maker to create an alternative and to analyze an alternative in a effective way. Operations research methods are very often used in management science, economic, mathematics, industrial engineering and statistics. The main objective of operations research deals to get the best course of action of a decision-making problem under certain constraints. Inventory models are the mathematical framework which is used to control and manage the inventory to overcome the shortages. The primary goal of the inventory problem is to determine the optimum order quantity with the specified ordering cost, holding cost etc.

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Zadeh [9] introduced the idea of fuzzy set theory to manage many decision making problems and plays a vital role to overcome vagueness .In many decision making analysis fuzzy sets helps to yield the better results. Here fuzzy numbers determine the closest approach to the traditional inventory models. R.M.Rajalakshmi and Michael Rosario[6] studied the fuzzy inventory model with allowable shortages using different fuzzy numbers. S. Hemalatha and K.

Annadurai [4] determined the optimum inventory value using pentagonal fuzzy numbers. Many researches[2,3,6,7,8] had spread their ideas in optimising the inventory cost.

Here the fuzzy inventory model with ordering cost and holding cost were being considered in Hexagonal fuzzy number which are further defuzzified into crisp value by using

Magnitude and Pascal's ranking technique .The results obtained were compared with traditional inventory model and further with the other ranking techniques ,it was observed that the result yield by using Magnitude ranking method yield better results. Further sensitivity analysis for various demands were also done.

Fuzzy number

Let $\tilde{A}$ be a fuzzy set defined on R the set of real numbers is a Fuzzy number if
$\mu\tilde{A}(x)$: R→ [0,1] is called the membership function if it satisfies the following characteristics.
(i) $\tilde{A}$ is normal. It means there exists an $x \epsilon R$ such that $\mu\tilde{A}(x) = 1$ (ii) $\tilde{A}$ is convex. It means for every $x1, x2 \epsilon R$
$\mu\tilde{A}(px1 + (1 - p)x2) \geq \min\{\mu\tilde{A}(x1), \mu\tilde{A}(x2)\}$ where $p \epsilon \{0,1\}$ (iii) $\mu\tilde{A}(x)$ is an upper semi – continuous membership function.

Fuzzy set

Let X be a universal set. A Fuzzy Set $\tilde{A}$ is defined by a membership function $\mu\tilde{A}(x): X \rightarrow$ [0,1] where $\mu\tilde{A}$ is the degree of membership of $x \in X$ in the set $\tilde{A}$ and is denoted by
$\tilde{A} = \{(x, \mu\tilde{A}(x)); x \in X\}$ Hexagonal fuzzy number:
A fuzzy number $\tilde{H}\ HFN = (a1, a2, a3, a4, a5, a6)$ is a hexagonal fuzzy number and its membership function is given by.

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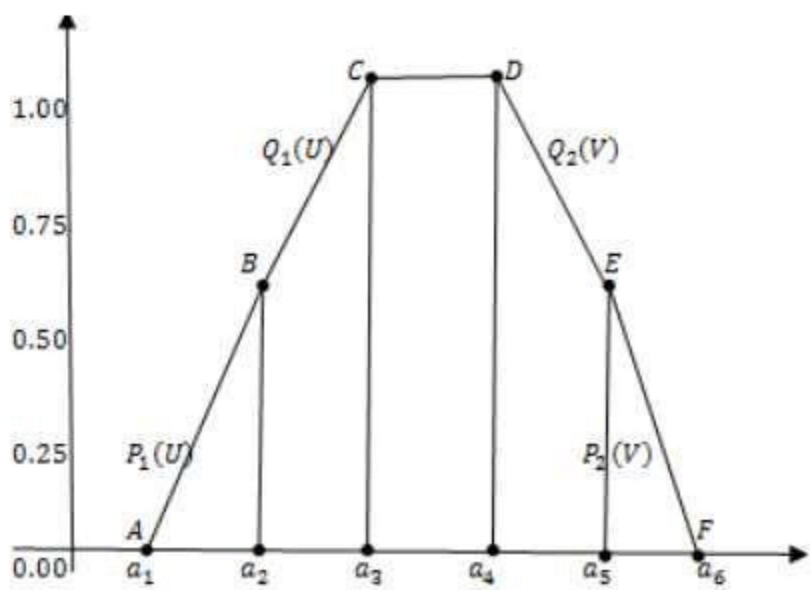
$$\mu_{\breve{H}^{HFN}}(x) = \begin{cases} 0 \; for \; x < a_1 \\ \frac{1}{2}(\frac{x-a_1}{a_2-a_1}) \; for \; a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) for \; a_2 \le x \le a_3 \\ 1, a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) \; for \; a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) \; for \; a_5 \le x \le a_6 \\ 0 \; for \; x > a_6 \end{cases}$$

where are real numbers.



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Arithmetic operations of Hexagonal fuzzy number

Let $\tilde{P} = (p_{1'}, p_{2'}, p_{3'}, p_{4'}, p_{5'}, p_{6'})$ and $\acute{Q} = (q_{1'}, q_{2'}, q_{3'}, q_{4'}, q_{5'}, q_{6'})$ be two hexagonal fuzzy numbers, then

Addition

$\check{P} + \acute{Q} = (p_{1'} + q_{1'}, p_{2'} + q_{2'}, p_{3'} + q_{3'}, p_{4'} + q_{4'}, p_{5'} + q_{5'}, p_{6'} + q_{6'})$

Subtraction

$\check{P} - \acute{Q} = (p_{1'} - q_{1'}, p_{2'} - q_{2'}, p_{3'} - q_{3'}, p_{4'} - q_{4'}, p_{5'} - q_{5'}, p_{6'} - q_{6'})$

Multiplication

$\check{P}\acute{Q} = (p_{1'} \times q_{1'}, p_{2'} \times q_{2'}, p_{3'} \times q_{3'}, p_{4'} \times q_{4'}, p_{5'} \times q_{5'}, p_{6'} \times q_{6'})$

Notations and Assumptions

Here the proposed model is executed under the following notations

TC : Total Inventory Cost

H : Holding cost per unit time

Q : Optimum Order Quantity per cycle

K : Ordering cost per unit time

D : Demand rate

T : Total length of the plan

$Q*$:Fuzzy Optimum Order Quantity

$\tilde{H}$: Fuzzy Holding cost per unit time

$\tilde{K}$: Fuzzy Ordering cost per unit time

FTC : Fuzzy Total Inventory Cost Assumptions: o Demand rate is constant o Ordering cost, Holding cost , are being taken as hexagonal fuzzy numbers. o Shortage cost is not allowed o Time plan is constant

Proposed Inventory Model

Assuming the following cost:

KD

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Holding Cost $= \frac{}{Q}$

Ordering Cost $= \frac{HQ}{2}$

Total Inventory Cost : $TC = \frac{KD}{Q} + \frac{HQ}{2}$ …………(1)

In order to find the total cost equate $\frac{\partial TC}{\partial Q} = 0$ and $\frac{\partial^2 TC}{\partial Q} > 0$

$$\frac{\partial TC}{\partial Q} = -\frac{KD}{Q^2} + \frac{H}{2}$$

$$-\frac{KD}{Q^2} + \frac{H}{2} = 0$$

Getting the value of $Q = \sqrt{\frac{2DK}{H}}$ ……….(2)

Ranking Methods :

Magnitude Ranking Technique : $R(HFN) = \frac{a_1+2a_2+2a_3+2a_4+2a_5+a_6}{10}$

Pascal's Ranking Technique : $R(HFN) = \frac{a_1+5\,a_2+10a_3+10a_4+5a_5+a_6}{32}$

Proposed Fuzzy Inventory Model

In this model holding cost and ordering cost are taken as Hexagonal fuzzy numbers. $\tilde{K} = (K1, K2, K3, K4, K5, K6)$, $\tilde{H} = (H1, H2, H3, H4, H5, H6)$,The fuzzy inventory model are given as

$$\mathrm{TC} = \left\{\left(\frac{K_1D}{Q} + \frac{H_1Q}{2}\right), \left(\frac{K_2D}{Q} + \frac{H_2Q}{2}\right), \left(\frac{K_3D}{Q} + \frac{H_3Q}{2}\right), \left(\frac{K_4D}{Q} + \frac{H_4Q}{2}\right), \left(\frac{K_5D}{Q} + \frac{H_5Q}{2}\right), \left(\frac{K_6D}{Q} + \frac{H_6Q}{2}\right)\right\}$$

$$\mathrm{FTC} = \frac{1}{10}\left\{\left(\frac{K_1D}{Q} + \frac{H_1Q}{2}\right) + 2\left(\frac{K_2D}{Q} + \frac{H_2Q}{2}\right) + 2\left(\frac{K_3D}{Q} + \frac{H_3Q}{2}\right) + 2\left(\frac{K_4D}{Q} + \frac{H_4Q}{2}\right) + 2\left(\frac{K_5D}{Q} + \frac{H_5Q}{2}\right) + \left(\frac{K_6D}{Q} + \frac{H_6Q}{2}\right)\right\}$$

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$$\mathrm{FTC} = \frac{(\mathrm{K_1 + 2K_2 + 2K_3 + 2K_4 + 2K_5 + K_6})\mathrm{D}}{10\mathrm{Q}} + \frac{(\mathrm{H_1 + 2H_2 + 2H_3 + 2H_4 + 2H_5 + H_6})\mathrm{Q}}{20} \ldots (3)$$

By equating $\frac{\partial(FTC)}{\partial Q} = 0$ the fuzzy optimum cost were obtained

$$\frac{1}{10}\left\{-\frac{\mathrm{D}}{\mathrm{Q}^2}(\mathrm{K_1 + 2K_2 + 2K_3 + 2K_4 + 2K_5 + K_6}) + \frac{1}{2}(\mathrm{H_1 + 2H_2 + 2H_3 + 2H_4 + 2H_5 + H_6})\right\} = 0$$

(i.e) $\mathrm{Q}^* = \sqrt{\frac{2\mathrm{D}(\mathrm{K_1+2K_2+2K_3+2K_4+2K_5+K_6})}{(\mathrm{H_1+2H_2+2H_3+2H_4+2H_5+H_6})}}$ …….(4)

Algorithm:

Step 1: Calculate optimum inventory model using traditional inventory method. Step 2:Defuzzify the parameters given in Hexagonal fuzzy numbers using Magnitude

Ranking technique,

Step 3:Determine fuzzy total cost (FTC) using fuzzy arithmetic operations on fuzzy carrying and holding cost which is taken as heptagonal fuzzy numbers. Obtain the value of fuzzy optimum quantity by finding the first derivatives of FTC and equating to zero, where the second derivatives of FTC is positive.
Step 4:Compare the optimum value with fuzzy model and traditional inventory model.
Step 5: Sensitivity analysis for various demand rate were calculated.

Numerical Example

(i) Under Magnitude Method

Let us assume the data with D =700, K =9, = 0.7 find the optimum order quantity and total cost using inventory model. Here K, $H$are considered as hexagonal fuzzy numbers. $\tilde{K}$ =

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(3,7,8,10,12,13), $\tilde{H}$ = (0.2,0.4,0.6,0.8,1,1.2) then find the optimal order quantity and total cost under fuzzy environment.

Solution :

Step 1: Given with D =700, K = 9, H = 0.7

$$Q = \sqrt{\frac{2DK}{H}} = 134.16$$

$$TC = \frac{KD}{Q} + \frac{HQ}{2} = 93.91$$

Step 2: Fuzzy Optimum Order Quantity

$$\tilde{Q}^* = \sqrt{\frac{2D(K_1 + 2K_2 + 2K_3 + 2K_4 + 2K_5 + K_6)}{(H_1 + 2H_2 + 2H_3 + 2H_4 + 2H_5 + H_6)}} = 134.16$$

Step3 : Fuzzy Model

$$FTC = \frac{1}{10}\left\{\left(\frac{K_1D}{Q} + \frac{H_1Q}{2}\right) + 2\left(\frac{K_2D}{Q} + \frac{H_2Q}{2}\right) + 2\left(\frac{K_3D}{Q} + \frac{H_3Q}{2}\right) + 2\left(\frac{K_4D}{Q} + \frac{H_4Q}{2}\right) + 2\left(\frac{K_5D}{Q} + \frac{H_5Q}{2}\right) + \left(\frac{K_6D}{Q} + \frac{H_6Q}{2}\right)\right\}$$

$$FTC = \frac{(K_1 + 2K_2 + 2K_3 + 2K_4 + 2K_5 + K_6)D}{10Q} + \frac{(H_1 + 2H_2 + 2H_3 + 2H_4 + 2H_5 + H_6)Q}{20}$$

FTC = 93.90

Step 4: Comparative analysis

Traditional Method: Q = 134.16. TC = 93.91

Fuzzy Method : $Q*$ = 134.16 , FTC = 93.90

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Sensitivity Analysis for Demand

S.No	D	Q	TC	Q∗	FTC
1	800	143.42	100.39	143.42	100.32
2	900	152.12	106.48	152.12	106.48
3	1000	160.35	112.24	160.35	112.21
4	1100	168.18	117.72	168.18	117.72
5	1200	175.66	122.96	175.66	122.96

The sensitivity analysis reveals that when the demand rate increases, both the economic lot size and inventory cost increases. But remains same for both the classical method and fuzzy environment.

(ii)Under Pascal's Ranking Method

Let $\tilde{K}$ = ($K1$, $K2$, $K3$, $K4$, $K5$, $K6$), $\tilde{H}$ = ($H1$, $H2$, $H3$, $H4$, $H5$, $H6$), The fuzzy inventory model are given as

$$TC = \left\{\left(\frac{K_1D}{Q}+\frac{H_1Q}{2}\right),\left(\frac{K_2D}{Q}+\frac{H_2Q}{2}\right),\left(\frac{K_3D}{Q}+\frac{H_3Q}{2}\right),\left(\frac{K_4D}{Q}+\frac{H_4Q}{2}\right),\left(\frac{K_5D}{Q}+\frac{H_5Q}{2}\right),\left(\frac{K_6D}{Q}+\frac{H_6Q}{2}\right)\right\}$$

$$FTC = \frac{1}{32}\left\{\left(\frac{K_1D}{Q}+\frac{H_1Q}{2}\right)+5\left(\frac{K_2D}{Q}+\frac{H_2Q}{2}\right)+10\left(\frac{K_3D}{Q}+\frac{H_3Q}{2}\right)+10\left(\frac{K_4D}{Q}+\frac{H_4Q}{2}\right)+5\left(\frac{K_5D}{Q}+\frac{H_5Q}{2}\right)+\left(\frac{K_6D}{Q}+\frac{H_6Q}{2}\right)\right\}$$

$$FTC = \frac{(K_1+5K_2+10K_3+10K_4+5K_5+K_6)D}{32Q}+\frac{(H_1+5H_2+10H_3+10H_4+5H_5+H_6)Q}{64} \quad ....(5)$$

By equating $\frac{\partial(FTC)}{\partial Q}=0$ the fuzzy optimum cost were obtained

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$$\frac{1}{32}\left\{-\frac{D}{Q^2}(K_1+5K_2+10K_3+10K_4+5K_5+K_6) + \frac{1}{2}(H_1+5H_2+10H_3+10H_4+5H_5+H_6)\right\}=0$$

$$\text{(i.e) } Q^* = \sqrt{\frac{2D(K_1+5K_2+10K_3+10K_4+5K_5+K_6)}{(H_1+5H_2+10H_3+10H_4+5H_5+H_6)}} \ldots\ldots\ldots(6)$$

The same algorithm will be followed for the Pascal's ranking technique .In the above numerical example step 1 gives the same results.

Step 2: Fuzzy Optimum Order Quantity

$$\widetilde{Q}^* = \sqrt{\frac{2D(K_1+5K_2+10K_3+10K_4+5K_5+K_6)}{(H_1+5H_2+10H_3+10H_4+5H_5+H_6)}} = 134.16$$

Step3 : Fuzzy Model

$$FTC = \frac{1}{32}\left\{\left(\frac{K_1D}{Q}+\frac{H_1Q}{2}\right)+5\left(\frac{K_2D}{Q}+\frac{H_2Q}{2}\right)+10\left(\frac{K_3D}{Q}+\frac{H_3Q}{2}\right)+10\left(\frac{K_4D}{Q}+\frac{H_4Q}{2}\right)+5\left(\frac{K_5D}{Q}+\frac{H_5Q}{2}\right)+\left(\frac{K_6D}{Q}+\frac{H_6Q}{2}\right)\right\}$$

$$FTC = \frac{(K_1+5K_2+10K_3+10K_4+5K_5+K_6)D}{32Q} + \frac{(H_1+5H_2+10H_3+10H_4+5H_5+H_6)Q}{64}$$

FTC = 94.40

Step 4: Comparative analysis

Traditional Method: Q = 134.16, TC = 93.91

Fuzzy Method : $Q*$ = 134.16, FTC = 94.40

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3	1000	160.35	112.24	161.18	112.81
4	1100	168.13	117.72	169.05	118.33
5	1200	175.66	122.96	176.57	123.59

The sensitivity analysis reveals that when the demand rate increases, both the economic lot size and inventory cost increases. But there is variations in the fuzzy environment.

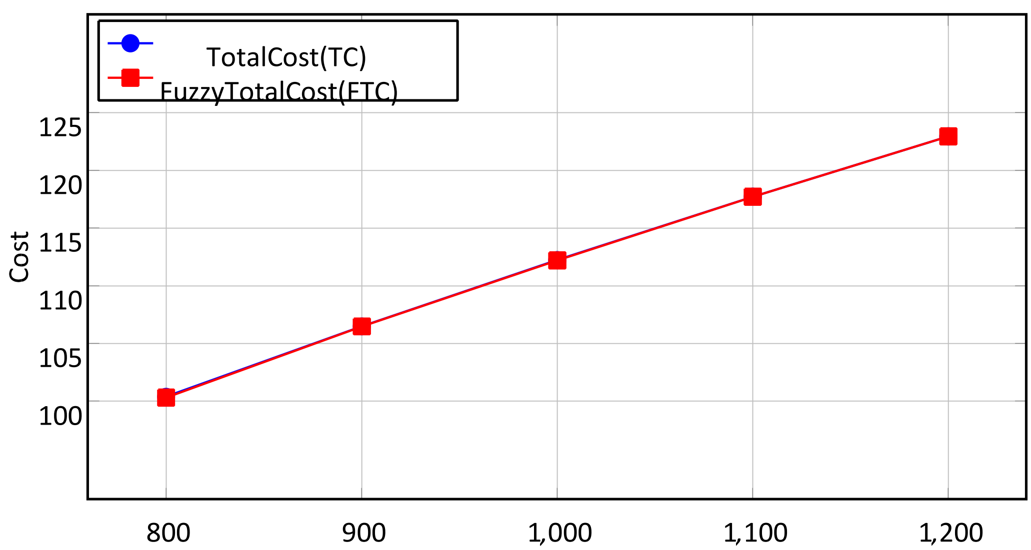

Figure 1:Graphical Representation for sensitivity analysis for demand under Magnitude ranking technique.

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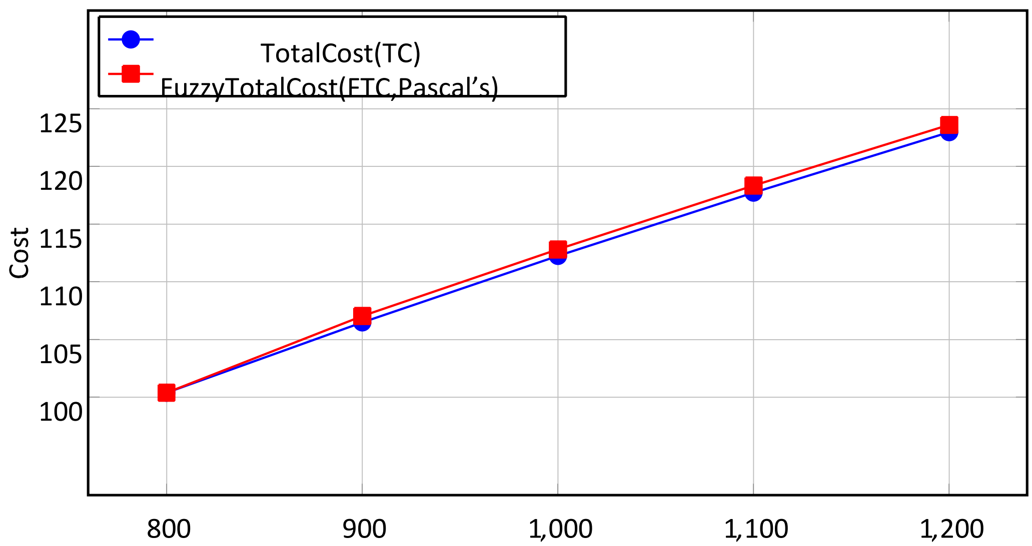

Figure 2:Graphical Representation for sensitivity analysis for demand under Pascal's ranking technique

Conclusion

Here the total inventory cost was minimized under Hexagonal fuzzy numbers under various defuzzification techniques. Comparison were done in conventional method and fuzzy environment the results depict that the fuzzy environment under magnitude method yields better results when compared with the Pascal' s ranking technique.

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