Modular Multiplicative Divisor Labeling in Vertex Switched Jellyfish Graph

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***Abstract*— This paper investigates the Modular Multiplicative Divisor (MMD) labeling of jellyfish graphs under vertex switching when the number of tentacles is even. A vertex labeling function $f(v)$ is defined to assign distinct positive integers to each vertex, ensuring that adjacent vertices satisfy the modular conditions required for MMD labeling. Correspondingly, an edge labeling function $f^*(uv)$ is derived from the vertex labels such that each edge label respects the MMD criteria. It is formally shown that the total sum of all edge labels is congruent to zero modulo the total number of vertices, confirming the consistency of the labeling scheme. The results rigorously establish that vertex-switched jellyfish graphs admit a valid MMD labeling, providing a mathematical framework for modular graph transformations.**

***Keywords*— Jellyfish graph, Vertex switching graph, Graph labeling and Modular Multiplicative Divisor (MMD) labeling.**

I. INTRODUCTION

The Seven Bridges of Königsberg problem, solved by Leonhard Euler in 1736, is considered the foundation of graph theory. This field plays a vital role in both theoretical and applied sciences, serving as a valuable tool for developing techniques in discrete mathematics. Mathematically, a graph is a structure consisting of a set of vertices and edges that connect specific couples of vertices, representing relationships involving them. Edges serve the function of depicting the relationships or connections between vertices. Bondy and Murty define a spanning tree as a minimal connected subgraph that includes all vertices of a graph without forming cycles. The significance of spanning trees is emphasized in applications such as network optimization and reliability analysis. While primarily focused on classical graph properties, the study acknowledges that graph labeling plays a role in combinatorial mathematics and structural analysis [1]. In authors [2], Modular Multiplicative Divisor (MMD) graphs are characterized, focusing on path, cycle, star, and wheel graphs. Conditions for MMD labeling are established, and the impact of structural modifications, like graph joining and edge alterations, on these properties is analyzed. The study includes proofs and examples, contributing to the broader understanding of labeled graphs. In [3], authors explored Super Vertex Graceful graphs and their behavior under certain graph operations. Conditions for graphs to admit SVG labeling are analyzed, and the impact of operations like graph union, join, and subdivision on these properties is investigated. The authors in [4] investigated five new cordial graphs, proving that the Shadow graph, splitting graph, and Degree Splitting graph of the star graph are cordial. Further, it is established that the Jewel graph and Jellyfish graph also admit cordial labeling. Each theorem in the paper rigorously defines and proves the cordiality of these graphs, supported by illustrations to enhance understanding of the labeling patterns. The findings contribute to the study of graph labeling and cordial graphs. The authors [5] established that the Jewel graph (for both odd and even values) and the Even Arbitrary Super Subdivision (EASS) graph of the Jewel graph admit Modular Multiplicative Divisor (MMD) labeling. MMD labeling is defined as a bijection from vertices to $\{1, 2,\ldots,n\}$ with an induced edge function satisfying divisibility conditions. The findings confirm that these graphs satisfy the necessary conditions for MMD labeling, contributing to graph labeling theory. Additionally, open problems related to this work are presented for further exploration. The authors [6] investigate Modular Multiplicative Divisor (MMD) labeling for various graph classes. It is proved that the triangular book with n-pages, triangular snake, and the graph obtained by duplicating every edge with a vertex in a cycle admit MMD labeling. Additionally, it is established that graphs formed by switching end vertices in a path and switching a vertex in a cycle also satisfy MMD labeling, with some exceptions. This work expands the study of MMD labeling by analyzing new graph structures and their labeling properties. The concept of graph labeling was first began by Rosa in 1967 [7]. The authors proved that modifications of the Jewel graph, including vertex switching, path union, and cycle formation, admit cordial labeling. The study extended the application of cordial labeling to these transformations, contributing to the classification of cordial graphs [8]. The authors [9] explored prime labeling in graphs obtained by switching a vertex and examined whether certain standard graphs retain their prime labeling properties. The authors proved that the graphs resulting from vertex switching in path graph and star graph admit prime labeling. The author analyzed the wheel graph and its prime labeling behavior under vertex switching, highlighting challenges due to the scattered nature of prime numbers. A conjecture related to prime labeling in such transformations is proposed. Additionally, it is proved that the Lilly graph admits Prime Cordial Labeling (PCL), extending to its variants under vertex switching, duplication, degree splitting, and barycentric subdivision [10]. Recent studies have applied Modular Multiplicative Divisor (MMD) labeling to diverse graph structures with promising outcomes. Kalarani and Revathi [15] applied MMD labeling techniques for jellyfish graph with even number of tentacles. [14] focused on the EASS of the Cartesian product of two graphs, showing that MMD labeling can be effectively extended to product graphs with complex topology.

The authors [11] proved that the Triangular Book, Triangular Book with Bookmark, and Jewel Graph are k-cordial. The authors [12] discussed self-vertex switching and duplication self-vertex switching in self-centered graphs, defining conditions where vertex switching or duplication results in an isomorphic graph. The existence of such graphs with these properties is established. The authors J. Meena and T.N.M. Malini Mai focused on Roman and Double Roman Domination in graphs, using Python for computational analysis. In [13] examined Roman domination in complete binary trees, Their studies provide a strong foundation for algorithmic approaches like Modular Multiplicative Divisor (MMD) labeling in vertex-switched Jewel graphs for network security.

Section 2 provides definitions of all key terms along with relevant examples. Section 3 focuses on the MMD labeling of the Jellyfish Graph under vertex switching when the number of tentacles is even. The study applies MMD labeling to the vertex-switched Jellyfish graph, with the primary objective of exploring and presenting its properties in the even case. Figure 1 shows the graphical representation of the proposed strategy.

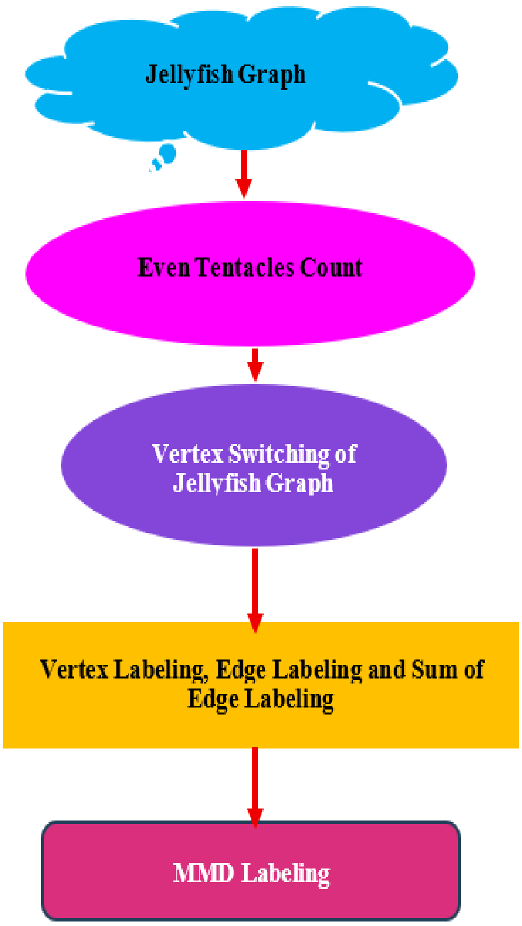

Fig. 1. Graphical representation of the proposed strategy

A. *Novelty of this study*

The paper presents a novel approach to Modular Multiplicative Divisor (MMD) labeling in vertex-switched jellyfish graphs, addressing a previously underexplored area. It introduces a structured bijection-based vertex labeling function that ensures edge labels comply with MMD conditions. A significant result is that the total sum of edge labels remains congruent to zero modulo the total number of vertices, which is mathematically established and supported with numerical verification. This work extends the scope of MMD labeling to vertex-switched graph transformations, providing new insights into systematic graph modifications and the underlying modular properties of such structures.

II. *PRELIMINARIES*

A. *Jellyfish graph*

The JF graph, marked as J (θ,ω), is formed by expanding a cycle C_4:pqrsp which consists of four points named as p,q,r and s, where the vertices q and s are connected by a bridge. Pendent lines θ and ω are appended to the vertices p and r respectively. This construction yields a graph resembling the shape of a Jellyfish, with the central cycle C_4 as the body and the attached pendent lines as the tentacles. The corresponding graph shown in figure2.

B. *Vertex switching of the graph*

The vertex switching $\boldsymbol{R_v}$ of a graph R is defined as the transformation of R into a new graph by modifying the adjacent relationships of a selected vertex v. Specifically, this process involves removing all edges directly connected to v in R and then connecting v to every vertex that was previously not adjacent to it in R. The result is a graph where the neighborhood of v is complemented, while the rest of the graph remains unchanged. Figure2 represents graph $\boldsymbol{R_v}$ and figure3 denotes the switching of the vertex v_4 of Graph R_v

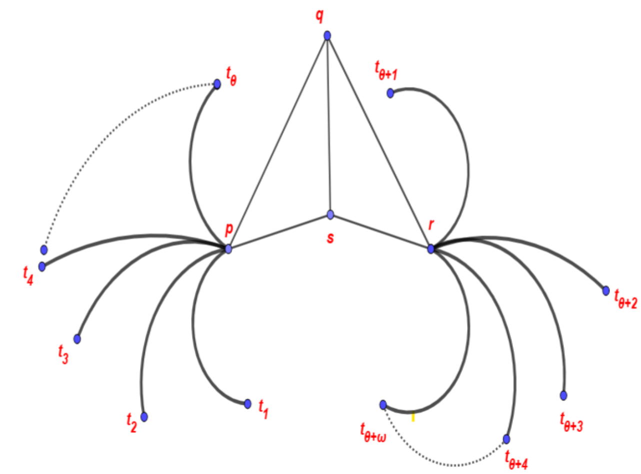

Fig.2. Jellyfish graph

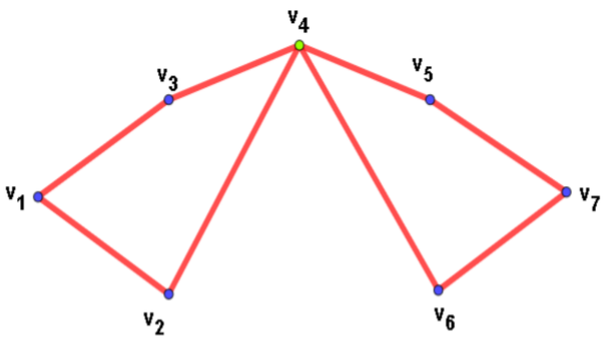

Fig.3. Graph R_v

C. *Graph labeling*

Assigning integers to points, lines, or both while adhering to predetermined guidelines in the graph is known as graph labeling. To provide a visual representation, Figure 3 displays the MMD assignment of labels to a graph with 7 points.

D. MMD labeling

The MMD graph *R* with *C* points. This graph involves a bijection *Z* from V(*R*) to the set {1,2,3,…,*C*}, along with an induced function y assigned to the edges of *R*, taking values from {0,1,2,3,…,*C-1*}. s Specifically, for any edge ab in *R*, the function *y(ab)* is determined by the equation y(ab) = *Z* (a) * *Z* (b) (mod *C*). It is important to note that the summation of the labels assigned to all the edges of this graph results in a value that is a multiple of *C*. The number obtained by the addition of all labels on edges of this labeling is 3+4+0+0+1+6+0+0 =14 ≡ 0 (mod 7).

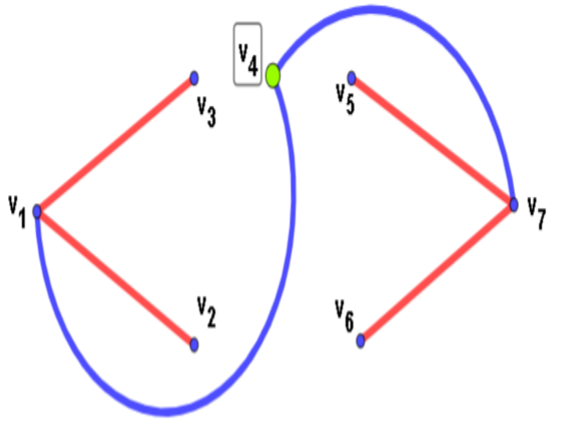

Fig.4. Switching of the vertex v_4 of Graph R_v

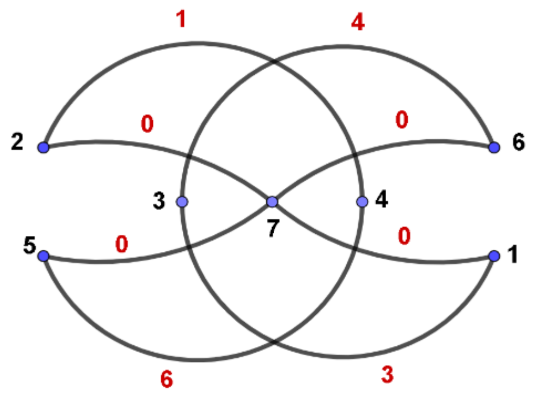

Fig.5. MMD labeling

III. Discussion of MMD Labeling for Jellyfish Graph

This study examines MMD labeling in vertex-switched Jellyfish graphs $j'_{\theta,\omega}$ with an even number of tentacles (θ *and* ω). It establishes a structured bijection-based vertex labeling to maintain MMD conditions. The research confirms that the sum of edge labels remains congruent to zero modulo the total vertex count. A systematic approach is used to validate MMD properties under vertex switching transformations. The algebraic proof and numerical verification confirm the correctness of the proposed method. The findings highlight how graph modifications preserve modular arithmetic properties. The study also establishes a relationship between tentacles count, vertex set, and edge labels. These results extend graph labeling techniques to new structured transformations.

A. Effect of Vertex Switching on MMD Labeling

Vertex switching in Jellyfish graphs with an even number of tentacles preserves MMD labeling by assigning a bijective vertex labeling from 1 to s^V, where s^V is the number of vertices. The edge labels, calculated as the product of vertex labels modulo s^V, continue to satisfy the MMD condition. As established in Theorem 3.1, the sum of all edge labels remains congruent to zero modulo s^V even after switching. This transformation retains the essential labeling properties and is verified through both algebraic formulation and Python-based computation.

Theorem :3.1

A Jellyfish Graph with an even number of tentacles admits Modular Multiplicative Divisor (MMD) labeling under vertex switching. There exists a bijective function on the vertex set that induces valid edge labels satisfying MMD conditions. Additionally, the total sum of edge labels remains a multiple of the vertex count after switching.

Proof:

The graph J (θ,ω) is obtained by extending a cycle C_4: $pqrsp$ (p, q, r *and* s are points of the cycle C_4), where the vertices q and s are connected by a bridge. Pendent lines θ and ω are appended to the vertices p and r respectively. This construction yields a graph resembling the shape of a J (θ,ω), with the central cycle C_4 as the body and the attached pendent lines as the tentacles. The number of tentacles θ of J (θ,ω) graph joined from the vertex *p* in cycle $C_4$and $\theta$is the number of tentacles of J (θ,ω) graph joined from the vertex *r* in cycle C_4. The degree of vertex q and s is 3, while the degree of vertex *p* is 2+θ and degree of vertex *r* is 2+ω. Let t_i, where $1 \le i \le \theta$ is a vertex with θ tentacles in the J (θ,ω) graph and its degree is one. On the other hand, $t_i$where $\theta + 1 \le i \le \theta + \omega$ is a vertex with ω tentacles and its degree is one. $V = t_1$,t_2,t_3,…, $t_{\theta+\omega}$,p,q,r,s are the vertices of J (θ,ω)graph.

Let J (θ,ω) be the jellyfish graph with the node set V(J (θ,ω)= $\{\mathrm{p,q,r,s}\,, t_i\!: 1 \le i \le \theta + \omega\}$ and E(J_η) $= \{pq, qr, rs, sp\ , pt_i, rt_i\!: i = 1\ to\ \theta + \omega\}$. The number of vertices follows the formula $\theta + \omega + 4$, while the number of edges is given by $\theta + \omega + 5$.

Let us consider q as a switching vertex in the J (θ,ω). Remove all the edges pq, qr, qs which are incident with p and make p to be adjacent with all the vertices which are not initially adjacent to it. The resultant graph termed as $j'_{\theta,\omega}$, the vertex switching of jewel graph. The vertex set of j'_η, is denoted as V($j'_{\theta,\omega}$) $= \{\mathrm{p,q,r,s}\,, t_i\!: 1 \le i \le \theta + \omega\}$ and E($j'_{\theta,\omega}$) $= \{ps, sr, pt_i;\ 1 \le i \le \theta$ and $rt_i\!: \theta + 1 \le i \le \theta + \omega\}$ is the edge set of $j'_{\theta,\omega}$ which is described in figure 6. The number of vertices $j'_{\theta,\omega}$ is denoted as $j^V = \theta + \omega + 4$ and the number of edges in $j'_{\theta,\omega}$ is denoted as $j^E = 2(\theta + \omega + 1)$. The labeling function for the vertices is defined as $: V(j'_{\theta,\omega}) \to \{1,2,3, \dots, \theta + \omega + 4\ \}$, with the following assignments: are

$$j(t_{2i-1}) = j(p) + i = 1 + i; 1 \le i \le \frac{\theta + \omega}{2},$$

$$j(t_{2i}) = j(s) - (i+1); 1 \le i \le \frac{\theta+\omega}{2},$$

$$j(p) = 1, j(r) = j^V - 1, f(q) = \frac{j^V}{2}, f(s) = j^V$$

The number of edges is $j^E = \left|E\,[j'_{\theta,\omega}]\right| = 2(\theta+\omega+1)$. . The edge labeling function is defined as $j^*(u'v') = j(u')j(v')(mod\ j^V)$ for every line $e = (u'v') \in E[j'_{\theta,\omega}]$.

Let total of all edges ϕ then,

$$\phi = \sum_{i=1}^{i=\frac{\theta}{2}} j(p)[j(p)+i] + \sum_{i=1}^{i=\frac{\theta}{2}} j(p)[j(s)-(i+1)]$$

$$+ \sum_{i=\frac{\theta}{2}+1}^{i=\frac{\theta+\omega}{2}} j(r)[j(p)+i] + \sum_{i=\frac{\theta}{2}+1}^{i=\frac{\theta+\omega}{2}} j(r)[j(s)-(i+1)] +$$

$$\sum_{i=1}^{i=\frac{\theta}{2}} j(q)[j(p)+i] + \sum_{i=1}^{i=\frac{\theta}{2}} j(q)[j(s)-(i+1)]$$

$$+ \sum_{i=\frac{\theta}{2}+1}^{i=\frac{\theta+\omega}{2}} j(q)[j(p)+i] + \sum_{i=\frac{\theta}{2}+1}^{i=\frac{\theta+\omega}{2}} j(q)[j(s)-(i+1)] +$$

$$j(p)j(s) + j(s)j(r)$$

$$= 1\left[(1+1)+(1+2)+\cdots+\left(\frac{\theta}{2}+1\right)\right] +$$

$$1\left\{(j^V-2)+(j^V-3)+\cdots+\left[j^V-\left(\frac{\theta}{2}+1\right)\right]\right\} +$$

$$(j^V-1)\left\{1+\left(\frac{\theta}{2}+1\right)+\cdots+\left[1+\left(\frac{\theta+\omega}{2}\right)\right]\right\} +$$

$$(j^V-1)\left\{j^V-\left(\frac{\theta}{2}+2\right)+\cdots+\left[j^V-\left(\frac{\theta+\omega}{2}+1\right)\right]\right\} +$$

$$\frac{j^V}{2}\left[(1+1)+(1+2)+\cdots+\left(\frac{\theta}{2}+1\right)\right] +$$

$$\frac{j^V}{2}\left\{(j^V-2)+(j^V-3)+\cdots+\left[j^V-\left(\frac{\theta}{2}+1\right)\right]\right\} +$$

$$\frac{j^V}{2}\left\{1+\left(\frac{\theta}{2}+1\right)+\cdots+\left[1+\left(\frac{\theta+\omega}{2}\right)\right]\right\} +$$

$$\frac{j^V}{2}\left\{j^V-\left(\frac{\theta}{2}+2\right)+\cdots+\left[j^V-\left(\frac{\theta+\omega}{2}+1\right)\right]\right\} +$$

$$1.j^V + j^V.(j^V-1) \qquad (1)$$

By using (1), it is shown that the total sum of edge labels ϕ is congruent to zero modulo the number of vertices j^V, confirming that the MMD labeling condition is satisfied.

The sum of edge labels ϕ is calculated based on the given mathematical expression. To confirm the MMD labeling condition, the program checks whether μ_1 is divisible by j^V. If the condition holds, the labeling is considered valid.

This confirms that graph with an even number of tentacles $j'_{\theta,\omega}$ admits Modular Multiplicative Divisor (MMD) labeling, ensuring that the total of its edge labels remain congruent to 0 modulo j^V. Figure 6 shows the generalised MMD labeling of vertex switching of jellyfish graph.

TABLE I.
TRUCTURAL PARAMETERS OF $j'_{(\theta,\omega)}$ JELLYFISH GRAPH

θ	ω	Number of vertices	Number of edges
2	4	10	14
2	6	12	18
2	8	14	22
2	10	16	26
4	2	10	14
4	6	14	22
4	8	16	26
4	10	18	30
6	2	12	18
6	4	14	22
6	8	18	30
6	10	20	34
8	2	14	22
8	4	16	26
8	6	18	30
8	10	22	38
10	2	16	26
10	4	18	30
10	6	20	34
10	8	22	38

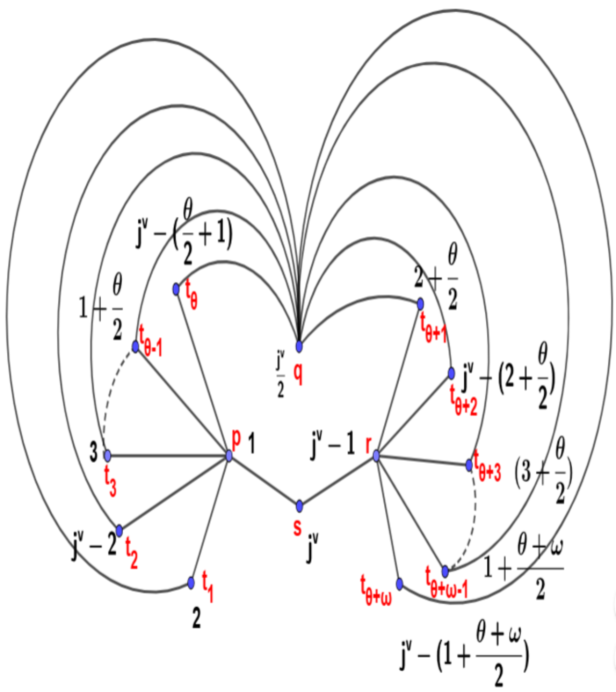

Fig.6 Generalised MMD labeling of vertex switching of jellyfish graph

Table 1 presents the structural parameters of the jellyfish graph $j'_{(\theta,\omega)}$ based on different even integer values of θ and ω,

where $\theta \neq \omega$.
The number of vertices j^V and number of edges j^E are determined by the relations:

$$j^V = \theta + \omega + 4, j^E = 2(\theta + \omega + 1)$$

These expressions describe how the topological size of the jellyfish graph expands as the parameters θ and ω increase. This table is fundamental in understanding the growth pattern of the jellyfish graph and serves as the basis for applying Modular Multiplicative Divisor (MMD) labeling

The figure 7 illustrates a comparative 3D column chart showing the relationship between the number of vertices and the number of edges in the jellyfish graph $j'_{(\theta,\omega)}$ for various even values of θ and ω.
The green bars represent the number of vertices

$$j^V = \theta + \omega + 4,$$

and the red bars represent the number of edges

$$j^E = 2(\theta + \omega + 1).$$

From the visual trend, it is evident that the number of edges consistently exceeds the number of vertices for all parameter combinations, showing a linear growth pattern as both parameters increase. This pattern confirms the proportional expansion of the jellyfish graph's structure with respect to its defining parameters. The figure effectively demonstrates how graph complexity rises with increasing θ and ω, which is crucial for understanding the scaling behavior under Modular Multiplicative Divisor (MMD) labeling.

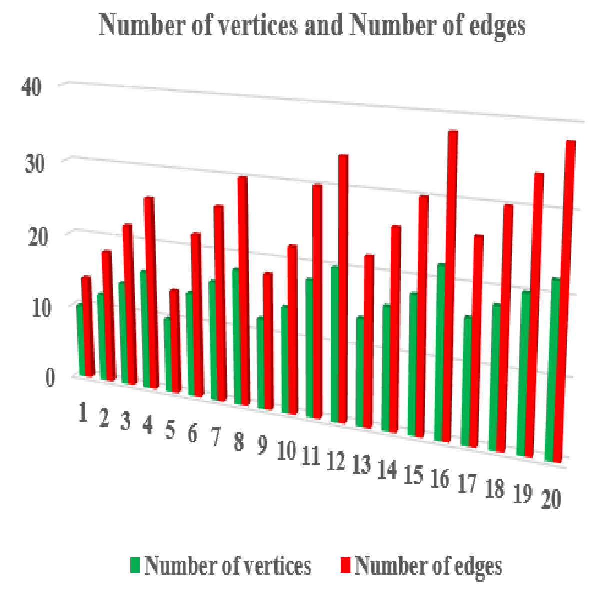

Fig. 7 Number of Vertices and Number of Edges in the Jellyfish Graph $j'_{(\theta,\omega)}$

IV. *Comparison of Existing Survey and Present Work*

Table II. presents a comparative analysis between this research and earlier studies. In [14], the authos confirmed that the vertex switching of the Jewel Graph, as well as the path union of its vertex switching, exhibit Cordial labeling. In contrast, our work establishes the MMD labeling for the vertex switching of the jellyfish graph in the even case. The table provides a comparative overview of past findings and our present research.

A. *Techniques to Enhance Accuracy in MMD Labeling*

Enhancing the accuracy of Modular Multiplicative Divisor (MMD) labeling in vertex-switched Jellyfish graphs involves several key techniques. Firstly, establishing a clear bijective function that maps vertices to a specific set of integers is crucial. This precise mapping ensures that each vertex receives a unique label, which is fundamental for accurate edge labeling. Secondly, the induced edge labeling function should be meticulously defined to assign labels based on the product of the labels of adjacent vertices, taken modulo the total number of vertices. This approach maintains consistency and adheres to MMD labeling rules. Additionally, rigorous verification processes, such as algebraic proofs and computational checks, are essential to confirm that the sum of all edge labels is congruent to zero modulo the number of vertices. These verification steps help in identifying and rectifying any discrepancies in the labeling process, thereby enhancing overall accuracy.

TABLE II. Comparison of Present and Previous Results on Labeling Techniques for Vertex Switching of the Jewe

Aspect	Present result	Previous result
Title	Modular Multiplicative Divisor Labeling in Vertex Switched Jellyfish Graph	Cordial Labeling on the Vertex Switching of Jewel Graph
Graph Types Investigated	Vertex switching of Jellyfish graph (Tentacles count is an even)	vertex switching of the jewel graph
Labeling Technique	MMD labeling	Cordial Labeling
Objective	Explored the novel application of MMD labeling in the vertex switching of a Jellyfish Graph.	Proved that the vertex switching of jewel is Cordial labeling
Labeling Function	The function f maps the vertices of the graph L to the set 1 to n, while the function f* maps the edges of the graph L to the set 0 to n – 1.	f: V(G) → {0, 1}
Main Results	- MMD labeling method is developed for vertex switching in the Jellyfish graph. graph with MMD labeling demonstrated.	- Vertex switching of the jewel graph is Cordial labeling

B. *Strategies for Enhancing MMD Labeling Performance*

To improve the performance of Modular Multiplicative Divisor (MMD) labeling in vertex-switched Jellyfish graphs, the paper adopts several algorithmic and structural strategies. A key enhancement involves the implementation of automated Python-based algorithms that systematically assign labels to vertices and edges, significantly reducing manual effort and increasing computational efficiency. Additionally, the modular decomposition of the graph into smaller subgraphs allows for localized labeling, which can later be recombined to preserve the overall MMD labeling properties. The use of computational tools also facilitates the processing of large and complex graphs, enabling accurate and faster validation.

C. *Future Scope*

In future work, we aim to switch the remaining vertices that admit MMD labeling and analyze their structural impact. We will explore the effects of these transformations on various graph properties. Additionally, we plan to investigate the existence of other labeling schemes applicable to Jewel graphs. The study can be extended to different classes of graphs to examine similar switching behaviors. Furthermore, algorithmic approaches for efficiently identifying and transforming such graphs will be developed.

D. *Impact of Vertex Switching on MMD Labeling Preservation*

Vertex switching in Jewel graphs with an odd number of jewels results in a transformation that preserves the Modular Multiplicative Divisor (MMD) labeling properties. Specifically, the operation modifies the adjacent relations of a selected vertex (e.g., vertex q), leading to a new but isomorphic structure where the original topology is altered, yet the graph retains a bijective labeling function. The induced edge labels still satisfy the MMD condition, i.e., the total sum of edge labels remains congruent to zero modulo the number of vertices. This transformation demonstrates that MMD labeling is robust under structural modifications like vertex switching. The result extends to a broader class of isomorphic graphs derived from the original Jellyfish graph, confirming that the MMD labeling property is invariant under such isomorphic transformations when carefully constructed. This reinforces the structural stability and applicability of MMD labeling in dynamic network environments.

E. *Congruency in Recursive Expansions and Graph Products*

The congruency condition on edge label sums in Jellyfish graphs remains valid under vertex switching due to structured labeling and modular arithmetic. For recursive expansions or graph product operations, the condition can still hold if vertex labels are reassigned properly and the labeling rule (product modulo total vertices) is maintained. However, since these operations alter the graph's structure, the labeling must be recalculated and verified for each case. Algorithmic validation is essential to ensure the MMD condition continues to be satisfied.

V Conclusion

The Modular Multiplicative Divisor (MMD) labeling was established for the vertex switching of the jellyfish graph $J'_{(\theta,\omega)}$ when both θ and ω are even. A well-defined labeling function was introduced to ensure that the sum of all edge labels remains congruent to zero modulo the total number of vertices. This work extends the applicability of MMD labeling to new jellyfish graph structures and highlights the modular consistency achieved through vertex switching. The results contribute to the understanding of structured graph transformations and provide a basis for further exploration of modular labeling techniques in complex graph systems.

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