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Eccentric Distance Number and Eccentric Distance Sequence in Product Graph

Khusbhu Chourashia[1]
Research Scholar, Department of Mathematics,
Vels Institute of Science, Technology and Advanced Studies (VISTAS),
Pallavaram, Chennai, India

S. Meenakshi[2]
Associate Professor, Department of Mathematics,
Vels Institute of Science, Technology and Advanced Studies (VISTAS),
Pallavaram, Chennai, India

Abstract: An unordered pair (v_i,v_j) of vertices is associated with each e_i of a graph G, where V is a collection of vertices (V = {v_1,v_2,v_3,...}) and E is a set of edges (E = {e_1,e_2,e_3,...}). The eccentric distance number (EDN) is the number of vertices in G that are at maximum separation from vertex v_i. The Eccentric Distance Sequence (EDS) is a list or enumeration of each vertex's EDN in the graph. Eccentricity is the distance of a farthest vertex from a given vertex. In this paper EDS of product graph of path graph and complete graph is competitive. Based on the computation and generalisation of the outcome, the EDS can be further introduced in other fields.

Keywords: Eccentric distance sequence (EDS), Eccentric distance number (EDS), Path graph, Complete graph, Eccentricity, Sub graph, Cartesian product

1. Introduction

Graph theory, which involves the exploration of the sequences of many graphs, has taken a major platform to create a futuristic scope in the field of graph sequences. Sequences are more informative in graph theories than an invariant [1]. In the field of graph theory many sequences are now representing the graphs more precisely. There are numerous sequences that will be introduced in the future for graph theory, including path, distance, eccentric, status, degree, and many others that have already been introduced [2].

The first sequence Degree sequence established by Havel and Hakimi [3]. Introduced by Ostrand, Behzad, Simpson, Lesniak (with a major contribution), and Nandakumar, eccentric sequences for undirected graphs are widely recognised concepts [11]. Followed by Randiac, whose contribution in the field of distance-related sequences like degree sequences for paths and distances is the major turning point in extent graph theory in different dimensions [4].

Eccentric distance sequence is a combination both eccentric sequence and graphs distance-degree sequences. Graphs eccentric distance sequences, such as the complete graph K_n and path graph P_n have been computed in this paper [9].

Based on that product of path graph and complete graphs were presented in the paper. Eccentric distance number is the proceeding section to obtain Eccentric distance sequence. In this paper, the basic idea is to find EDS

[1]khusbhuchourashia@gmail.com, [2]meenakshikarthikeyan8@gmail.com

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(eccentric sequence distance) of product graph of path graph and complete graph. Also based on the above study, further extension of the concept to be implemented on the product of subgraphs of graph [5, 8].

In order to clarify the concept and determine the EDS, the product graph of two distinct graphs has been taken into consideration. A binary functions on graphs and is well known [6].

In more specific terms it is a function which receives any 2 graphs say G_A as well as G_B moreover produces a graph H, similarly, the vertex set of H is the cartesian product of $V(G_A) \times V(G_B)$; where $V(G_A)$ and $V(G_B)$ are the vertex set of G_A and G_B [7, 9].

2. Preliminaries

a) *Graphs:*

Graph G is a pair (V, E) of non-empty sets in which V ={v_1,v_2,v_3,...} indicates the group of vertices as well as E ={e_1,e_2,e_3,.}is a set in which each element ek of E is identified with an pair of vertices(vi, vj) where by vi, vj is not considered.[11]

b) *Distance:*

Shortest path among two vertices u and v is such that the smallest distance between two vertices "u" and "v". For this distance, we have an expression for the representation d(u,v). [3]

c) *Sequence of Vertex Degrees in a Graph*

The order in which the vertices of a graph have their degrees is the degree sequence of the graph. The degree of a vertex of the graph is the number of edges adjecent to the vertex [11].

d) *Eccentricity:*

The eccentricity of a vertex u is defined as the greatest distance from u to any other vertex in the graph [10].

e) *Eccentric sequence:*

Graph G's eccentric sequence is a collection of its vertices' eccentricities ordered in a non-decreasing order [11].

f) *Eccentric distance number:*

The n(v) at a max. distance for any vertex, let's say u in G, is known as the vertex u's EDN [1].

g) *Eccentric distance sequence:*

The graph's eccentric distance sequence is the list or enumeration of each vertex's EDN as a series. EDS is the symbol for it [1].

To clarify, the EDS refer to the EDN of each vertex of the graph represents in a sequence, while the EDN is calculated for each vertex of the graph.

h) *Path graph:*

The tree of the route graph has two nodes with degree 1 vertices and the other nodes with degree 2 vertices. A route graph is one that may be shown with every vertex and edge on a single straight line [2],[3].

i) *Complete graph:*

Complete graph is a particular kind of graph where each pair of graph vertices is connected by an edge, denoted by Kn [10][9].

j) *Cartesian Product of two graphs:*

Cartesian product of two graphs is a combination of vertices of two original graphs Each vertex is in the new graph created using Cartesian product of two given graphs. One component of the new network shares the two vertices with the rest of the components and the rest of the components are close enough to the original graphs of the two vertices [5] [10].

3. Main Outcomes

Theorem 1: The Product graph that comprehend the Eccentric distance sequence between the two path graphs of same even number of vertices must have EDS as { 1, 1, 1, ... to n^2 times }.

Evidence: Let's use the provided series of eccentric distances is {1, 1, 1,... n^2} times}. Assume G be the basic finite graph that satisfies the specified EDS. Here are n^2vertices in the Product graph G. For the EDS, 1 entails the maximum amount of vertices having maximum distance from a particular vertex Vi.

If we take path graphs with even number of vertices, our purpose will be served, as each vertex will get only one vertex in the product graph having maximum distance with it.

Algorithm for the result

Requirement: Construction and characterization of product of path graphs with even number of vertices to establish the result, by getting Eccentric distance number EDN of each vertex.

Input: Both path graphs taken have same extent of vertices (even number)

Configuration: It has n^2 vertices in G; v1, v2, ... vi. n is even and i=n^2 number of vertices.

Iteration

Step 1: Set up the vertex set v1, v2, ... vn. Make certain the EDN of each vertex is one. If so, go on to the next phase. Otherwise, go on to Step 5.

Step 2: Cartesian product of path graphs with even number of vertices has been computed. The number of vertices for both path graphs is taken same.

Step 3: To get the EDN, the EDN of each vertex must be calculated.

Step 4: Eccentric distance sequence of eccentric distance number is computed. The sequence obtained claims the result.

Step 5: End the calculation.

Evidence of accuracy

We use inductive mathematics to demonstrate the algorithm's soundness.

G(n) be the simple finite, product graph, obtained from two path graphs of same number (even number) of vertices.

Then G(n) eccentric distance number must be {1, 1, 1, ... to n^2 times}.

Proof: We demonstrate the technique for G(n)'s n_2 vertices.

Fundamental case: Since number of vertices taken is even. So n = 2 (Path graph P2)

The maximum distance of vertex v1 is with v4.

Since only one vertex v4 is at maximum distance with v1. Therefore its Eccentric distance number is 1.

Similarly for v_2, v_3, v_4 EDS is 1 respectively. Since EDS is {1, 1, 1, 1}.

Hence the assertion is true for n = 2 vertices (1).

Induction Step: Suppose (1) holds then the graph that comprehend the EDS can hold for (n-1) vertices for each path graph [here n must be odd then (n-1) will be even]

Since each vertex has only one vertex as maximum distance, so each vertex has EDN 1.

We now consider the EDS as {1, 1, 1, ... to $(n-1)^2$ times}

It is hypothesized that the approach is valid for (n-1) vertices.

Expanding each route graph by one vertex, the EDS by hypothesis will be {1, 1, 1, ... to n^2 times}.

Theorem 2: The product graphs between Path graphs Pn having an even number of vertices and Complete graph K_3 will give EDS { 2,2,2 …….to 3n times}

Proof:

Let the given Eccentric distance sequence be {2, 2, 2,……..to 3n times}. The finite graph G is the one that satisfies the specified EDS. The product graph G has 3n vertices. The max. number of vertices with a maximum distance from a vertex V_k in the EDS is implied by 2. If we take path graph with even number of vertices, our purpose will be served. As the product graph between path graph and complete graph will gives eccentric distance number for every vertex to compute its EDS.

Algorithm for the result

Requirement: To establish the desired result, complete graph K3 and Path graphs till n(even) vertices are computed.

Input: Both Path graph and complete graph product graph serves as the input for EDS.

Configuration: The resultant graph G will have 3n vertices, with V_1, V_2, V_3,... V_{3n} vertices.

Iteration

Step-I Initialize the vertices v_1, v_2, ... v_{3n}. Ensure the EDN for each vertex to be 2. If so, go on to the next phase. Otherwise, go on to step 5.

Step II Cartesian Product of path graph with Complete graph has been computed. Path graph Considered has even number of vertices.

Step III Computation of EDN of each vertices to generate the sequence

Step IV The sequence obtained can Claim the result

Step V End of the calculation.

Evidence of Correctness

We use Mathematical Induction to demonstrate the algorithm's correctness.

Let G be the product graph obtained. EDS of Product graph G must be [2,2,2.... to 3n times].

Proof: We prove the algorithm 3n vertices of G.

Base case: Path graph P2 and Complete graph K3 computation of product graph to be initiated (Fig. 61.1, 61.2 and 61.3).

Fig. 61.1 Path graph P_2

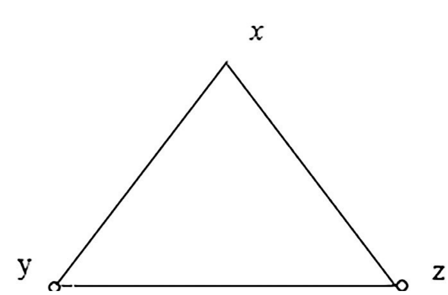

Fig. 61.2 Complete graph K_3

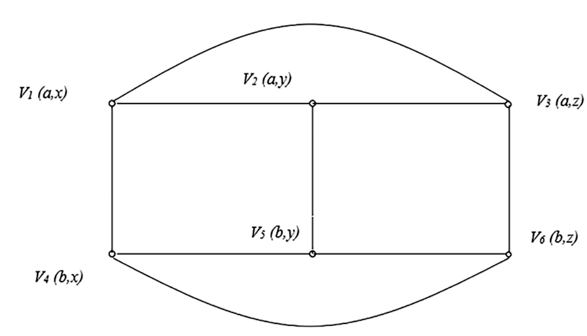

Fig. 61.3 Product graph G

The maximum distance of vertex V1 is with vertex V5 and V6. Hence EDN of V1 is 2. Similarly for vertex V2, V3, V4, V5, V6 the EDN is 2, respectively.

Since EDS of product graph is {2,2,2,2,2,2,2}.

Hence the assertion is true for n=2 vertex in path graph with complete graph K3 (1)

Induction Steps: Suppose (1) holds, then the graph that realizes EDS can hold for (n-1) vertices in Path graph (Fig. 61.4).

Each vertex has only 2 vertex at maximum distance so each vertices has EDN 2.

Since we now consider EDS as {2,2,2,2,… to 3(n-1) times}.

According to the speculation, the technique works for route graphs with (n-1) vertices. Hence result hold for pn path graph with complete graph k3 (Fig. 61.5).

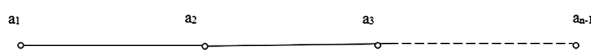

Fig. 61.4 Path graph P_{n-1}

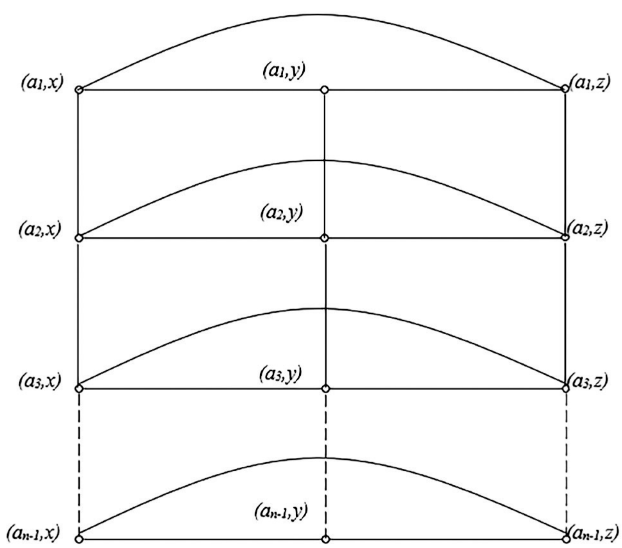

Fig. 61.5 Product graph G

Theorem 3: The EDS obtained from the product graph between the path graphs having odd numbers of vertices will give two necessary results or conditions.

(i) The vertex at the middle point of the graph will always have EDN as 4.

(ii) The other vertices present on the intersecting line, which have the vertex at the middle with EDN 4, will have EDN as 2.

Proof: Based on Theorems 1 and 2, we may infer that the segments of eccentric distance exhibit a certain elemental pattern in the sequence. So we can conclude (i) and (ii) based on our results obtained in Theorem 1 and 2. Using Mathematical induction to prove (i) and (ii) Condition.

Let us consider two path graphs with odd numbers of vertices and derive the results. Hence singular graph has no result (Fig. 61.6 and 61.7).

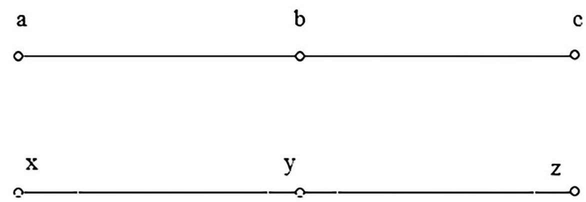

Fig. 61.6 Path graph p3

The product graph between two path graphs

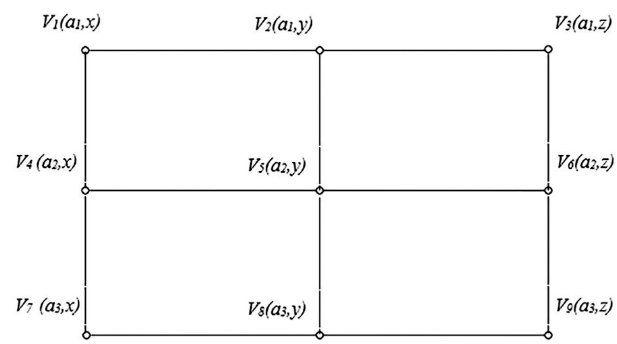

Fig. 61.7 Product graph G

Here vertex V5at the centre of the graph is at maximum distance with vertex V1, V3, V7, and V9 so its EDN is 4, which proves our condition (i).

Now the vertices V2, V4, V6, V8 which present on the intersecting path having centre vertex, have EDN as 2 from each vertex. Vertex V2 is at maximum distance V7 and V9 so its EDN is 2. Similarly, vertex V4 is at maximum distance with V3 and V9, similarly vertex V6 is at maximum distance with V1 and V7, and vertex V8 is at maximum distance with V1 and V3. Hence our condition (ii) holds.

If the condition holds for P3 path graphs, then it must hold from Pn-1 path graphs (Fig. 61.8). Here (n-1) number of vertices must be odd.

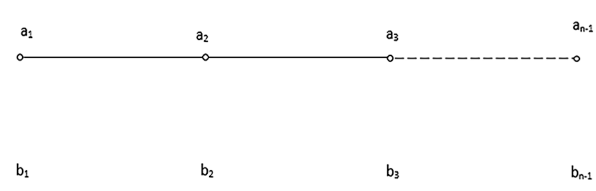

Fig. 61.8 Path graph P_{n-1}

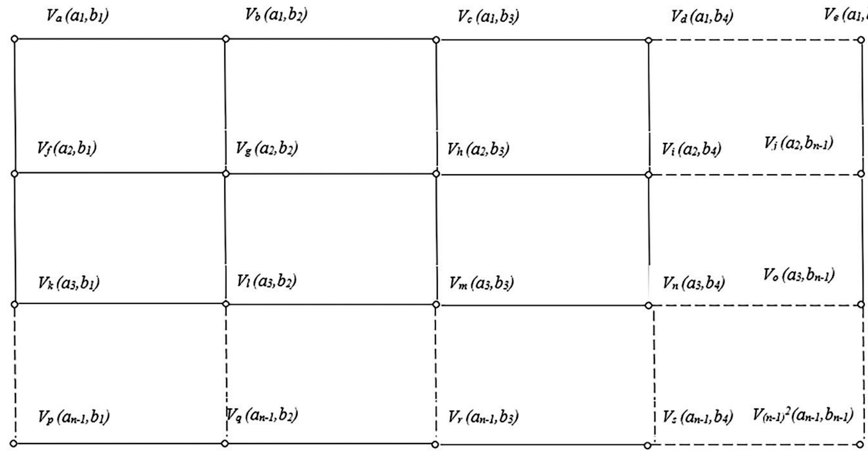

Fig. 61.9 Product of path graph G_{n-1}

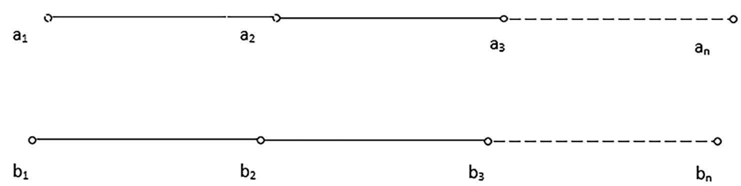

Fig. 61.10 Path graph P_n

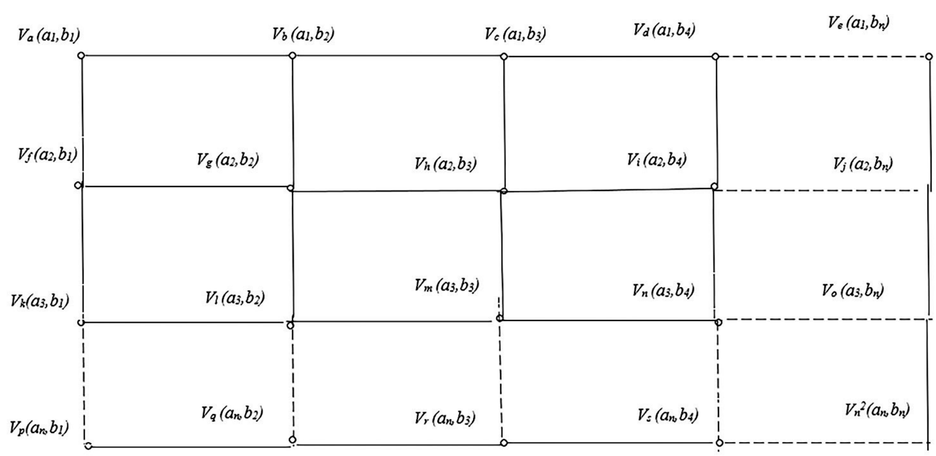

Fig. 61.11 Product graph P_n

The condition (i) and holds for the Fig. 61.9. The vertex at the centre have EDN as 4. And the vertices present on the intersecting path of the central vertices have EDN 2.

Thus by hypothesis we can conclude that if the results hold for (n-1) vertices, then it will hold good for ‘n’ vertices of path graph where n must be odd (Fig. 61.10).

The product graph between path graph Pn (Fig. 61.11).

Hence the result hold for path graph p_n when n is odd.

4. Conclusion

Significant advancement in the study of sequence has been noted. Numerous distance related sequences, such as EDN, EDS were used in the study of molecular and chemical structures in the field of chemistry. Finding the EDN of specific vertices and making the sequence EDS can be used in several networking process, also it can have a wide approach in Biomedical field (since the several organ join makes a system). We can find its application in chemical structure when two compounds are joining to make a resultant compound. The use of EDN in several field and its application in natural science is yet to establish in further presentation.

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Note: All the figures in this chapter were made by the authors.