Chapter-11

Neutrosophic Bishop Graphs: Geometric and Computational Reasoning Under Uncertainty

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ABSTRACT

Neutrosophic Bishop Graphs (NBGs), a graph-theoretic framework for making decisions in situations with ambiguity, inconsistency and multi-criteria dependence are presented in this study. Neutrosophic Bishop Graphs (NBGs), model inference as path-based propagation of truth, indeterminacy and falsehood, building on Neutrosophic logic and a geometric constraint inspired by diagonal movement in chess. Formal definitions, dominance-based inference, aggregate semantics, a worked diagnostic case and comprehensive theoretical analysis are all provided. Specifically, we prove NP-completeness of precise inference, establish monotonicity properties and derive fixed-parameter tractability results and ε-approximation guarantees. The framework provides a computationally valid and accessible substitute for fuzzy and probabilistic decision models.

Keywords: Neutrosophic Graphs, Neutrosophic logic, Neutrosophic Bishop Graphs, Uncertainty Modelling.

INTRODUCTION

The expectations of comprehensive, regular and uninterrupted information are not often fulfilled by reasoning in intricate decision systems. Evidence is frequently lacking, conflicting and context-dependent in fields involving specialized systems, standardized medical diagnosis, error analysis and reasoning in law. The structure of uncertainty and disagreement is often obscured by classical decision frameworks, whether they are fuzzy [1], probabilistic or logical which flatten these epistemic subtleties into a single scalar variable. By clearly differentiating between degrees of truth, indeterminacy and falsity, Neutrosophic logic [2] offers a more comprehensive epistemic basis.

Logic by itself, yet, is unable to explain how these epistemic values spread via organized systems of relationships. Conversely, graph-based reasoning [3], which usually depends on numeric edges values, offers a suitable paradigm for relational assessment. Neutrosophic Bishop Graphs which combine geometric constraints, graph theory and Neutrosophic logic are proposed in this chapter. The main point is that valid inference frequently necessitates simultaneous change across several decision dimensions, which are represented as diagonal movement in a decision-space. Neutrosophic Bishop Graphs provide for expressive, comprehensible and computationally rigorous decision reasoning by combining this restriction with Neutrosophic edge valuations.

BACKGROUND

The fundamental ideas of uncertainty modeling presented by fuzzy [1]and intuitionistic frameworks [4] serve as the basis for the development of Neutrosophic graph theory. Zadeh's groundbreaking work [1, 5] developed linguistic variables and fuzzy sets to deal with ambiguity, and Atanassov expanded this to intuitionistic fuzzy sets by adding non-membership. Building on these constraints, Smarandache [2, 6] developed Neutrosophic sets, which provide a progressively sophisticated framework for managing inconsistent and indeterminate data by incorporating truth, indeterminacy, and falsity components. Theoretical development and applications of neutrosophic sets and their extensions [7-10] and different hybrid sets [11-14] were presented in the studies [15-23]. Fuzzy and intuitionistic aggregation techniques were reinforced by later theoretical advances by Klir and Yuan [24] and Xu and Yager [25] which served as the foundation for sophisticated uncertainty models.

By introducing single -valued, bipolar and interval-valued Neutrosophic graphs, Broumi et al. [26-28] formulized the shift from Fuzzy to Neutrosophic graphs and greatly improved the depiction of uncertain relationships. These structures were later extended into pentapartitioned and quadripartitioned Neutrosophic graphs by Das et al. [29] and Muhiuddin et al. [30] and overview of Fermatean Neutrosophic graphs was given by Raut et al. [31], showing their adaptability in simulating higher order uncertainty. The increasing complexity of graph models is demonstrated by recent developments such as complex t-Neutrosophic graphs for biodiversity preservation by Kaviyarasu et al. [32], complement properties of Pythagorean co-Neutrosophic graphs by Govindan et al. [33] and irregular Pythagorean Neutrosophic graphs by Chellamani and Ajay [34].

Additionally, Fujita [35, 36] focused on theoretical generalization while revisiting bipolar and interval-valued Neutrosophic graphs and helping to unify uncertain combinatorics. The structural and operational characteristics of Neutrosophic graphs, Such as operations on interval-valued graphs [28], connectivity indices [37], dominating path colouring and chromatic numbers [38] and the topological structures with Neutrosophic bridges using computational tools like MATLAB [39], have also been thoroughly studied.

Classical graph theory and complexity such as NP-Completeness theory by Garey and Johnson [40], computational complexity by Papadimitriou [41], parameterized complexity frameworks by Cygan et al. [42] and Downey and Fellows [43], combinatorial optimization techniques by Korte and Vygen [44] and approximation algorithms by Hassin [45] serve as the foundation for algorithmic and computational viewpoints. Practical applications, such as Prim's technique for minimum spanning tree evaluation in Neutrosophic graphs, have been made possible by these theoretical bases. Neutrosophic graphs have been used extensively in real-world applications such as computer network analysis [37], biodiversity conservation [32], election analysis utilizing Wiener index in Fermatean Neutrosophic graphs [46] and earthquake response systems.

Also, when intelligent systems that are faced with uncertainty, Reals Probabilistic reasoning frameworks [47] supplement Neutrosophic methods. The transition from Fuzzy to Neutrosophic graph models is highlighted in comprehensive evaluations like Vetrivel et al. [48] and their relevance in engineering and decision-making contexts is shown by Abd et al. [39] and Al-Omeri et al.[46, 49].Although there are still issues with computational efficiency, standardization and large-scale real-world implementation, the literature generally shows a progressive shift from classical and fuzzy graphs theories toward highly generalized Neutrosophic structures capable of modeling complex, uncertain and indeterminate systems.

Neutrosophic Bishop Graph Model

This section establishes Neutrosophic Bishop Graph Model and its fundamental mathematical properties of Neutrosophic Bishop Inference.

Definition 1. A Neutrosophic Bishop Graph is a tuple $\mathcal{G} = (V, E, \mu)$,where:

- $V$is a finite set of vertices representing states, hypotheses, or decisions,
- $E \subseteq V \times V$is a set of edges,
- $\mu: E \to [0,1]^3$assigns to each edge ea neutrosophic weight

$\mu(e) = (T_e, I_e, F_e)$, denoting degrees of truth, indeterminacy, and falsity.

Definition 2. Let $\phi: V \to \mathbb{R}^k$ be an embedding of vertices into a k-dimensional decision space. An edge $(v_i, v_j) \in E$is bishop-admissible if the vector $\phi(v_i) - \phi(v_j)$ has at least two non-zero components. This constraint enforces multi-criteria inference, distinguishing NBGs from ordinary weighted graphs.

Definition 3. Given a path $P = (v_0, v_1, \dots, v_n)$,its neutrosophic valuation is defined as $\mu(P) = (T_P, I_P, F_P)$, where

$$T_P = \min_{e \in P} T_e,$$
$$I_P = \max_{e \in P} I_e,$$
$$F_P = \max_{e \in P} F_e.$$

These operators reflect weakening confidence, accumulating uncertainty, and persistent contradiction.

Definition 4. Given two paths P_1 and P_2, P_1 dominates P_2 if $T_{P_1} \geq T_{P_2}, I_{P_1} \leq I_{P_2}, F_{P_1} \leq F_{P_2}$, with at least one strict inequality. Dominated paths are never rationally preferable.

Example 1.
Consider a simplified medical diagnosis scenario with vertices: {Fever,Fatigue,Inflammation,Diagnosis}. Edges encode neutrosophic causal relations derived from expert knowledge. Two distinct inference paths connect Fever to Diagnosis. Although both paths support the diagnosis, one exhibits higher truth and lower indeterminacy, and therefore dominates the other. Crucially, the inferior path is not discarded because it is "wrong," but because it is epistemically weaker. Contradiction and uncertainty remain explicit, enabling transparent explanation of the decision.

Proposition 1. For any Neutrosophic path P, extending the path cannot increase its truth value.

Proof. By definition, $T_P = \min_{e \in P} T_e$. Adding an additional edge introduces a new candidate for the minimum, which cannot increase the result.

Proposition 2. Indeterminacy along a path is non-decreasing under path extension.

Proof: Since $I_P = \max_{e \in P} I_e$, adding edges can only maintain or increase the maximum.

Proposition 3. Between any two connected vertices in a finite NBG, at least one non-dominated path exists.

Proof: The set of simple paths is finite. Dominance defines a partial order on this set, which must contain at least one maximal element.

Definition 5. Given thresholds (α, β, γ), determine whether there exists a path P such that

$T_P \geq \alpha, I_P \leq \beta, F_P \leq \gamma.$

Proposition 4. The Neutrosophic Bishop Path Decision Problem is NP-complete.

Proof: Verification path is polynomial. NP-hardness follows from a polynomial reduction from the multi-constrained path problem, which is known to be NP-complete.

Definition 6. For $\varepsilon > 0$, path P_1 ε-dominates P_2 if

$$T_{P_1} \geq T_{P_2} - \varepsilon,$$
$$I_{P_1} \leq I_{P_2} + \varepsilon,$$
$$F_{P_1} \leq F_{P_2} + \varepsilon.$$

Proposition 5. For fixed ε, neutrosophic bishop inference admits a fully polynomial-time approximation scheme.

Proof: Each valuation component lies in $[0, 1]$. Discretizing the space into $O(1/\varepsilon^3)$ buckets bound the number of retained path states per vertex, yielding polynomial runtime in input size and $1/\varepsilon$.

Proposition 6. Neutrosophic bishop inference is fixed-parameter tractable with respect to maximum path length k.

Proof: All paths of length at most k can be enumerated in $O(m^k)$ time, which is polynomial for fixed k.

Proposition 7. If the underlying graph has bounded treewidth τ, the problem is fixed-parameter tractable in τ.

Proof: Dynamic programming over a tree decomposition aggregates neutrosophic values locally and combines them efficiently across bags.

Proposition 8. Let $P = (v_0, \ldots, v_k)$ be a neutrosophic bishop path, and let $P' = (v_0, \ldots, v_k, v_{k+1})$ be its extension by one admissible edge. Then $T_{P'} \leq T_P$.

Proof: By Definition 3, the truth value of a path is defined as: $T_P = \min_{e \in P} T_e$.
The extended path P' contains all edges of P, plus one additional edge $e_{k+1} = (v_k, v_{k+1})$.

Hence, $T_{P'} = \min\left(\min_{e \in P} T_e, T_{e_{k+1}}\right)$. Since the minimum of a set augmented by an additional element cannot exceed the minimum of the original set, we conclude that $T_{P'} \leq T_P$.
Therefore, extending a path cannot increase its truth value.
Proposition 9. For any path extension $P \subseteq P'$, $I_{P'} \geq I_P$ and $F_{P'} \geq F_P$.
Proof: From Definition 3, $I_P = \max_{e \in P} I_e, F_P = \max_{e \in P} F_e$.
The extension P' introduces a new edge e_{k+1}.

Thus, $I_{P'} = \max(I_P, I_{e_{k+1}}), F_{P'} = \max(F_P, F_{e_{k+1}})$.
Since the maximum of a set cannot decrease when an element is added, both inequalities follow directly:

$I_{P'} \geq I_P, F_{P'} \geq F_P$. Hence, uncertainty and contradiction are non-decreasing along inference chains.

Proposition 10. Between any two connected vertices in a finite neutrosophic bishop graph, there exists at least one non-dominated path.

Proof: Let $\mathcal{P}$ denote the set of all simple bishop-admissible paths between two fixed vertices s and t. Since the graph is finite, $\mathcal{P}$ is finite. Define a binary relation $\preceq$ on $\mathcal{P}$ by: $P_1 \preceq P_2 \iff T_{P_1} \leq T_{P_2}, I_{P_1} \geq I_{P_2}, F_{P_1} \geq F_{P_2}$. This relation is reflexive, antisymmetric and transitive. Hence, $(\mathcal{P}, \preceq)$ is a partially ordered set. Every finite poset contains at least one maximal element. A maximal element under $\preceq$ is precisely a *non-dominated path.*

Proposition 11. The Neutrosophic Bishop Path Decision Problem is NP-complete.

Proof:

Membership in NP

Given a candidate path P, its Neutrosophic valuation (T_P, I_P, F_P) can be computed by scanning the edges of P, which takes time linear in $|P| \leq |V|$. Checking threshold constraints is constant time. Hence, the problem is in NP.

NP-Hardness

We reduce from the Multi-Constrained Path Problem (MCPP), known to be NP-complete. Given an instance of MCPP with constraints c_1, c_2, c_3, construct an NBG where each original edge maps to a bishop-admissible edge, constraints map to Neutrosophic components: $T_e = 1 - c_1(e)$, $I_e = c_2(e)$, $F_e = c_3(e)$. Thresholds (α, β, γ) are chosen accordingly.

A feasible constrained path exists in MCPP if and only if a path satisfying Neutrosophic thresholds exists in the constructed NBG. Thus, the decision problem is NP-hard. Since the problem is both in NP and NP-hard, it is NP-complete.

Proposition 12. For any fixed $\varepsilon > 0$, ε-approximate neutrosophic bishop inference can be solved in polynomial time.

Proof: Each Neutrosophic component lies in [0, 1]. Partition this interval into $\left\lceil \frac{1}{\varepsilon} \right\rceil$ equal-width buckets.

Thus, the total number of distinct ε-equivalence classes is bounded by $O\left(\frac{1}{\varepsilon^3}\right)$. At each vertex, we retain at most one path per ε-class, discarding ε-dominated paths. Path extension generates polynomial many, and pruning ensures the state space remains polynomial bounded. Therefore, total runtime is polynomial in $|V|, |E|, \frac{1}{\varepsilon}$. Hence, ε-approximate inference is polynomial-time solvable.

Proposition 13. The NBG inference problem is fixed-parameter tractable with respect to maximum path length k.

Proof: Any admissible inference path of length at most k contains at most k edges. The total number of such paths is bounded by $O(|E|^k)$. For fixed k, this quantity is polynomial in input size. Neutrosophic aggregation for each path takes $O(k)$ time. Thus, the total runtime is $O(f(k) \cdot \text{poly}(|V|))$, with $f(k) = |E|^k$. Hence, the problem is fixed-parameter tractable in k.

Proposition 14. If the underlying graph has bounded treewidth τ, then neutrosophic bishop inference is fixed-parameter tractable in τ.

Proof: Let $(T, \mathcal{B})$be a tree decomposition of width τ. Each bag contains at most $\tau + 1$vertices. Dynamic programming proceeds bottom-up, storing for each bag all feasible Neutrosophic summaries of partial paths crossing the bag. Since each summary is a triple discretized by ε (or exact for bounded bags), the number of states per bag is bounded by a function of τ alone. Thus, runtime is: $O(|V| \cdot g(\tau))$,for computable g.Hence, the problem is FPT in treewidth.

Results and Discussions

Neutrosophic Bishop Graphs provide a principled way to preserve epistemic nuance while enabling rational decision making. Unlike probabilistic or fuzzy models, NBGs do not suppress contradiction or uncertainty but incorporate them structurally into inference. The complexity results clarify both the limitations of exact reasoning and the regimes in which efficient inference is possible. This balance between expressiveness and tractability is essential for real-world expert systems.

Conclusion

Neutrosophic Bishop Graphs were introduced in this chapter as a cohesive framework for computational, logical and geometric reasoning under uncertainty. By using formal definitions, proofs and complexity analysis, we were able to show that Neutrosophic Bishop Graphs are useful in both theory and practice. In situations that are unpredictable and inconsistent, the framework provides additional avenues for reliable and explainable decision-making.

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