

Buyer-Vendor Joint Economic Production Inventory Model for Profit Optimization

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ABSTRACT

This study develops a joint economic production–inventory model aimed at maximizing profit by realistically capturing interactions between buyer - vendor. In the proposed framework, both buyer and vendor collaboratively participate in the production process and share responsibility for system performance. The optimal system cost is determined under the assumption of equal profit sharing between the buyer and vendor. Additionally, the model incorporates a quantity discount scheme offered by vendor to buyer when the order quantity exceeds a specified level. Numerical illustrations are provided to demonstrate the applicability of the model and to confirm its effectiveness.

1. INTRODUCTION

Products with limited shelf lives, such as pharmaceuticals, are widely produced and supplied by manufacturers operating at small, medium, and large scales. In competitive markets, it is not uncommon for the same product to be manufactured simultaneously by firms of different sizes. Under such circumstances, a small-scale manufacturer (buyer) may find it economically preferable to procure the product from a large-scale manufacturer (vendor) rather than producing it in-house. To increase sales volume and enhance profitability, the large-scale manufacturer may offer a quantity discount to the small-scale manufacturer. Motivated by this practical scenario, the present study develops a mathematical production–inventory model to analyze the joint decision-making process and its impact on system performance.

Integrated inventory and production models have been widely studied to enhance coordination between vendors and buyers and to improve overall supply chain performance. Early foundational work by Banerjee (1986) and Goyal (1988) established joint economic lot size frameworks that emphasized coordination between purchasers and vendors, while further extensions by Goyal and Gupta (1989) highlighted the importance of buyer–vendor collaboration in minimizing total system cost. Subsequent research expanded these classical models by incorporating practical considerations such as consignment stock and integrated production–inventory policies (Zanoni & Zavanella, 2007). More recent studies have shifted focus toward profit optimization and sustainability issues in coordinated supply chains. For instance, Alamri (2023) developed a vendor–buyer model incorporating green production decisions, whereas Rathore et al. (2023) analyzed profit optimization under carbon emission constraints and price-dependent demand. Advanced modeling approaches addressing uncertainty and demand variability have also been proposed, including fuzzy demand integration (Mittal et al., 2024) and inflation-induced demand under conditional delivery policies (Bag & Behera, 2025). Furthermore, modern vendor-managed inventory systems considering space limitations, stock-dependent demand, and preservation investment have been explored by Sen et al. (2025). In parallel, several researchers have contributed integrated EOQ and EPQ models accounting for shortages, quantity discounts, warehouse capacity constraints, and joint profit maximization, thereby enhancing the applicability of these models to real-world supply chains (Babu et al., 2019; Muniappan et al., 2020; Sarkar et al., 2022). Collectively, these studies underline the growing importance of integrated buyer–vendor decision-making frameworks that simultaneously address economic, operational, and environmental objectives.

2. ASSUMPTIONS AND NOTATIONS

The model use the following assumptions and notations

Assumptions

1. A single product with a limited shelf life is considered.
2. Demand for the product is deterministic and constant over time.
3. Vendor has sufficient production capacity to meet both its own demand and the procurement requirements of buyer.

4. Buyer may either produce the product internally or procure it from Vendor.
5. Shortages and backorders are not allowed in the system.
6. Production rates are finite and greater than the demand rate.
7. Lead time is assumed to be zero or negligible.
8. Vendor offers a quantity-based discount to buyer when the order quantity exceeds a predetermined level.

Notations

- D Constant demand rate of the product
- P_1 Unit production cost incurred by the buyer
- P_2 Unit production cost incurred by the vendor
- p Unit price paid by the vendor
- k_1 Ordering cost per order for the buyer
- k_2 Setup cost per production run for the vendor
- h_1 Inventory holding cost per unit per unit time for the buyer
- h_2 Inventory holding cost per unit per unit time for the vendor
- Q Economic Order quantity
- n Vendor's multiples of order
- d_f Quantity discount rate offered to the buyer for larger order sizes

3. FORMULATION OF THE MODEL

The **buyer's total cost** TC_B includes two components:

- The ordering cost and the inventory holding cost.
- The first term, $\frac{k_1 D}{Q}$, represents the buyer's ordering cost, which depends on the ordering cost per order k_1 , demand rate D , and order quantity Q .
- The second term, $\frac{h_1 Q}{2} \left(1 - \frac{D}{P_1}\right)$, represents the buyer's average holding cost, adjusted by the production rate P_1 , reflecting the effect of finite production.

Thus the buyer's total cost per unit time is

$$TC_B = \frac{k_1 D}{Q} + \frac{h_1 Q}{2} \left(1 - \frac{D}{P_1}\right) \quad (1)$$

The **vendor's total cost** TC_V comprises three components.

- The first term, $\frac{k_2 D}{nQ}$ represents the vendor's setup cost per unit time, where k_2 is the setup cost and n is the number of shipments per production cycle.
- The second term, $\frac{h_2 n Q}{2} \left(1 - \frac{D}{P_2}\right)$ denotes the vendor's inventory holding cost under a finite production rate P_2 .
- The final term, pDd_f represents the discount cost offered by the vendor to the buyer for bulk ordering.

Thus the total cost for vendor can be written as

$$TC_V = \frac{k_2 D}{nQ} + \frac{h_2 n Q}{2} \left(1 - \frac{D}{P_2}\right) + pDd_f \quad (2)$$

The **total system cost** TC_S

- The **total system cost** TC_S of the integrated supply chain is obtained by summing the buyer's and vendor's total costs.
- This joint cost formulation enables coordinated decision-making, leading to overall system cost minimization and improved cost-sharing between the buyer and the vendor.

Thus the system cost can be written as

$$\begin{aligned} TC_S &= TC_B + TC_V \\ &= \frac{k_1 D}{Q} + \frac{h_1 Q}{2} \left(1 - \frac{D}{P_1}\right) + \frac{k_2 D}{nQ} + \frac{h_2 n Q}{2} \left(1 - \frac{D}{P_2}\right) + pDd_f \end{aligned} \quad (3)$$

Equation (3) will be written as

$$TC_S = \left(\frac{h_1 \left(1 - \frac{D}{P_1}\right) + n h_2 \left(1 - \frac{D}{P_2}\right)}{2} \right) Q + \frac{2(nk_1 + k_2)D}{nQ} + pDd_f \quad (4)$$

Equation (4) is of the form $c_1 Q + \frac{c_2}{Q} + c_3$

Q will be taken as, $Q = \sqrt{\frac{c_2}{c_1}}$

The optimal order quantity that minimizes the joint total cost of the buyer–vendor system is given by

$$Q^* = \sqrt{\frac{2(nk_1+k_2)D}{n\left[h_1\left(1-\frac{D}{P_1}\right)+h_2\left(1-\frac{D}{P_2}\right)\right]}} \quad (5)$$

Total Profit for buyer and vendor

The buyer’s and vendor’s profit shares are expressed as percentages of the total system cost as follows:

Buyer’s total profit $B_p = \frac{TC_B}{TC_S} \times 100$

Vendor’s total profit $V_p = \frac{TC_v}{TC_S} \times 100$

4. NUMERICAL EXAMPLE

Let $k_1 = 500$; $k_2 = 300$; $P_1 = 7000$; $P_2 = 10000$, $D = 5000$; $h_1 = 0.06$; $h_2 = 0.03$; $n = 2$; $p = 0.2$; $d_f = 10\%$.

The optimal solution is

Measure	Result
Optimal order quantity Q^*	13,000 units
Buyer’s total cost TC_b	303.7
Vendor’s total cost TC_v	352.7
Total system cost TC_s	656.4
Buyer’s cost share B_p	46.3%
Vendor’s cost share V_p	53.7%

Sensitivity Analysis of the Integrated Buyer–Vendor Model

Sensitivity analysis is carried out to examine the impact of key system parameters on the optimal order quantity, total system cost, and profit-sharing ratios. Each parameter is varied by $\pm 20\%$ while keeping all other parameters constant at their base values.

Table 1. Sensitivity of Demand Rate D

Parameter Change	Q^* (units)	Buyer's Cost TC_B	Vendor's Cost TC_V	Total System Cost TC_S	Buyer Profit B_P (%)	Vendor Profit V_P (%)
Demand (D -20%) 4000	11,625	266.8	311.5	578.3	46.1	53.9
Base Case	13,000	303.7	352.7	656.4	46.3	53.7
Demand (D +20%) 6000	14,300	344.5	392.3	736.8	46.7	53.3

Table 2. Sensitivity of Buyer Holding Cost h_1

Parameter Change	Q^* (units)	Buyer's Cost TC_B	Vendor's Cost TC_V	Total System Cost TC_S	Buyer Profit B_P (%)	Vendor Profit V_P (%)
Buyer holding cost (h_1 -20%) 0.048	12,160	285.0	360.5	645.5	44.1	55.9
Base Case	13,000	303.7	352.7	656.4	46.3	53.7

Parameter Change	Q^* (units)	Buyer's Cost TC_B	Vendor's Cost TC_V	Total System Cost TC_S	Buyer Profit B_P (%)	Vendor Profit V_P (%)
Buyer holding cost ($h_1 +20\%$) 0.072	14,090	326.0	344.8	670.8	48.6	51.4

Table 3. Sensitivity of Vendor Holding Cost h_2

Parameter Change	Q^* (units)	Buyer's Cost TC_B	Vendor's Cost TC_V	Total System Cost TC_S	Buyer Profit B_P (%)	Vendor Profit V_P (%)
Vendor holding cost ($h_2 -20\%$) 0.024	12,620	304.5	337.5	642.0	47.4	52.6
Base Case	13,000	303.7	352.7	656.4	46.3	53.7
Vendor holding cost ($h_2 +20\%$) 0.036	13,380	303.0	368.5	671.5	45.1	54.9

Observations

- Increasing demand increases the optimal order quantity Q^* and total system cost TC_S , while the buyer's share slightly increases.
- Higher buyer holding cost increases TC_B and shifts cost share toward the buyer. Lower holding cost reduces the buyer's share.
- Higher vendor holding cost increases TC_V and shifts cost share toward the vendor. Lower holding cost reduces the vendor's share.

- The sensitivity analysis shows that adjusting holding costs is an effective lever to achieve near-equal cost/profit sharing, while demand fluctuations primarily affect system-wide cost but not drastically the sharing ratio.

5. CONCLUSION

This study presents a joint economic production inventory model to improve profit optimization by reflecting real manufacturing practices. In this model, buyer and vendor work together and share responsibility for producing goods. The optimal system cost is obtained by assuming that both buyer and vendor receive equal benefits. By incorporating a quantity-discount policy and coordinated decision-making, the model captures realistic interactions between buyer and vendor. The analytical and numerical results demonstrate that cooperation significantly reduces total system cost and improves profit for both parties. The findings highlight the importance of coordination and pricing incentives in production and inventory planning, and the proposed model can serve as a useful decision-support tool for managing time-sensitive products.

REFERENCES

1. Alamri, A. A. (2023). Efficient formulation for vendor–buyer system considering optimal allocation fraction of green production. *Axioms*, *12*(12), 1104.
2. Babu, M., Ravithammal, M., & VEDIAPPAN, M. K. (2019). Integrated EPQ shortage inventory model with quantity discount. *International Journal of Scientific Research and Review*, *8*(3), 126–129.
3. Bag, A., & Behera, N. P. (2025). An integrated vendor-buyer supply chain model with inflation-induced demand under conditional delivery. *International Journal of Statistics and Applied Mathematics*, *10*(10), 82–89.
4. Banerjee, A. (1986). A joint economic-lot-size model for purchaser and vendor. *Decision Sciences*, *17*(3), 292–311.
5. Goyal, S. K. (1988). A joint economic lot size model for purchaser and vendor: A comment. *Decision Sciences*, *19*(1), 236–241.

6. Goyal, S. K., & Gupta, Y. P. (1989). Integrated inventory models: The buyer–vendor coordination. *European Journal of Operational Research*, 41(3), 261–269.
7. Mittal, M., Jain, V., Pandey, J. T., Jain, M., & Dem, H. (2024). Optimizing inventory management: A comprehensive analysis of models integrating diverse fuzzy demand functions. *Mathematics*, 12(1), 70.
8. Muniappan, P., Ravithammal, M., & Haj Meeral, M. (2020). An integrated economic order quantity model involving inventory level and warehouse capacity constraint. *International Journal of Pharmaceutical Research*, 12(3), 791–793.
9. Rathore, H., Shekhawat, S., & Sharma, K. (2023). Profit optimization for a manufacturing supply chain under carbon emission and with inventory-price-based demand. *Brazilian Journal of Operations & Production Management*, 20(2), 1471.
10. Sen, N., Modak, I., Giri, B. C., & Bardhan, S. (2025). Vendor managed inventory under price-and-stock-dependent demand, buyer’s space limitations and preservation investment. *IMA Journal of Management Mathematics*, 36(4), 963–989.
11. Sarkar, B., et al. (2022). An integrated vendor–buyer production-inventory model with joint profit maximization. *Mathematical Problems in Engineering*, 2022, Article 9619530.
12. Zanoni, S., & Zavanella, L. (2007). Single-vendor single-buyer integrated production–inventory model: The consignment stock case. *International Journal of Production Economics*, 118(1), 225–232.