MATHEMATICAL MODELING AND ITS APPLICATIONS

Editor names:
Dr.P.Balaganesan
Dr.J.Renuka
Dr.E.Poongothai
Dr. V. Maheswari

HSRA
PUBLICATIONS

Published by
HSRA Publications 2025
#02, Sri Annapoorneshwari Nilaya,
1st Main, Byraveshwara Nagar, Laggere,
Bangalore – 560058
Sales Headquarters – Bangalore

ISBN: 978-93-6850-545-7

First Edition 2026

No. of Pages – 146

Size : ¼ Demy

TABLE OF CONTENTS

1. The Basics of Mathematical Modelling and Its Applications

[1]Adhil Basha D, *[2]Ramanathan P, [3]R Sengothai,[4] Subramanian R

[1,3]Department of Mathematics, AMET Deemed to be University, Chennai
[2]Department of Mechanical Engineering, AMET Deemed to be University, Chennai
[4]Department of Mathematics, Assistant professor, Sri Sairam Engineering College, Chennai - 44

Abstract

The contemporary methods of scientific research are founded on mathematical modeling, which not only provides a proper approach for the mathematical formulation of real world phenomena but also provides a proper framework for the mathematical representation of real world problems. The basic principles of mathematical modelling are defined in this chapter along with its theoretical underpinnings and methods. First, we introduce readers to the basic modelling process that leads us to the development of mathematical models and their redevelopment. Strict derivations of basic mathematical ideas such as differential equations and stochastic models are also covered in this chapter. The power of mathematical methods is affirmed through real world examples in the applied sciences. This chapter equips the reader with basic knowledge and concepts required for the development and analysis of mathematical models in their disciplines.

Keywords: mathematical modelling, numerical methods, differential equations, optimisation, model validation, and real world applications

1.Introduction

The process of translating physical, biological, economic, or social phenomena and observations into mathematical terms, reducing irrelevant aspects to emphasize relevant ones, has been described as mathematical modelling. It has made possible mathematical analysis, prediction, and optimisation through abstraction, which otherwise could not have been achieved by qualitative thought or experimentation.

One cannot overestimate how important mathematical modelling is for contemporary research and engineering. Mathematical models are the indispensable tool in understanding complex systems and enlightening decision makers, everything from climate change and the development of sustainable energy systems to smoothing out supply chains to simulating infectious diseases.

The COVID-19 pandemic put into sharp focus the importance of mathematical epidemiological models for informing public health policy decisions on a global scale.

1.1. The Modeling Cycle

All mathematical modelling projects entail an iterative process involving several key stages:

1. **Problem Formulation:** Identify the problem itself, and state the goals and scope thereof.

2. **Model Construction:** Utilize equations, inequalities, or other mathematical constructs to model.

3. **Mathematical Analysis:** The results and predictions should be calculated using analytical or numerical procedures.

4. **Validation & Verification**: The accuracy of the model can be checked by comparing the predictions obtained from the model with actual values.

5. **Model Development:** Use validation findings to refine model assumptions, alter assumptions, or alter model parameters.

6. **Implementation:** Implement the validated model for predictions of results, process improvements, and decision-making.

1.2 Theoretical Background

1.2.1 Classification of Mathematical Models

Mathematical models can be grouped based on a number of criteria, with different classifications underscoring different aspects of the models' structure and dynamics.

Deterministic vs. Stochastic Models

Deterministic systems always generate the same output for any set of initial conditions and parameters. These systems involve complete knowledge and lack randomness. Some of the examples of such systems are Newton's laws of physics and classical physics models.

Stochastic models include random variables and probability distributions to express stochastic uncertainties or variations. Many applications of stochastic models include finance, quantum physics, and population genetics.

Static and Dynamic Models

Static models: These are models of a system when it's in equilibrium, without any time dependence. Optimization instances, or structural analysis, commonly utilize static models.

Dynamic models involve explicit consideration of time evolution solutions with equations that involve differences or differentials. Dynamic models play a central role in analyzing transient responses, oscillations, and asymptotes.

Linear vs. Non-Linear Models

Linear systems obey the superposition principle and can be solved analytically very efficiently. But this is not the case when it comes to real-world applications. Most phenomena are nonlinear.

Nonlinear models can demonstrate very complex behavior such as having several equilibria, limit cycles, chaos, and bifurcations. Although nonlinear models are more difficult to study, they can offer more accurate descriptions of real-world systems.

1.2.2 Fundamental Modeling Principles

There are several basic principles for making good mathematical models:

Parsimony: Models must be as simple as possible in order to account for most of their features (Occam's Razor).

Conservation Laws: Mass, energy, momentum, and charge conservation laws can be very constraining.

itizenoidal Integrity: All terms involved in an equation should have equivalent dimensions.

Scale Separation: Separations of time or space scales that are vastly different from one another often lead to decoupling.

Empirical Foundation: The model should be founded on experimental findings and physical principles.

1.3 Mathematical Formulations

1.3.1 Ordinary Differential Equations

Dynamic modelling is based on ordinary differential equations (ODEs). The general form of a first-order ODE is as follows:

$$\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{f(x, t)} \quad (1.1)$$

If t is time, f is the rate of change, and x(t) is the state variable.

For systems with multiple state variables, we use vector notation:

$$\frac{dx}{dt} = f(x, t) \quad (1.2)$$

where $X = (x_1, x_2, \ldots\ldots, x_n)^T$ is the state vector.

Example: Exponential Growth Model

The most basic population model makes the assumption that population size and growth rate are proportionate:

$$\frac{\mathrm{dN}}{\mathrm{dt}} = \mathrm{rN} \quad (1.3)$$

where N(t) is population size and r is the growth rate constant. The analytical solution is:

$$N(t) = N_0 e^{rt} \quad (1.4)$$

where $\mathrm{N}_0 = \mathrm{N}(0)$is the initial population.

1.3.2 Logistic Growth Model

Unlimited exponential growth is unrealistic due to resource constraints. The logistic model incorporates carrying capacity *K*:

$$\frac{\mathrm{dN}}{\mathrm{dt}} = \mathrm{rN}(1 - \mathrm{N}/\mathrm{K} \quad (1.5)$$

This equation exhibits density-dependent growth, slowing as *N* approaches ***K***. The analytical solution is:

$$N(t) = K/(1 + (K - N_0)/N_0)N_0 e^{-rt}) \quad (1.6)$$

1.3.3 Systems of Linear Equations

Solving linear systems is the solution to many equilibrium problems:

$$\mathrm{AX} = \mathrm{b} \quad (1.7)$$

If **x** is the unknown vector, **b** is the right-hand side vector, and A is a m × n coefficient matrix. When rank(A) = rank([A|**b**]), there are solutions.

1.3.4 Optimization Models

Optimization problems seek to maximize or minimize an objective function subject to constraints. The general form is:

minimize f(x)

subject to $g_i(X) \leq 0$, i=1...m

$h_j(X) = 0$, j=1....p (1.8)

For unconstrained optimization, necessary conditions for a local minimum are:

$$\nabla f(X^*) = 0 \quad \text{(first-order)} \quad (1.9)$$

$$\nabla^2 f(X^*) > 0 \quad \text{(second-order)} \quad (1.10)$$

where $\nabla^2 f$ denotes the Hessian matrix and > 0 indicates positive definiteness.

1.4 Worked Examples

1.4.1 Example 1: Radioactive Decay

Problem Statement

The half-life of a radioactive material is 5,730 years (carbon-14). How much is left over after 10,000 years if we begin with 100 grammes?

Solution

Step 1: Formulate the model. Radioactive decay follows first-order kinetics:

$$\frac{dN}{dt} = -\lambda N \quad (1.11)$$

where λ is the decay constant.

Step 2: Relate decay constant to half-life. The half-life $t_{1/2}$ satisfies $N(t_{1/2}) = N_0/2$. From the solution

$$N(t) = N_0 e^{(-\lambda t)}$$

$$\frac{N_0}{2} = N_0 e^{(-\lambda t_{1/2})}$$

$$90n\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$\lambda = \ln(2)/t_{\frac{1}{2}} = 0.693147/5730 \approx 1.21 \times 10^{-4}\ year^{-4} \tag{1.12}$$

Step 3: Calculate the amount remaining:

$$N(10000) = 100 \times e^{(-1.21 \times 10^{-4} \times 10000)}$$

$$N(10000) = 100 \times e^{(-1.21)}$$

$$N(10000) \approx 100 \times 0.298 = 29.8\ grams \tag{1.13}$$

Answer: Approximately 29.8 grams of carbon-14 remain after 10,000 years.

1.4.2 Example 2: Mixing Problem

Problem Statement

A tank contains 1000 liters of water with 50 kg of salt dissolved in it. Water containing 0.1 kg/L of salt flows in at 10 L/min, and the well-mixed solution flows out at 10 L/min. Find the salt concentration as a function of time.

Solution

Step 1: Define variables. Let $S(t)$be the amount of salt (kg) in the tank at time *t* (minutes).

Step 2: Apply conservation principle:

$$\frac{ds}{dt} = (rate\ in) - (rate\ out) \tag{1.14}$$

Rate in = 0.1 kg/L × 10 L/min = 1 kg/min

Rate out = (S/1000) kg/L × 10 L/min = S/100 kg/min

Therefore:

$$\frac{ds}{dt} = 1 - s/100 \tag{1.15}$$

Step 3: Solve the differential equation. This is a first-order linear ODE. Rearranging:

$$\frac{ds}{dt} + \frac{S}{100} = 1$$

Using integrating factor $\mu(t) = e^{(t/100)}$

$$\frac{d}{dt}\left[Se^{\left(\frac{t}{100}\right)}\right] = e^{t/100}$$

$$Se^{(t/100)} = 100e^{(t/100)} + c$$

$$S(t) = 100 + Ce^{(-t/100)} \tag{1.16}$$

Step 4: Apply initial condition S(0) = 50:

50=100 + C → C = -50

$$S(t) = 100 - 50^{\left(-\frac{t}{100}\right)} = 100(1 - 0.5e^{(-t/100)}) \tag{1.17}$$

The concentration is . $C(t) = \frac{S(t)}{1000} = 0.1\left(1 - 0.5e^{\left(-\frac{t}{100}\right)}\right)\ kg/L$

Equilibrium Analysis: As $t \to \infty$, $S \to 100$ kg and $C \to 0.1$ kg/L, matching the input concentration.

1.4.3 Example 3: Chemical Reaction Kinetics

Problem Statement

Consider a second-order reaction where two molecules of substance A combine to form product P: 2A → P. If the initial concentration of A is 2 mol/L and the rate constant is k = 0.5 L/(mol·min), find the concentration after 5 minutes.

Solution

Step 1: Write the rate law. For second-order kinetics:

$$\frac{d[A]}{dt} = -k[A]^2 \qquad (1.8)$$

Step 2: Separate variables and integrate:

$$\frac{d[A]}{[A]^2} = -k\,dt$$

$$\int \frac{d[A]}{[A]^2} = -k\int dt$$

$$\frac{-1}{[A]} = -kt + C \qquad (1.19)$$

Step 3: Apply initial condition: $[A](0) = [A]_0 = 2\ mol/L$

$$-\frac{1}{2} = C$$

$$\frac{1}{[A]} = kt + \frac{1}{[A]_0}$$

$$[A](t) = \frac{[A]_0}{(1+kt[A]_0} \qquad (1.20)$$

Step 4: Calculate concentration at t = 5 minutes:

$$[A](5) = 2/(1 + 0.5 \times 5 \times 2) = 2/(1 + 5) = 2/6 = 0.333 \text{ mol/L} \qquad (1.21)$$

Answer: The concentration of A after 5 minutes is approximately 0.333 mol/L.

1.5 Real-World Applications

1.5.1 Epidemiological Modelling: SIR Model

A key component of mathematical epidemiology, the Susceptible-Infected-Recovered (SIR) model is widely applied during disease outbreaks, such as COVID-19. Three sections comprise the population:

S(t): Susceptible individuals who can contract the disease

I(t): Infected individuals who can transmit the disease

R(t): Recovered individuals with immunity

Model Equations

The dynamics are governed by:

$$\frac{dS}{dt} = -\beta SI/N \qquad (1.22)$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I \qquad (1.23)$$

$$\frac{dR}{dt} = \gamma I \qquad (1.24)$$

where $N = S + I + R$ is the total population (constant), β is the transmission rate, and γ is the recovery rate.

Basic Reproduction Number

The basic reproduction number R_0 represents the average number of secondary infections from one infected individual in a completely susceptible population:

$$R_0 = \beta/\gamma \tag{1.25}$$

Epidemic threshold: If $R_0 > 1$, an epidemic occurs; if $R_0 < 1$, the disease dies out.

Application Example

For influenza, typical parameters might be: $\beta = 0.5\ day^{-1}$, $\gamma = 0.1\ day^{-1}$ (recovery period ~10 days). This gives $R_0 = 5$,indicating each infected person infects 5 others on average. Public health interventions aim to reduce β (through social distancing, masks) or increase γ (through treatment) to bring R_0 below 1.

1.5.2 Environmental Science: Lake Pollution Model

Consider a lake receiving pollutant from industrial discharge and losing pollutant through natural degradation and water outflow.

Model Development

Let *P(t)* be pollutant concentration (mg/L). The mass balance gives:

$$V\left(\frac{dp}{dt}\right) = Q_{in}P_{in} - Q_{out}P - kVP \tag{1.26}$$

where V is lake volume, Q_{in} and Q_{out} are inflow and outflow rates, P_{in} is input concentration, and k is the degradation rate constant.

Assuming steady-state volume (Q_{in} = Q_{out}= Q):

$$\frac{dP}{dt} = \left(\frac{Q}{V}\right)(P_{in} - P) - kP \tag{1.27}$$

Equilibrium Analysis

At equilibrium ($\frac{dP}{dt} = 0$):

$$P^* = \frac{P_{in}}{1+\frac{kV}{Q}} = \frac{P_{in}}{1+k\tau} \tag{1.28}$$

where $\tau = \frac{V}{Q}$ is the residence time (average time water stays in the lake).

Management Implications

This model reveals that equilibrium concentration depends on: (1) Input concentration P_{in}, which can be reduced through pollution controls; (2) Degradation rate k, which might be enhanced through bioremediation; and (3) Residence time τ, which can be decreased by increasing flow rate. For a lake with $V = 10^6\ m^3V$, $Q = 100\ m^3/day$ and $k = 0.1 day^{-1}$, the residence time is

$\tau = 10{,}000$ days, and a 50% reduction in equilibrium concentration requires reducing P_{in} by 50%.

1.5.3 Engineering: Heat Transfer in a Rod

Heat conduction in solid materials is described by partial differential equations, but under certain conditions reduces to ODEs.

Steady-State Heat Conduction

For a thin rod of length L with constant cross-sectional area A, thermal conductivity k, perimeter P, and convection coefficient h, the steady-state temperature distribution $T(x)$ satisfies:

$$\frac{d^2T}{dX^2} - m^2(T - T_a) = 0 \tag{1.29}$$

where $m^2 = hp/(kA)$ and T_a is ambient temperature.

Boundary Conditions and Solution

With fixed end temperatures $T(0) = T_0$ and $T(L) = T_1$, the solution is:

$$\mathrm{T(x) = T_a + C_1 \sinh(mx) + C_2 \cosh(mx)} \qquad (1.30)$$

where constants C_1 and C_2 are determined from boundary conditions.

Practical Application

This model is applicable to designing thermal bridges, electronic component heat sinks, and cooling fins in heat exchangers. The equilibrium between convection to the environment and conduction along the rod is described by the parameter m. Strong convection in comparison to conduction is indicated by a large m, which causes a quick drop in temperature from the base.

1.6 Figures and Tables

The important tables and figures that would go with this chapter in a full textbook are described in this section.

Table 1.1: Common Mathematical Models and Their Applications

Model Type	**Mathematical Form**	**Applications**
Exponential Growth	$\frac{dy}{dt} = ky$	Population growth, compound interest, radioactive decay
Logistic Growth	$\frac{dy}{dt} = ky(1 - y/K)$	Limited population growth, tumor growth, epidemic spread
Harmonic Oscillator	$\frac{d^2y}{dt^2} + \omega^2 y = 0$	Spring-mass systems, pendulums, electrical circuits
Heat Equation	$\frac{\partial u}{\partial t} = \alpha \nabla^2 u$	Heat conduction, diffusion processes, options pricing
Wave Equation	$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$	Sound waves, electromagnetic waves, vibrating strings
Linear Programming	$\min c.X, AX \leq b$	Resource allocation, supply chain optimization, scheduling

Table 1.2: Comparison of Numerical Methods for ODEs

Method	**Order**	**Stability**	**Computational Cost**
Euler's Method	O(h)	Conditionally stable	Low
Runge-Kutta 2	$O(h^2)$	Good	Moderate
Runge-Kutta 4	$O(h^4)$	Excellent	Moderate-High
Adams-Bashforth	Variable	Moderate	Low (multistep)
Backward Euler	O(h)	Unconditionally stable	High (implicit)

Figure Descriptions

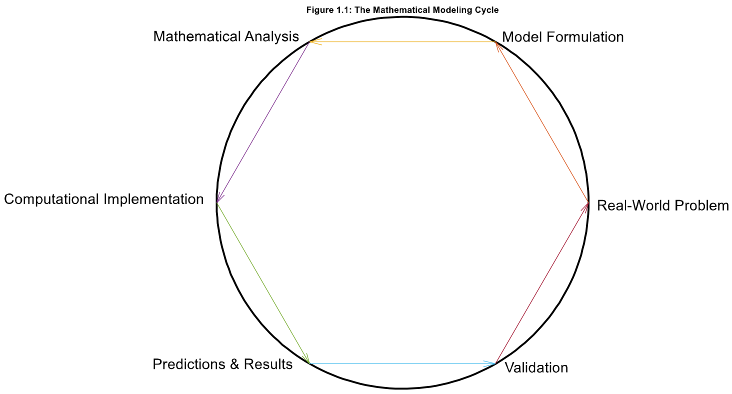

Figure 1.1

The Mathematical Modeling Cycle

A circular figure that demonstrates how mathematical modelling is iterative. Six interconnected stages are depicted in the cycle: (1) real-world problem; (2) model formulation (assumptions and simplifications); (3) mathematical analysis; (4) computational implementation; (5) predictions and results; and (6) validation against data. When models need to be improved, feedback loops from validation back to formulation are shown by arrows.

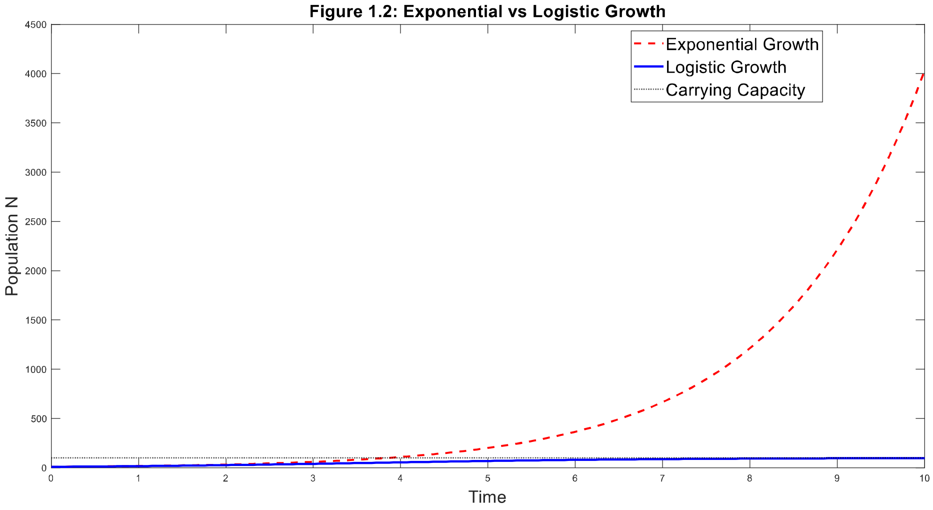

Figure 1.2

Comparison of Exponential and Logistic Growth

A chart representing how the number of people (N) in a certain population changes over time (t) displays two different shapes of the graph: an exponential growth shape $\left(\mathrm{N} = \mathrm{N}_0 \mathrm{e}^{(\mathrm{rt})}\right)$, which indicates that this population will continue to grow without a stopping point, and a logistic growth shape $\left(\mathrm{N} = \frac{\mathrm{K}}{1+\mathrm{Ae}^{(-\mathrm{rt})}}\right)$ that starts with a straight line like an exponential shape but levels out and approaches K as its limit. The logistic growth curve will show one point where the rate of growth is half of $K \left(N = \frac{K}{2}\right)$

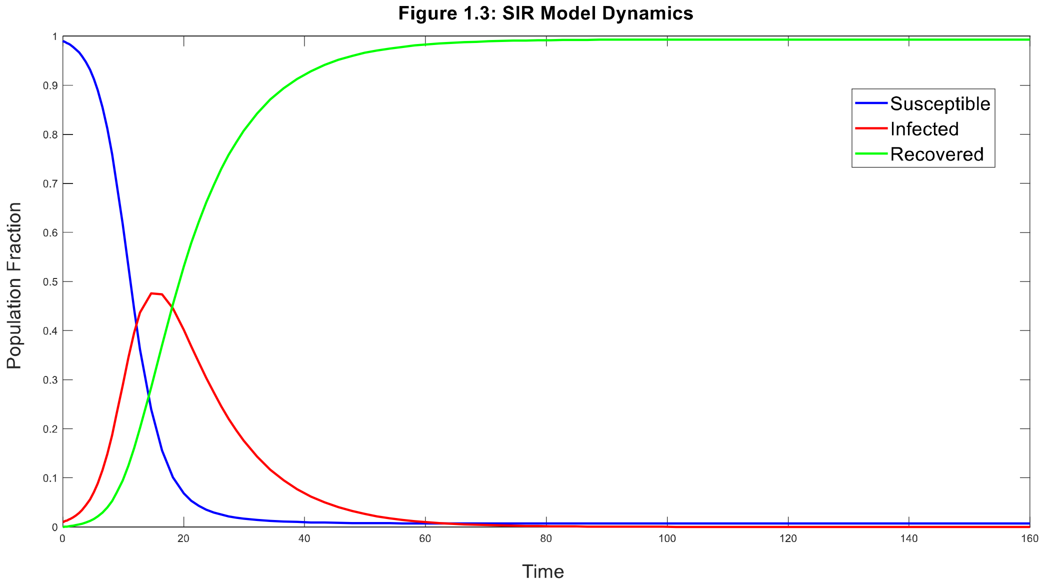

Figure 1.3

SIR Model Dynamics

Time series graphs of S(t), I(t), and R(t) during an epidemic. The susceptible population (S) always declines from its initial level. The infected population (I) increases, then peaks, before it ultimately decreases to zero as the epidemic progresses. The recovered population (R) starts at zero and goes up to its final amount as more people recover from illness. Parameters presented: $\beta = .05$; $\gamma = .10$; $R_0 = 5$; the initial conditions: $S(0) = 0.99$, I $I(0) = 0.01$, $R(0) = 0$.

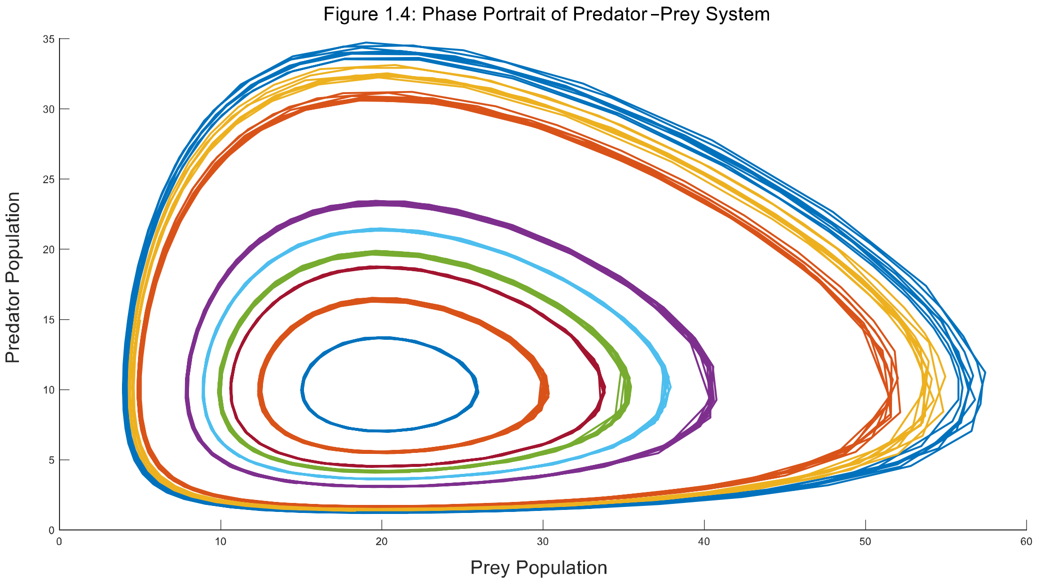

Figure 1.4

Phase Portrait of Predator-Prey System

The Lotka-Volterra model represents the flow of prey (X-axis) against Predator (Y-axis) using a phase plane diagram. The closed orbit around an equilibrium point shows the oscillations occur on a periodic basis. Various trajectories that are shown on this diagram all begin at different points in space but create closed loops around the equilibria indicating that the model is conservative in nature. Directions of how an object moves through a given trajectory are represented by arrows.

1.7 Discussion

1.7.1 Model Validation and Verification

The difference between validation and verification is an important but frequently disregarded part of mathematical modelling. "Are we solving the equations correctly?" is the question of verification. In contrast, validation poses the question, "Are we solving the correct equations?"

Verification includes conducting grid refinement studies, ensuring conservation properties are maintained, and comparing computational implementations to analytical solutions. For example, we confirm that S + I + R = N stays constant during numerical integration in the SIR model.

Validation Model predictions must be compared to independent experimental or observational data in order to be validated. Goodness of fit is measured statistically using metrics like the coefficient of determination (R^2), root mean square error (RMSE), and Akaike information criterion (AIC). However, multiple competing models may fit the same data equally well, so agreement with data does not prove a model is correct.

1.7.2 Limitations and Assumptions

Every mathematical model is predicated on presumptions that restrict its range of applicability. Comprehending these constraints is just as crucial as comprehending the model itself.

Almost all actual populations defy the exponential growth model's assumptions of limitless resources and a steady per capita growth rate. This is improved by the logistic model, which incorporates carrying capacity but ignores age structure, portrays the environment as static, and assumes an instantaneous response to density. Depending on the application, more complex models that include age structure, regional heterogeneity, or stochasticity can be required.

In a similar vein, the SIR model presupposes permanent immunity, homogenous mixing (everyone has an equal chance of coming into contact), and no births or deaths from non-disease-related causes. These presumptions might make sense for short-term epidemic dynamics in small populations. Extensions such as the SEIR model (which adds an Exposed compartment) or metapopulation models may be necessary for long-term endemic diseases in structured populations.

1.7.3 Sensitivity Analysis and Uncertainty Quantification

Rarely are model parameters completely known. In order to determine which parameters have the most impact on predictions, sensitivity analysis looks at how model outputs react to changes in parameters. The sensitivity coefficient with regard to parameter p_i for a model output $\boldsymbol{y = f(p_1, p_2, \ldots., p_n)}$is:

$$\boldsymbol{S_i} = \left(\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{p_i}}\right) \times \left(\boldsymbol{p}_{\frac{i}{y}}\right)(1.31)$$

The percentage change in y per percentage change in p_i is represented by this dimensionless number. Careful measurement and uncertainty quantification are necessary for parameters with

$|\boldsymbol{S_i}| \gg \boldsymbol{1}$ Beyond sensitivity analysis, uncertainty quantification propagates parameter uncertainties across the model to generate probabilistic forecasts as opposed to deterministic ones. Frameworks for quantifying uncertainty include Bayesian inference, polynomial chaos expansions, and Monte Carlo techniques.

1.7.4 Computational Considerations

Computational techniques are central to modern mathematical modeling, where analytical methods provide insight and function as benchmarks, while most realistic models require numerical techniques.

When selecting a numerical method, modelers need to identify an appropriate balance among accuracy, stability and computational cost. Although simple and straightforward to implement, explicit techniques such as Runge-Kutta will often require small time steps for stable solutions. In contrast, implicit approaches such as backward Euler can be computed using any time step and are therefore always stable, but require an iterative solution to nonlinear equations at each time step. In addition, specialized methods such as BDF (backward differentiation formulas) or exponential integrators might be needed for stiff systems (systems containing multiple time scales).

Robust software frameworks such as MATLAB and Python with SciPy, and Julia all offer advanced ODE solvers featuring adaptive time-stepping capabilities and error control that enable the modelers focus primarily on their modelling tasks instead of the technical aspects of implementing them. Nevertheless, modelers need to maintain an understanding of the underlying numerical techniques in order to interpret their results correctly and to diagnose the causes of model failure.

1.7.5 The Art of Simplification

Though all models are flawed in one way or another, they can still serve a purpose according to the famous statistician, George Box. In creating a mathematical model, the first thing you need to consider is how much detail you want to include. You do not want to add so much detail that the model is impossible to work with. On the other hand, if you do not add enough detail, you risk missing out on important information about the phenomenon being modeled.

Adding factors to a model does not necessarily improve the model's ability to produce accurate predictions. For example, if you add too much random variation into the model it could become overly complicated to use. As a result, this complexity could lead to the model underfitting the training set and obscuring the prediction mechanisms. Therefore, if you have multiple possible explanations of the results you have observed, then the most straightforward explanation for those results is usually assumed to be the correct one.

By constructing progressively complicated models, one can produce a hierarchy of models that will allow one to attain an understanding. As new information is derived from these model comparisons, add to these models until a finite amount of new information can be incorporated. Prior to adding a new element to the modeling, it should be based on evidence of improved predictive agreement between the predicted output and actual data and in order to represent phenomena that cannot be adequately represented by the previous two levels of representation.

1.8 Conclusion

This chapter established the core ideas of mathematical modeling, showing how abstract mathematics contains entrée keys to comprehend, predict, and optimize real-world systems. We discussed the modeling cycle; we considered various classes of models; and we developed three central mathematical formulations: the ordinary differential equation, the system of equations, and the optimization problem.

We have seen through worked examples (from radioactive decay to chemical kinetics) how mathematical formulations for general principles can yield specific mathematical formulations to use as a basis for numerical results. Real-world applications in epidemiology, environmental science and engineering demonstrate the breadth of applicability and importance of mathematical modelling across many industries.

Our conversation reveals a number of important themes:

1.**Iterative modeling:** Models evolve through multi-step processes involving creation, examination, and confirmation.

2.**Presumptions count:** All models include axial presumptions used to achieve certain bounds of usefulness

3.**Model verification:** Establishing concordance between model predictions and actual measurements instills belief, but does not authenticate the model

4.**Symmetric representation:** The right mix of theory and explanation is the most beneficial model usage

5.**Computation empowers exploration:** The ability to compute provides tools for modelling real-world behaviours in ways that cannot be captured using mathematics or graphical representations alone. By mastering how to convert verbal information into mathematical expressions, develop insights beyond simple answers through the construction of systems of equations, apply numerical techniques to solve equations, and critically evaluate the assumptions made when building a mathematical model, one acquires a core set of skills that is essential for applied mathematicians, engineers, and scientists. By mastering mathematical modeling, you are able to develop and understand complex systems in virtually all fields of human endeavour.

References

[1] Edelstein-Keshet, L. (2005). *Mathematical Models in Biology*. SIAM, Philadelphia, PA.

[2] Murray, J.D. (2002). *Mathematical Biology I: An Introduction*, 3rd Edition. Springer, New York.

[3] Strogatz, S.H. (2015). *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*, 2nd Edition. Westview Press, Boulder, CO.