# A Comprehensive Study on Fuzzy Labeling Techniques for Wheel Graphs Using Root Square Mean

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## Abstract

In this paper explores the concept of Root Square Mean (RSM) labeling in the fuzzy environment for wheel-related graphs. By integrating fuzziness into the structure of graphs, this study introduces fuzzy weights to vertices and edges, extending classical RSM labeling into a more versatile framework. The paper provides definitions, methodology, and illustrative examples of fuzzy RSM labeling for wheel-related graphs, emphasizing the theoretical and practical applications in network theory, fuzzy modeling, and decision-making processes.

## Keywords

Root Square Mean (RSM) Labeling, Fuzzy Graphs, Wheel Graphs, Graph Labeling, Fuzzy Environment, Fuzzy Sets, Network

## 1. Introduction

**Graph theory** has broad applications in fields like **computer science**, **chemistry**, **biology**, and **social networks**. In particular, **Graph labeling** assigns numerical values to vertices, edges, or both, under certain conditions to encode meaningful information. **Root Square Mean (RSM) labeling** is a relatively novel graph labeling method that assigns weights derived from the square root mean of the vertex or edge labels [1][2].

In real-world problems, data often involves uncertainty, imprecision, or vagueness. A **fuzzy graph**, introduced by Rosenfeld (1975), allows incorporating such uncertainties by associating membership degrees with vertices and edges [3]. This paper investigates how **RSM labeling** can be extended to **fuzzy graphs**, particularly focusing on **wheel-related graphs**, which are widely used in modeling communication and transportation networks [4][5].

## 1. Preliminaries

### 1.1 Graph

A **graph** $G=(V,E)$ consists of:

- A set of vertices V,
- A set of edges E, where $E \subseteq \{(u,v) | u,v \in V, u \neq v\}$.

### 1.2 Wheel Graph

A **wheel graph** $W_n$ for $n \geq 4$ is a graph consisting of n vertices:

(i). One central vertex $v_o$.

(ii). $n-1$ Peripheral vertices forming a cycle $C_{n-1}$.

The edge set is given by

$$E = \{(v_0, v_i) | \ i = 1,2,3....n-1\} \cup .\{(v_i, v_{i+1}) | \ i = i = 1,2,3....n-2\} \cup .\{(v_{n-1}, v_1)\}.$$

### 1.3 Fuzzy Graph

A fuzzy graph $G^F = (V, E, \mu V, \mu E)$ is defines as

$\mu V : V \to [0,1]$: a function assigning membership degrees to vertices.

$\mu E : E \to [0,1]$: a function assigning membership degrees to edges.

Satisfying:

$$\mu_E(u,v) \leq \min(\mu V(u), \mu V(v)) \quad \forall (u,v) \in E.$$

### 1.4 Root Square Mean (RSM) Labeling

For a graph $G=(V,E)$ an RSM labeling is a mapping $f : V \to \Re^+$ such that for any edge $e=(u,v)$:

$$w(e) = \sqrt{\frac{f(u)^2 + f(v)^2}{2}}$$

### 1.5 Fuzzy Root Square Mean (FRSM) Labeling

For a fuzzy graph $G^F = (V, E, \mu V, \mu E)$ an FRSM labeling assign

Fuzzy membership $\mu V(v)$ to each vertex $v \in V$

Fuzzy membership $\mu E(u,v)$ to each edge $(u,v) \in E$, such that

$$w^F(e) = \sqrt{\frac{\mu_V(u)^2 + \mu_V(v)^2}{2}}, \mu_E(u,v) = w^F(e)$$

**1.6 Admissibility Conditions**

**Injective RSM Labeling:** No two edges have the same weight

$$w^F(e_i) \neq w^F(e_j) \ \ for \ \ e_i, e_j \in E.$$

**Total FRSM Labeling:** Both vertices and edges are labeled using RSM criteria.

**Proper Fuzzy Graph:** Membership degrees satisfy

$$\mu_E(u,v) \leq \min(\mu V(u), \mu V(v)) \quad \forall (u,v) \in E.$$

## 2. Methodology for Fuzzy Root Square Mean Labeling in Wheel Graphs

**Step 1**: Understand the Structure of the Wheel Graph $W_n$

A **wheel graph** $W_n$ is defined for $n \geq 4$ as a graph consisting of n vertices:

(i). One **central vertex** $v_o$.

(ii). $n-1$ Peripheral vertices forming a **cycle** $C_{n-1}$.

The edge set is given by

$$E = \{(v_0, v_i) \mid i = 1,2,3....n-1\} \cup .\{(v_i, v_{i+1}) \mid i = i = 1,2,3....n-2\} \cup .\{(v_{n-1}, v_1)\}.$$

**Step 2**: **Assign Fuzzy Membership Values to Vertices**

Define a fuzzy membership function $\mu_V : V \to [0,1]$ for each vertex $v_i \ in \, V$ based on •

- Importance of the vertex in the graph.
- Application-specific criteria (e.g., reliability, weight, or connectivity in a network).

**Step 3**: **Define the Fuzzy RSM Edge Weight Function**

1. For any edge $e = (u,v) \in E$, compute the fuzzy root square mean weight $w^F(e)$:

$$w^F(e) = \sqrt{\frac{\mu_V(u)^2 + \mu_V(v)^2}{2}}$$

2. Assign the edge fuzzy membership values $\mu_E(u,v) = w^F(e)$.

**Step 4: Construct the Weighted Graph**

1. Represent $W_n^F = (V, E, \mu V, \mu E)$ such that

   $\mu V(v)$ is the fuzzy membership of each vertex v.

   $\mu_E(u,v)$ is the computed fuzzy membership for each edge $(u,v)$

2. is the computed fuzzy membership for each edge

$$\mu_E(u,v) \le \min(\mu V(u), \mu V(v)) \quad \forall (u,v) \in E.$$

**Step 5: Check Injectivity**

1. Ensure that all edge weights $w^F(e)$ are distinct to satisfy the injectivity condition:

$$w^F(e_i) \ne w^F(e_j) \ \ for \ \ e_i, e_j \in E.\ i \ne j$$

2. Adjust vertex fuzzy membership values if needed to resolve conflicts.

**Step 6: Validate and Analyze**

1. Verify that the labeling satisfies:

   (i). Fuzzy graph conditions,

   (ii). Injectivity of the labeling,

   (iii). Applicability to the problem domain (e.g., networks, communication systems, etc.).

2. Analyze the labeled graph for key properties:

   (i). Total weight: $W_{total} = \sum_{e \in E} w^F(e)$

   (ii). Connectivity and fuzzy influence of vertices.

## 3. Examples for Fuzzy Root Square Mean Labeling in Wheel Graphs

Fuzzy Root Square Mean (RSM) Labeling applied to a Wheel Graph $W_4$

1. **Graph Description**

$W_4$ consist of 4 vertices

$v_0$ : Central Vertex

$v_1, v_2, v_3$ : Peripheral vertices forming a triangle cycle $C_3$.

Edges include:

Spokes: $(v_0, v_1)$, $(v_0, v_2)$, $(v_0, v_3)$

**Cycle edges:** $(v_1, v_2)$, $(v_2, v_3)$, $(v_3, v_1)$

2. **Fuzzy Membership Values**:

Central vertex $v_0$ : $\mu_V(v_0)$

Peripheral vertices: $\mu_V(v_0) = 1.0$

$\mu_V(v_1) = 0.9$

$\mu_V(v_2) = 0.8$

$\mu_V(v_3) = 0.7$

$\mu_V(v_4) = 0.6$

3. **RSM Weight Calculation**:

For any edge $e = (u, v) \in E$, the fuzzy root square mean weight $w^F(e)$

is given by $w^F(e) = \sqrt{\dfrac{\mu_V(u)^2 + \mu_V(v)^2}{2}}$

**Edge Weights:**

**Spoke Edges (Connecting $v_0$ to peripheral vertices):**

$$w^F(v_0, v_1) = \sqrt{\frac{(1.0)^2 + (0.9)^2}{2}} = 0.95$$

$$w^F(v_0, v_2) = \sqrt{\frac{(1.0)^2 + (0.8)^2}{2}} = 0.90$$

$$w^F(v_0, v_3) = \sqrt{\frac{(1.0)^2 + (0.7)^2}{2}} = 0.86$$

$$w^F(v_0, v_4) = \sqrt{\frac{(1.0)^2 + (0.6)^2}{2}} = 0.80$$

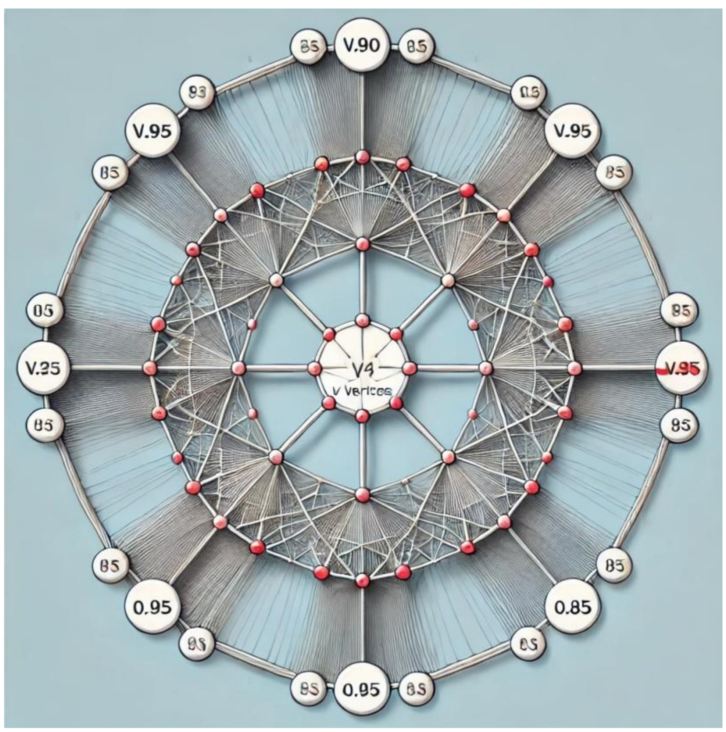
**Cycle Edges (Connecting peripheral vertices):**

$$w^F(v_1, v_2) = \sqrt{\frac{(0.9)^2 + (0.8)^2}{2}} = 0.85$$

$$w^F(v_2, v_3) = \sqrt{\frac{(0.8)^2 + (0.7)^2}{2}} = 0.75$$

$$w^F(v_3, v_4) = \sqrt{\frac{(0.7)^2 + (0.6)^2}{2}} = 0.65$$

$$w^F(v_4, v_1) = \sqrt{\frac{(0.6)^2 + (0.9)^2}{2}} = 0.74$$




## 4. Conclusion:

This research paper introduced Fuzzy Root Square Mean (RSM) Labeling for wheel-related graphs, extending traditional RSM methods to handle uncertainty and imprecision in graph connections. By using fuzzy membership values for vertices, we computed edge weights that reflect the strength of relationships under uncertainty. The method was demonstrated on the $W_4$ wheel graph, showcasing its applicability in real-world scenarios like network design and decision-making. Fuzzy RSM Labeling provides a more accurate representation of graph structures, offering significant potential for future research and practical applications in uncertain environments.

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